論 文

Differential Demodulation of GMSK in Rayleigh fading Channels

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ABSTRACT A new demodulation scheme for differentially encoded GMSK signal is introduced and described. In the proposed differential detection method, the signal is sampled at the center of the time slot, at which the effect of the intersymbol interference is relatively smaller than the edge of the time slot. The proposed differential detector takes this advantage. The error rate performance of the differential detector has been numerically calculated in the fast fading encountered in the land mobile radio channels. A comparison of performance with the differential detector for MSK signal is given. Finally the possibility of improving performance employing nonredundant error correction is studied.

Ⅰ.INTRODUCTION

In mobile radio communcations, modulation schemes that have constant envelope property to reduce the effect of fading and nonlinearity, and a compact power spectrum to reduce adjacent channel interference are preferred. Premodulation Gaussian filtered minimum shift keying (GMSK) has been proposed as a spectrum-efficient digital modulation for mobile radio[1]. The output power spectrum of MSK can be made compact by introducing a premodulation baseband low-pass filter before modulation. Premodulation Gaussian filtering suppresses the out-of-band spurious power for GMSK, while keeping constant envelope.

Extensive studies have been carried out to improve system performance. One method takes the advantage of the fact that the degradation effect due to the premodulation Gaussian filtering is smaller at the center of time slot than at the edge of the time slot[2]. In fact, the effect of the intersymbol interference due to the premodulation filtering is largest at the edge of time slot. Usually this point has been adopted as a sample timing in conventional coherent detection of GMSK, simply because it is the point of the maximum eye opening.

In this paper, we will take this advantage and use the center of time slot as a sampling instant in differential detection for differentially encoded GMSK signal. The error rate performance of the proposed differential detector in the fading channel has been numerically calculated. The approach used in this paper is similar to the one that calculated the error rate performance of duobinary coded MSK and TFM in(3). The possibility of improving performance for the proposed differential detector employing nonredundant error correction is investigated.

This paper begins in the next section with brief overview of GMSK signal followed in section Ⅲ by a consideration of differential detection for differentially encoded GMSK signal. In section Ⅳ, we calculate the error rate performance in the fading channel. The probability of error is obtained by numerical integration for slow and fast fading. Numerical

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resultsare presented in section V with concluding discussion.

II. GMSK SIGNAL

GMSK modulation that has a constant envelope and narrow band can be achieved by placing premodulation Gaussian baseband low pass filter before MSK modulator. The impulse response of the baseband Gaussian filter is given by

$$h(t)=A\sqrt{\frac{2\pi}{\ln 2}}\,B\exp\left\{-\frac{2(\pi B t)^2}{\ln 2}\right\} \quad (1)$$

where B is the 3dB bandwidth and A is constant. It's inverse Fourier transform of h (t) is given by

$$H(f)=A\exp\left\{-\left[\frac{f}{B}\right]^2\frac{\ln 2}{2}\right\} \quad (2)$$

Gaussian filtered input to the modulator can be calculated as

$$g(t)=\int h(\tau)\,a(t-\tau)\,d\tau \quad (3)$$

where a(t) is a unit rectangular pulse. The GMSK waveform of the modulator output can be expressed as

$$s(t)=\cos(2\pi f_c t+\phi(t)) \quad (4)$$

where

$$\phi(t)=2\pi f_d\int\sum g(\nu-nT)\,d\nu,\quad a_n=\pm 1 \quad (5)$$

f_c is the carrier frequency and f_d is the phase deviation constant. In the case of GMSK modulation index is 0.5 and T is the bit duration. If the excess phase of an MSK signal is plotted versus time, it follows that it is piecewise linear and increase or decrease exactly $\pi/2$ radian each bit duration. Lines with positive slope represent 1 and lines with negative slope represent 0. A premodulation Gaussian filtering carried out in transmitter would result in degradation of system performance. The digital binary data filtered by a Gaussian low pass filter before modulation experiences severe intersymbol interference (ISI). This leads to degradtion in the bit error rate.

As a specific example, let the input message sequence be 0111001··· represented by m_0, m_1, m_2, ···, and differentially encoded sequence be 1101000l··· repressented by a_0, a_1, a_2, ···. The resulting phase path ϕ_k is shown in Figure 1. Intersymbol interference from preceding and following symbols is largest at the sampling time mT(m=0, 1, 2, ···). On the other hand, the effect of ISI is relatively small at the center of time slot, which is denoted by black dot. We will take this advantage in detecting GMSK signal. Figure 2 shows the phase transition of the worst case i. e., ···, 0, 0, 0, 1, 1, 1, ···, which causes the maximum effect of intersymbol interference.

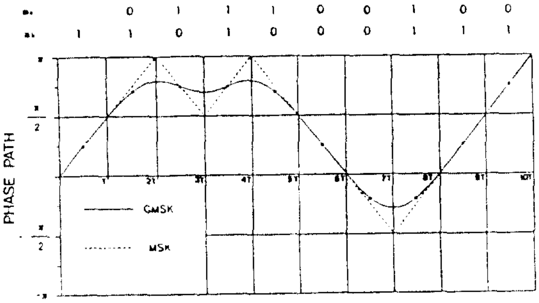


Fig. 1. Phase path in GMSK.

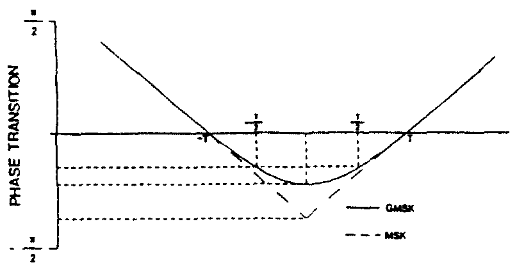


Fig. 2. Phase transition of the worst case.

Ⅲ. SYSTEM CONFIGURATION

The GMSK modulator with differential encoding is shown in Figure 3. Gaussian baseband filter suppreses well out-of-band spurious power for GMSK, while still keeping the constant envelope property. Figure 4 shows the differential detector with sampling instant (m+0.5)T. this sample timing is less sensitive to the effect of the intersymbol interference resulting from the Gaussian filter.

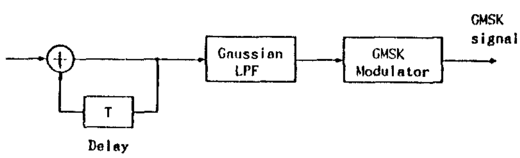


Fig. 3. GMSK modulator with differential encoding.

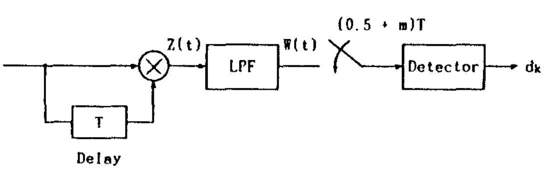


Fig. 4. GMSK demodulator with differential encoding sampled at t=(m+0.5)T.

In the absence of noise and fading, input to the low-pass filter is

$$\begin{aligned} Z(t) &= \cos[\omega_c t+\phi(t)]\cos[\omega_c(t-T)+\phi(t-T)] \\ &= \tfrac{1}{2}\cos[\omega_c T+\phi(t)-\phi(t-T)] \\ &\quad +\tfrac{1}{2}\cos[2\omega_c(t-T/2)+\phi(t)+\phi(t-T)] \end{aligned} \tag{6}$$

The double frequency terms are rejected by the low-pass filter leaving only the leading term.

$$W(t)=\tfrac{1}{2}\cos[\phi(t)-\phi(t-T)] \tag{7}$$

where $\omega_c T$=multiple of 2π, and the sampled output at the time instant (m+0.5)T is

$$W((m+0.5)T)=\tfrac{1}{2}\cos[\phi((m-0.5)T)-\phi((m+0.5)T)] \tag{8}$$

The output of differential detector for differentially encoded MSK signal at the sampling instant (m+0.5)T is

$$\begin{aligned} W((m+0.5)T)&=0 \quad \text{when } m_k=0 \\ W((m+0.5)T)&=\tfrac{1}{2} \quad \text{when } m_k=1 \end{aligned} \tag{9}$$

The decision threshold becomes 0.25. This decision threshold will be used later to calculate the probability of error for MSK signaling in the presence of fading and noise. If MSK signal shown in figure 1 is demodulated, phase of MSK signal, phase difference, W((m+0.5)T), and output of detector are resulted as shown in Table 1.

Let $\varphi_E(mT)$ be the phase difference between MSK signal and GMSK signal for the worst signal pattern at the sampling instant mT and $\varphi_c((m+0.5)T)$ be the phase difference between MSK signal and GMSK signal for the worst signal pattern at the sampling instant (m+0.5)T. $\varphi_E(mT)$ and $\varphi_c((m+0.5)T)$ can be calculated with parameter BT as shown in

Table 1. Possible values of MSK signal in Figure 1.

Phase $\phi(t)$	$\Delta\phi(t)$	W((m+0.5)T)	Output d_k
45°			
135°	90	0	0
135°	0	1/2	1
135	0	1/2	1
135°	0	1/2	1
45°	90	0	0
45	90°	0	0
45	0	1/2	1
45	90°	0	0
135	90	0	0

Figure 5. Without loss of generality, we assumed that $\phi(mT)$ and $\phi((m+0.5)T)$ are $\pi/2$ and $\pi/4$ respectively for the case of MSK.

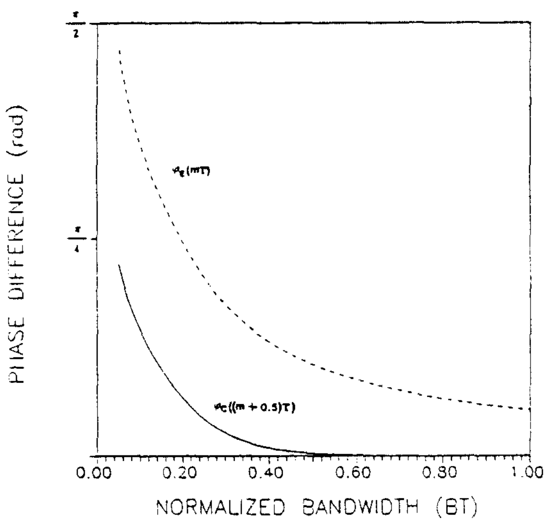
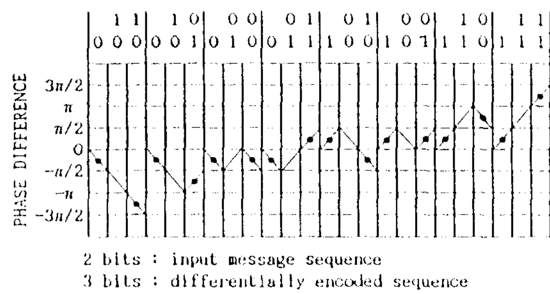


Fig. 5. Phase difference compared with MSK versus BT.

It was shown in(4) that MSK signal have the inherent properties that the outputs of a conventional differential detector and a differential detector with two time slot delay circuit sampled at mT correspond to the data and parity, respectively, of a single error correcting self orthogonal convolutional code. We will show that the proposed demodulation scheme also has this inherent nonredundant error correcting property. Figure 6 shows the relationship between the input message sequence and output of the 2 bit differential detector sampled at $t=(m+0.5)T$ denoted by black dot for the differentially encoded sequence. It is clear that when the output of 2 bit differential detector is $\cos(\pm\pi)=-1$, input message 11 is sent. When the output of the 2 bit differential detector is $\cos 0=1$, in input message sequence 00 is sent. Likewise when the output of the differential detector is $\cos(\pm\pi/2)=0$, input message sequence is 01 or 10.

We can obtain the parity of a single error correcting self orthogonal convolutional code from the output of 2 bit differential detector passed through the absolute value device and inverter (0→1, 1→0).



(a)

Output of differential detector	Input bits
$\cos(\pm\pi)=-1$	1 1
$\cos(\pm\pi/2)=+1$	0 0
$\cos(0)=0$	0 1, 1 0

(b)

Fig. 6. Relationship between the input message sequence and the output of the 2 bit differential detector sampled at $t=(m+0.5)T$. (a) Possible value of phase difference, (b) Output of 2 bit differential detector versus input bits.

Ⅳ. ERROR RATE PERFORMANCE

The received signal at the receiver input through the fast fading radio channel, s(t) is given by

$$s(t)=x_s(t)\cos[\omega_c t+\phi(t)] - y_s(t)\sin[\omega_c t+\phi(t)] \quad (10)$$

where $x_s(t)$ and $y_s(t)$ are independent zero mean Gaussian low pass processes. With the assumption of addtive Gaussian noise at the receiver input and using Gaussian bandpass filtering before differential detection, the total signal plus noise to the differential detector is

$$e(t)=s(t)+n(t) \quad (11)$$

where

$$n(t)=x_n(t)\cos\omega_c t-y_n(t)\sin\omega_c t \quad (12)$$

and $x_n(t)$, $y_n(t)$ are independent zero mean Gaussian lowpass processes of Gaussian spectrum. Then it can be shown that the output of the sampler at the time $t=(m+0.5)T$ is given by[5]

$$W((m+0.5)T)=\frac{1}{2}[R_1^2-R_2^2] \quad (13)$$

where R_1 and R_2 are jointly Rayleigh distributed [6][7].

$$P(R_1, R_2)=\frac{4R_1R_2}{\sigma^4(1-|\rho|^2)}\exp\left\{-\frac{(1-\rho_I)R_1^2+(1+\rho_I)R_2^2}{\sigma^2(1-|\rho|^2)}\right\} \cdot I_0\left[\frac{2\rho_r R_1R_2}{\sigma^2(1-|\rho|^2)}\right], \quad R_1, R_2 > 0 \quad (14)$$

$I_0(\cdot)$ is the modified Bessel function of the first kind and zeroth order defined by

$$I_0(\nu)=\frac{1}{2\pi}\int_0^{2\pi}\exp(\nu\cos\varphi)\,d\varphi \quad (15)$$

and

$$\rho=\rho_r+j\rho_I \quad (16)$$

$$\sigma^2\rho=\sigma_s^2\rho_s\exp(j(\theta_1-\theta_2))+\sigma_n^2\rho_n \quad (17)$$

where $\theta_1-\theta_2$ is the change in the desired signal phase, σ_s^2 and σ_n^2 are average signal and noise powers respectively, and ρ_s and ρ_n are corresponding normalized autocorrelation functions, and

$$\sigma^2=\sigma_s^2+\sigma_n^2 \quad (18)$$

$$\sigma_s^2\rho_s=\int_{-\infty}^{\infty}W_s(f)\exp(j2\pi fT)\,df \quad (19)$$

$$\sigma_n^2\rho_n=\int_{-\infty}^{\infty}W_n(f)\exp(j2\pi fT)\,df \quad (20)$$

where $W_s(f)$ and $W_n(f)$ are the equivalent baseband power spectra of s(t) and n(t) respectively. In order to analyze the error probability, let us assume that $W_s(f)$ and $W_n(f)$ in (19) and (20) are respectively given by

$$W_s(f)=\begin{cases}\dfrac{\sigma_s^2}{\pi\sqrt{(f_D^2-f^2)}}, & |f|\leq f_D\\ 0, & |f|>f_D\end{cases} \quad (21)$$

$$W_n(f)=\begin{cases}\dfrac{\sigma_n^2}{B\sqrt{\dfrac{\pi}{\ln 2}}}\exp\{-[\dfrac{f}{B}]^2\ln 2\}, & |f|\leq f_D\\ 0, & |f|>f_D\end{cases} \quad (22)$$

where f_D is the maximum Doppler frequency. We assumed that 3dB bandwidth of IF filter and premodulation baseband filter are same. ρ_s and ρ_n can then be obtained as

$$\rho_s = J_0(2\pi f_D T) \quad (23)$$

$$\rho_n = \exp\left\{-\frac{(\pi BT)^2}{\ln 2}\right\} \quad (24)$$

where $J_0(\cdot)$ is the zeroth order bessel function of the first kind. ρ is reduced to

$$\rho = \frac{\Gamma}{\Gamma+1}\exp\left\{-\frac{(\pi BT)^2}{\ln 2}\right\} \quad (25)$$

$$\Gamma = \frac{\sigma_s^2}{\sigma_n^2}$$

Assuming a priori probability of mark transmission in s(t) is denoted by p, the probability of error is given by

$$P_e = p \cdot Pr\{R_1^2 - R_2^2 < a^2 | mark\} + (1-p) \cdot Pr\{R_1^2 - R_2^2 \geq a^2 | space\}$$
$$= p\int P(z|mark)\,dz + (1-p)\int P(z|space)\,dz$$
$$= \frac{1}{2}\int\int P(R_1, R_2)\,dR_1\,dR_2 + \frac{1}{2}\int\int P(R_1, R_2)\,dR_1\,dR_2 \quad (26)$$

where $z = R_1^2 - R_2^2$ and a^2 is the value of threshold.

The integral in (26) is numerically calculated by integrating the density function $P(R_1, R_2)$ over appropriat integration region in Figure 7. Assuming a priori probabilities of mark and space to be equally likely, using the numerical method, we calculated the error probability for the worst signal pattern which causes the maximum ISI effect[8]. The value of threshold a^2 changes as the parameter BT changes. However the change of a^2 is relatively small, since the sampling time of (m+0.5)T is less sensitive to the effect of the intersymbol interference. The threshold value becomes 0.5 for MSK signaling. Calculated results are shown in figure 8, 9, 10 for BT=0.05, 0.25 and 0.5 respectively. MSK results are also shown in Figure 11 for comparison. Fading rate $f_D T$ is taken as a parameter for all plotting with values ranging from 0 to 0.005 corresponding to the vehicle speed ranging from 0 to 100 km/h at a bit rate of 16 kbps in 900 MHz band.

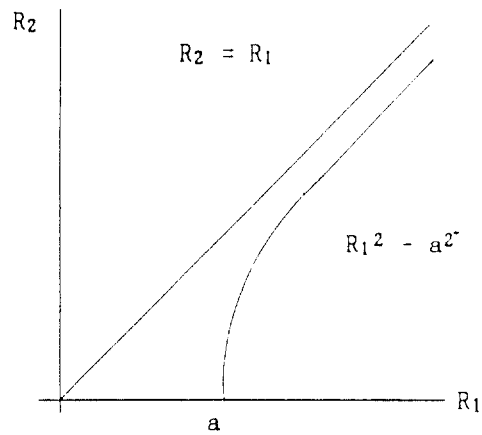


Fig. 7. Integration Region.

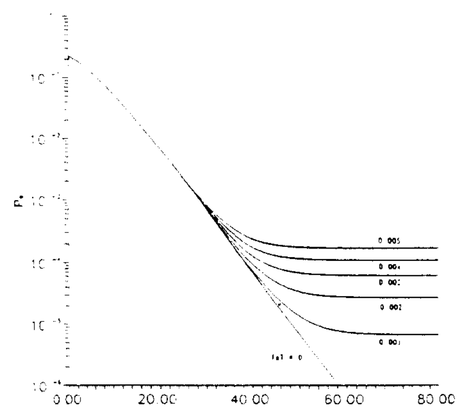


Fig. 8. Error rate performance of GMSK with differential detection with BT=0.05.

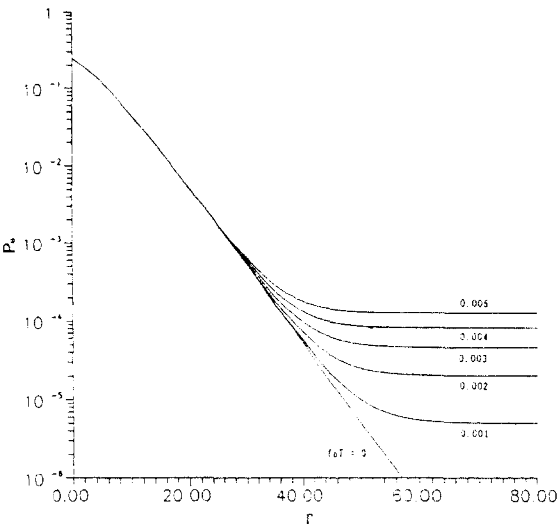


Fig. 9. Error rate performance of GMSK with differential detection with BT=0.25.

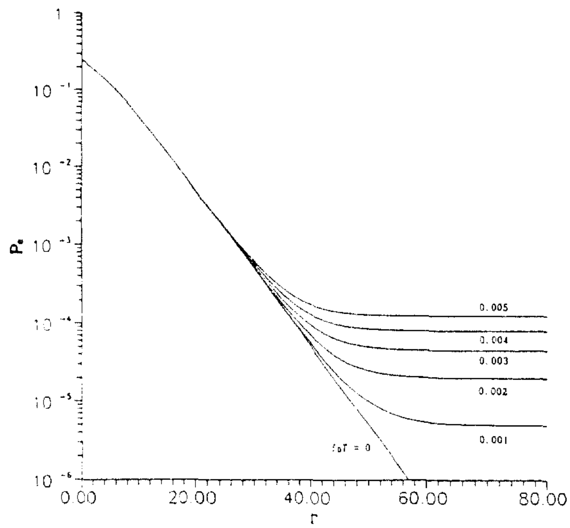


Fig. 10. Error rate performance of GMSK with differential detection with BT=0.5.

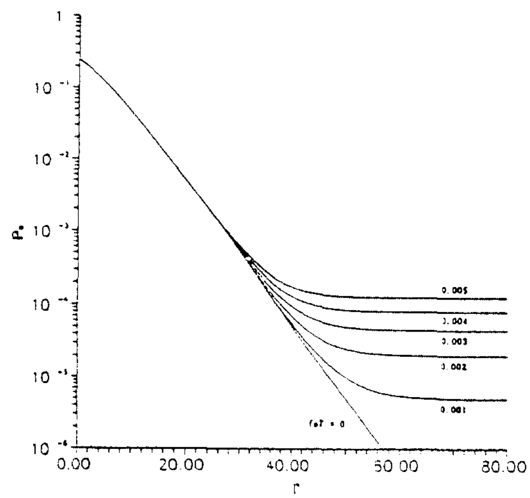


Fig. 11. Error rate performance of MSK with differential detection.

V. CONCLUSION

An improved differential demodulation method for GMSK signal has been presented. This method take advantage of the fact that the sample timing at the center of time slot is less sensitive to the effect of the intersymbol interference resulting from the Gaussian baseband low pass filtering prior to modulation. Error rate performance of the proposed differential demodulation of GMSK was calculated numerically in the mobile radio communication channel characterized by Rayleigh fading for the worst signal pattern. The plotted results show that the error rate of GMSK is always higher than that of MSK because of intersymbol interference. The degradations are slightly higher for the smaller values of BT, which are expected in the proposed demodulation method.

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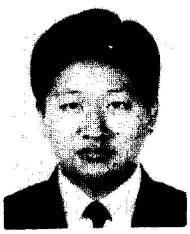

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