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論 文

적산성 잡음에서의 약한 확률적 신호 검파기의 검정통계량

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Test Statistics of a Detection Scheme for Weak Random Signals in Multiplicative Noise

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要 約 최근에 제안된 적산성 잡음을 포함하는 일반화된 관측 모델에서의 약한 확률적 신호검파를 다루었다. 적산성 잡음이 있을 경우, 확률적 신호를 검파하기 위한 국소최각 검파기의 검정통계량은 순수 가산성 잡음만 있을 경우의 국소최적검파기의 검정통계량이 확장된 것임을 보였다. 이는 이미 발표된 약한 알려진 신호 검파의 경우와 비슷한 결과이다. 널리 쓰이는 두 확률밀도 함수에 대해, 검정통계량을 구성하는 국소최적 비선형성들의 형태를 예시해 보였다.

ABSTRACT The problem of detecting weak random signals is addressed in a generalized observation model incorporating multiplicative noise which has recently been introduced. It is shown that the locally optimum random-signal detectors in the multiplicative-noise model are interesting generalizations of those which would be obtained in the purely-additive noise model. Examples of explicits results for the locally optimum detector test statistics are given for two typical cases of well-known pdfs.

1. INTRODUCTION

Detection of *random signals* is of interest in various situations. For example, in underwater applications it is often impossible to represent desired signals by known or parametric signal models because of random dispersion resulting from turbulence or inhomogeneities in the medium, and also because of the very nature of the signal source.

For the most part problems of random-signal detection have been idealized as being those of detecting (random) signals in *purely-additive noise*. The use of a purely additive noise model,

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which is usually an approximation of the physical mechanism producing noisy observations, may cause a considerable performance degradation in detection applications where the observations have significant non-additive noise components such as signal-dependent and multiplicative noise [1-6]. Explicit use of non-additive noise models is necessary in such situations. For example, additive signal and noise components may produce multiplicative or signal-dependent noise due to nonlinearities in the observation system.

A generalized observation model for noisy signals has recently been proposed in[7], in which the effects of multiplicative or signal-dependent noise can be represented. This model was used in studying locally optimum detection of *known signals* in[6]. In this paper we will mainly be concerned with finding test statistics of detection schemes for weak random signals in observations governed by the generalized noisy signal model introduced in[7]. More specifically, we will find the test statistics of locally optimum (LO) detectors, which are optimum in detecting signals in the case of weak signals ($SNR \to 0$), for the problem of random-signal detection in the generalized observation model. It should be noted that although much effort has been devoted to the problem of locally optimum detection of (known or random) signals, most of the results from the effort are based on the purely-additive noise model.

A special case of the generalized observation model to be used in this paper will be reviewed in Section 2. In Section 3 we will find the test statistics of locally optimum detectors for random-signal detection based on this observation model. Examples of the locally optimum nonlinearities constituting the test statistics of the locally optimum detectors for two specific pdfs will be considered in Section 4, which will be followed by a summary in Section 5.

2. THE MULTIPLICATIVE-NOISE MODEL

For an array of L receivers the multiplicative-noise model describes the discrete-time observation X_{ji} at time instant i for receiver j by

$$X_{ji} = \theta S_i + \theta S_i N_{ji} + W_{ji}, \qquad (1)$$

where $i=1,2,\cdots,n$ and $j=1,2,\cdots,L$. Here n is the size of the sample (number of observations) collected at each input channel; θ is the signal amplitude parameter, which may be zero (null-hypothesis) or positive (alternative hypothesis); W_{ji} is the purely-additive noise component in the j-th channel at the i-th sampling instant; S_i is a zero-mean random signal component with amplitude θ at each channel at the i-th sampling instant, which is assumed to have variance σ_i^2, $i=1,2,\cdots,n$. The joint probability density function (pdf) of $(S_1, S_2, \cdots, S_n)$ will be denoted by $f_S(s)$ and we will denote by $r_S(i,k)$ the covariance between S_i and S_k; that is, $r_S(i,k) = E\{S_i S_k\}$. The purely-additive noise components W_{ji} will be taken to be independent and identically distributed (i.i.d.) random variables with common pdf f_W and means zero and variances σ_W^2, with the signal and noise being statistically independent. The N_{ji} are i.i.d. random variables independent of the S_i but with N_{ji} and W_{ji} generally not independent. The random-signal term θS_i multiplies N_{ji} to produce an additional *multiplicative* noise term generally correlated with the purely-additive noise component W_{ji}. Let the variance of the N_{ji} be σ_N^2 and let f_N be the common pdf of the N_{ji}. We will denote by f_{NW} the common joint pdf of the (N_{ji}, W_{ji}), which are i.i.d. bivariate random variables for $i=1,2,\cdots,n$ and $j=1,2,\cdots,L$. Clearly, $\sigma_N^2=0$ yields the usual zero-mean random signal and purely-additive noise model. In this paper, we will focus on the random signal detection problem in the above multiplicative-noice model for noisy signal.

We can now formulate our detection problem as a statistical hypothesis testing problem of choosing either a null hypothesis H or an alternative hypothesis K describing the joint pdf $f(x|\theta)$ of the observation matrix $X=\{X_{ji}\}$, where x is a realization of X. Under H we have $\theta=0$ and under K we have $\theta>0$, with

$$f(x|\theta)=\int f_S(s)\prod_{i=1}^{n}\prod_{j=1}^{L}\int f_{NW}(n_{ji},\ x_{ji}-\theta s_i-\theta s_i n_{ji})\,dn_{ji}\,ds \quad (2)$$

Only $\theta>0$ is assumed to be unknown under the alternative hypothesis; $\theta=0$ in (2) yields the null hypothesis pdf.

Assumptions

In computing the test statistics of the locally optimum detectors in Section 3, the following quantities are assumed to exist and to be finite:

$$g_1(x)=-\frac{f_W'(x)}{f_W(x)}, \quad (3)$$

$$g_2(x)=-\frac{u'(x)}{f_W(x)}, \quad (4)$$

$$h_1(x)=\frac{f_W''(x)}{f_W(x)}, \quad (5)$$

$$h_2(x)=\frac{u''(x)}{f_W(x)} \quad (6)$$

and

$$h_3(x)=\frac{v''(x)}{f_W(x)}, \quad (7)$$

where

$$u(x)=\int n f_{NW}(n,x)\,dn = f_W(x)\,E\{N|W=x\} \quad (8)$$

and

$$v(x)=\int n^2 f_{NW}(n,x)\,dn = f_W(x)\,E\{N^2|W=x\} \quad (9)$$

The functions $g_2(x)$, $h_2(x)$ and $h_3(x)$ are generalizations of the usual locally optimum nonlinearities $g_1(x)$ and $h_1(x)$, and arise from the dependence of the two noise processes N and W through the weighted conditional average $u(x)$ and the weighted conditional variance $v(x)$. It should be noted that $E\{N|W\}\equiv 0$ implies that N and W are uncorrelated but the converse is not necessarily true; for example, $N=W^{2m}+Z$ with an m positive integer, Z and W independent and with symmetric pdf for W we have a counterexample against the converse. If, however, N and W are independent, $E\{N|W\}\equiv 0$. It should also be noted that $g_2(x)\equiv 0$ if and only if $E\{N|W=x\}\equiv 0$. We will also be assuming in the following development that the pdfs f_N, f_W and f_N are smooth enough and satisfy regularity conditions justifying mathematical operations such as interchanges in the order of differentiation with respect to a parameter and integral of a function.

3. TEST STATISTICS FOR RANDOM-SIGNAL DETECTION IN MULTIPLICATIVE NOISE

In this section we will find the locally optimum detector test statistics in the multiplicative-noise model (1) so that a test function for detecting random signals can be established. Detailed discussions on the general properties and concepts of locally optimum (LO) detectors can be found in a number of other available work (e.g. 8).

Now from the generalized Neyman-Pearson lemma[9,10] the locally optimum detector test statistic for $\theta=0$ versus $\theta>0$ is obtained as the ratio

$$T_{LO}(x)=\frac{\dfrac{d^{\nu} f(x\mid 0)}{d\theta^{\nu}}}{f(x\mid 0)} \tag{10}$$

In (10) ν is the first non-zero derivative of $f(x|\theta)$ at $\theta=0$; that is ν is defined by

$$\frac{d^{i} P_{d}(0\mid D)}{d\theta^{i}}=0,\quad i=1,2,\cdots,\nu-1, \tag{11}$$

for all D in D_{α} and

$$\frac{d^{\nu} P_{d}(0\mid D_{LO})}{d\theta^{\nu}}>0, \tag{12}$$

where D_{α} is the class of all detectors of size α for H versus K, $P_{d}(\theta|D)$ is the power function of a detector D for signal strength θ and D_{LO} is a locally optimum detector in D_{α}.

For random-signal detection in the purely-additive noise model, it can easily be shown that $\nu=2$ and that the locally optimum detector test statistic is[11].

$$T_{LO+}(X)=\sum_{i=1}^{n}\sum_{j=1}^{L}\sum_{k=1}^{n}\sum_{l=1}^{L} r_{S}(i,k)\,g_{1}(X_{ji})g_{1}(X_{lk}) +\sum_{i=1}^{n}\sum_{j=1}^{L}\sigma_{i}^{2}\{h_{1}(X_{ji})-g_{1}^{2}(X_{ji})\} \tag{13}$$

The detector whose test statistic is (13) will be called the LO+ (locally optimum detector in the purely-additive noise model). In the single-channel case $L=1$ in (13) gives the usual locally optimum detector test statistic.

We now consider locally optimum detection of random signals in multiplicative noise in addition to the purely-additive noise. It can be shown as in Appendix Ⅲ. 1 of [7] that $\nu=2$ and that (10) yields for the locally optimum detector test statistic in multiplicative noise the result

$$T_{LO}(X)=\sum_{i=1}^{n}\sum_{j=1}^{L}\sum_{k=1}^{n}\sum_{l=1}^{L} r_{S}(i,k)[g_{1}(X_{ji})+g_{2}(X_{ji})]\cdot[g_{1}(X_{lk})+g_{2}(X_{lk})] +\sum_{i=1}^{n}\sum_{j=1}^{L}\sigma_{i}^{2}[h_{1}(X_{ji}) +2h_{2}(X_{ji})+h_{3}(X_{ji})-\{g_{1}(X_{ji})+g_{2}(X_{ji})\}^{2}], \tag{14}$$

where $g_{1}, g_{2}, h_{1}, h_{2}$ and h_{3} are defined in (3)-(7) of Section 2 and $r_{S}(i,k)$ is the covariance function of S_{i} and S_{k} with $\sigma_{i}^{2}=r_{S}(i,i)$ being the variance of S_{i}. In (14) the terms g_{2}, h_{2} and h_{3} represent the effects of the multiplicative noise, and of the dependence between the two noise processes N and W on the test statistic. It should be noticed that even when $E\{N\}=0$, $g_{2}(x), h_{2}(x)$ and $h_{3}(x)$ do not vanish if N and W are correlated.

From the expression (14) for the locally optimum detector test statistic in multiplicative noise, the following general observations can be made:

(a) The locally optimum detector test statistic of (14) is of the same form as (13) for the LO+ detector, and would be obtained in the purely-additive noise model with $g_{1}(x)$ replaced by $g_{1}(x)+g_{2}(x)$ and $h_{1}(x)$ replaced by $h_{1}(x)+2h_{2}(x)+h_{3}(x)$. A similar observation has been made for the known signal detection problem in [6].

(b) The test statistic depends on the random signal through the covariance function $r_S(i,k)$ and the variances σ_i^2 but does not depend on the exact functional form of the pdf of random signal. This is a consequence of the assumption that the random signal is weak.

(c) Since the function $h_3(x)$ will not generally vanish identically, it will generally always exist in the test statistic. This implies that the multiplicative noise term will in most case have an effect on the locally optimum detector test statistic; it has an effect on the test statistic through g_2, h_2 and h_3 when $E\{N|W\}$ is not identically zero and through $h_3(x)$ even when $E\{N|W\}\equiv 0$.

(d) In general, the test statistics (13) and (14) are clearly not the same; they do, however, become equivalent as $L\to\infty$ if $E\{N|W\}\equiv 0$, because the quadruple summations in (13) and (14) dominate over the double summations. This implies that the detectors based on the test statistics (13) and (14) will have similar performance characteristics when L is large and $E\{N|W\}\equiv 0$.

Table 1 shows which of the functions g_1, g_2, h_1, h_2 and h_3 remain in the test statistics under different conditions.

Table 1. Locally Optimum Nonlinearities Contained in the Test Statistics for the Multiplicative-Noise Model.

	under quadruple summation	under double summation
$E\{N\|W\}\neq 0$	g_1, g_2	h_1, h_2, h_3
$E\{N\|W\}\equiv 0$	g_1	h_1, h_3

4. EXAMPLES OF TEST STATISTICS

We will now consider two specific examples to illustrate some of the above results. First, let us take a bivariate Gaussian pdf $\eta(0,0,s,1,r)$ for f_{NW} with $\eta(m_1, m_2, s_1, s_2, r)$ denoting a bivariate Gaussian pdf for which $E\{N\}=m_1$, $E\{W\}=m_2$, $\sigma_N^2=s_1^2$, $\sigma_W^2=s_2^2$ and r is the correlation coefficient between N and W In this case we can easily obtain

$$g_1(x)=x, \tag{20}$$

$$g_2(x)=rs(x^2-1), \tag{21}$$

$$h_1(x)=x^2-1, \tag{22}$$

$$h_2(x)=rs(x^3-3x), \tag{23}$$

and

$$h_3(x)=s^2\{r^2x^4+(1-6r^2)x^2+3r^2-1\}. \tag{24}$$

Note that for bivariate Gaussian pdf f_{NW}, $E\{N|W\}\equiv 0$ if and only if N and W are uncorrelated, and $h_3(x)$ with $r=0$ has the same functional form as $g_2(x)$ with $r\neq 0$. The locally optimum detector test statistics can now be found from (14) using (30)-(24); that is, we get explicity

$$T_{LO}(X)=\sum_{i=1}^{n}\sum_{j=1}^{L}\sum_{k=1}^{n}\sum_{l=1}^{L} r_S(i,k)\{X_{ji}+rs(X_{ji}^2-1)\}\cdot\{X_{lk}+rs(X_{lk}^2-1)\}+\sum_{i=1}^{n}\sum_{j=1}^{L}\sigma_i^2[r^2s^2-8rsX_{ji}+s^2(1-8r^2)X_{ji}^2] \tag{25}$$

as the locally optimum detector test statistic. The terms containing s in (25) represent the effects of the multiplicative noise on the test statistics.

As the second example, let us consider a vibariate t-distribution $t(m_1, m_2, s_1, s_2, r, k)$ whose joint pdf is given by (12)

$$f_{NW}(x,y)=\frac{1}{2\pi s_1 s_2 (1-r^2)^{1/2}}$$
$$[1+\frac{1}{k(1-r^2)}\{\frac{(x-m_1)^2}{s_1^2}-\frac{2rxy}{s_1 s_2}+\frac{(y-m_2)^2}{s_2^2}\}]^{-\frac{k+2}{2}}. \quad (26)$$

Here $E\{N\}=m_1$, $E\{W\}=m_2$, $\sigma_N^2=\frac{ks_1^2}{k-2}$, $\sigma_W^2=\frac{ks_2^2}{k-2}$, r is the correlation coefficient between N and W, k is a parameter that determines the algebraic rate of decay of the pdf and the variances σ_N^2 and σ_W^2 are defined only for $k>2$; if $k=2$ (26) represents a bivariate Cauchy pdf and for $k\to\infty$ (26) tends to a bivariate Gaussian pdf. If we take the pdf corresponding to $(0,0,s,1,r,k)$ for for which $E\{N\}=E\{W\}=0$, $\sigma_N^2=\frac{ks^2}{k-2}$, and $\sigma_W^2=\frac{k}{k-2}$, we obtain

$$g_1(x)=\frac{(k+1)x}{k+x^2}, \quad (28)$$

$$g_2(x)=\frac{rsk(x^2-1)}{k+x^2}, \quad (29)$$

$$h_1(x)=\frac{(k+1)\{(k+2)x^2-k\}}{(k+x^2)^2}, \quad (30)$$

$$h_2(x)=\frac{rsk(k+1)(x^3-3x)}{(k+x^2)^2} \quad (31)$$

and

$$h_3(x)=\frac{ks^2[\{1+(k-2)r^2\}x^4+\{(1-6r^2)k-1\}x^2+k(3r^2-1)]}{(k+x^2)^2} \quad (32)$$

For the bivariate t-distribution also $g_2(x)\equiv 0$ if and only if $r=0$ and $h_3(x)$ with $r=0$ has the same functional expression as $g_2(x)$ with $r\neq 0$. Using (28)-(32) we can also obtain the locally optimum detector test statistics for bivariate t-distribution.

For the bivariate t-distribution function also the locally optimum detector test statistics have additional terms containing the factors which represent the effects of the multiplicative noise. We also observe that the results for the vibariate t-distribution become those for the Gaussian case for $k\to\infty$. A few important quantities for bivariate Gaussian and bivariate t-distribution functions are shown in Table 2

5. SUMMARY

In this paper we employed the generalized observation model introduced in a recent paper and addressed the problem of locally optimum

Table 2. Values of Key Quantities for Two Typical Bivariate Probability Density Functions.

	$\eta(0,0,s,1,r)$	$t(0,0,s,1,r,k)$
σ_N^2	s^2	$\frac{ks^2}{k-2}$, $k>2$
σ_W^2	1	$\frac{k}{k-2}$, $k>2$
$E\{N\mid W=x\}$	rsx	rsx
$E\{N^2\mid W=x\}$	$(rsx)^2+s^2(1-r^2)$	$\frac{s^2\{(1+(k-2)r^2)x^2+k(1-r^2)\}}{k-1}$, $k>1$
$E\{NW\}$	rs	$\frac{rsk}{k-2}$, $k>2$

detection of random signals in multiplicative noise. Morea specifically, the test statistics of the locally optimum detectors which were derived for the multiplicative-noise model for detecting random signals were shown to be generalizations of those for the purely-additive noise model. For situatio=ns in which the strength of the multiplicative noise term is different from that of the signal term, the locally optimum detectors can be obtained after derivations similar to those given for signal-dependent noise situations in[6].

Study of adaptive and robust detection schemes for good overall performance when exact specifications of parameter values and characterization of pdfs cannot be determined *a priori* would be in interesting investigation to perform in extending these results.

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