論　文

The Error Rate Performance of Digital Radio Signals in Diversity Reception

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다이버시티 수신시의 제반 디지탈 신호의 오율특성

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ABSTRACT　The error rate equations of digital radio signals(CPSK, DPSK, FSK, MSK) transmitted through the Nakagami's m-distribution fading channel have been derived both for nondiversity reception and for diversity reception cases in the environmment of Gaussian noise. Using the derived equation, error rate performance has been evaluated and represented in figures as a function of carrier-to-noise ratio, fading figure, and the power correlation coefficient between two diversity branches. By comparing the diversity reception case with nondiversity reception case, we could measure the amount of error rate performance improvement of digital radio signals in fading environment.

要　約　m분포 페이딩의 영향을 받는 디지탈 무선 통신신호(CPSK, DPSK, FSK, MSK)를 그냥 수신시할때와 다이버시티 수신시할때의 오율식을 유도하여 반송파전력 대 잡음전력비, 페이딩지수, 두 가지(branch)신호 사이의 전력상관계수를 함수로 하여 수치계산한 결과를 그림으로 나타내어 페이딩에 의한 각 디지탈 무선 통신신호의 오율특성의 열화를 보였으며, 다이버시티 기법의 도입으로 인한 오율특성의 개선 정도를 정량적으로 비교할 수 있도록 하였다.

I. Introduction

When a steady state single frequency radio signal is transmitted over a long path, the envelope of the received signal is observed to fluctuate continuously in time. This phenomenon is known as fading and its existence constitutes one of the fatal conditions in radio system design(1).

It is known from experimental observations that the fading in radio channels which are sufficiently separated in space, frequency and time, is more or less statistically uncorrelated. This fact is utilized in diversity reception techniques which are extensively used on fading circuits. The objective of these diversity techniques is to make use of several received signal which constitute closely similar copies of some desired signal, in order to realize rather higher input signal level more economically than by increasing transmitter power(2). Up to now, almost all analyses related to system performance of digital radio signals in fading has assumed the Rayleigh distribution(3) (8). However, practical radio channel is being set up on the various modes of propagation. So the fading model may be much deeper or shallower than Rayleigh fading. In literature(9), the analysis of digital radio signal in Nakagami's m-distribution fading channel is done in the case of CPSK signal.

In this paper, the error rate equation of digital radio signals transmitted through the Nakagami's m-distribution fading channel has been derived both for nondiversity reception and for diversity reception cases as an extension of ref. (9) and derived a concise formular for various digital signals. The digital radio signals considered in this paper are Coherent Phase Shift Keying(CPSK), Differential Phase

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Shift Keying(DPSK), Frequency Shift Keying (FSK) and Minimum Shift Keying(MSK) signals. Using the derived equation, the error rate performance has been evaluated and represented in figures as a function of carrier-to-noise ratio, fading figure and the power correlation coefficient between two diversity branches to measure the amount of error rate performance improvement of digital radio signals by diversity reception.

II. Error Rate Performance in Gaussian Noise

Analysis model is shown in Fig.1. The radio signal is subjected to m distribution fading and Gaussian noise.

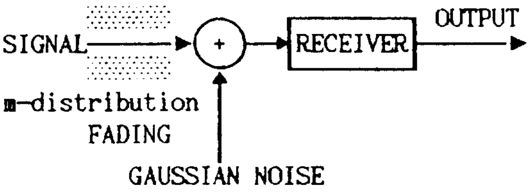

Fig 1. Analysis model.

The error rate performance of digital radio signals in Gaussian noise channel is already analyzed by many authors and is given in eq (1)(10) (11).

$$P_{EG} = \alpha \operatorname{erfc}(\beta\sqrt{\rho_G}) \qquad (1)$$

where, $\rho_G (= S^2 / 2\sigma_G^2)$: carrier-to noise ratio (CNR),

S : signal amplitude,
σ_G^2 : Gaussian noise power.

Here, α and β are the coefficients of error rate equation of N-ary CPSK, N-ary DPSK, FSK and MSK signals as given in table 1.

Table 1. Coefficients α and β of the error rate equation of digital radio signals.

Digital signals	α	β	Remarks
N ary CPSK	1	$\sin(\pi/N)$	N : the number of array
Binary CPSK	1/2	1	
N ary DPSK	1	$\sin(\pi/2N)$	
Binary DPSK	1/2	$\sin(\pi/4)$	
FSK	1/2	$1/\sqrt{2}$	
MSK	1/2	$\pi/4$	

III. Error Rate Performance in Fading Environments

In this section, we derive the error rate equations of digital radio signals(CPSK, DPSK, FSK, MSK) transmitted through the Nakagami's m distribution fading channel.

1. m-distribution Fading

There are two general classes of fading : fast fading and slow fading. The former is mainly caused by multipath transmission and the resultant amplitude variations are well described by a Rayleigh distribution. The latter results from the meteorological and climatological changes, and its fluctuations follow a log-normal distribution. These modes of fading are usually complicated by the simultaneous occurrence : that is, fast fading superimposed on the slow fading. Therefore the practical faded envelope has more diverse distribution(2).

In this paper, the m distribution is used as a general model of the faded envelope. It was formulated by Nakagami in his study of experimental data and its applicability to both ionospheric and tropospheric modes of propagation is well confirmed by some observations.

The probability density function(p.d.f.) of faded envelope R, p(R) is givenas[12].

$$p(R)=\frac{2m^m R^{2m-1}}{\Gamma(m)\ \Omega^m}\exp\left(\frac{-m R^2}{\Omega}\right)$$
$$=M(R, m, \Omega), \tag{2}$$

where, m ; fading figure($m \geqq 1/2$),

$\sqrt{\Omega}$; average envelope of faded signal.

Fig 2. shows the m-distribution p.d.f. for some values of m. m=1/2 and m=1 represent the one-sided Gaussian and Rayleigh p.d.f. respectively. The envelope fluctuation of faded signal becomes smaller as m becomes larger. And m→∞ corresponds to nonfading.

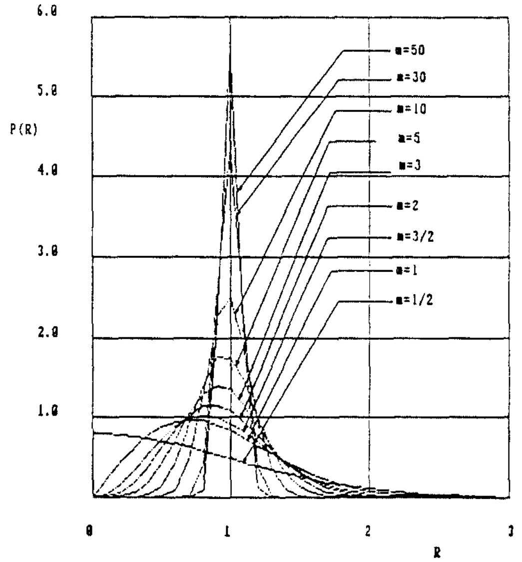

Fig 2. m-distribution(Ω=1).

2. Average Error Rate

In Gaussian noise and m-distribution fading environments, the error rate equation becomes(11)(13)(14).

$$P_{EF}=\int_0^\infty P_{EG}\cdot p(R)\ dR \tag{3}$$

Using eq.(1) and eq.(2), eq.(3) becomes

$$P_{EF}=\alpha\int_0^\infty \mathrm{erfc}\left(\beta\frac{R}{\sqrt{2}\,\sigma_G}\right)\frac{2m^m\ R^{2m-1}}{\Gamma(m)\ \Omega^m}\cdot\exp\left(\frac{-m\ R^2}{\Omega}\right)dR \tag{4}$$

Equation (4) has been solved as eq.(5) by using formula (A.1)~(A.3) in appendix.

$$P_{EF}=\alpha\left\{1-2\beta\sqrt{\frac{\rho_G}{\pi\cdot m}}\frac{\Gamma(m+1/2)}{\Gamma(m)}\cdot\left[1+\frac{\beta^2\rho_G}{m}\right]^{-(m+1/2)}{}_2F_1\left(1,m+1/2\ ;3/2\ ;\frac{\beta^2\rho_G}{m+\beta^2\rho_G}\right)\right. \tag{5}$$

Ⅳ. Error Rate Performance in Diversity Reception

The signal performance that is degraded by severe fading can be improved by increasing transmitted power, antenna size and height, etc., but these solutions are costly and sometimes impractical. Diversity reception techniques have been adopted as an excellent means of combating with fading. The important aspect of diversity reception is its ability to reduce the fraction of the time in which the signal drops down to unstable levels[2].

In this paper, three types of predetection diversity techniques are considered, that is, Selection Combining(SC), Equal Gain Combining (EGC) and Maximal Ratio Combining (MRC). For the practical reasons, only two branches diversity is treated.

1. The Joint Distribution of Two Diversity Inputs

Let the envelope of signals in the two diversity branches be $R_1(t)$ and $R_2(t)$, individually engaged in the m-distribution p.d.f. in (2). Then the joint p.d.f. of them is given in ref.(12) as

$$P_J(R_1,R_2)=\frac{4m^{m+1}R_1^{m}R_2^{m}}{\Gamma(m)\Omega^{m+1}k^{m-1}(1-k^2)}\exp\left[\frac{m}{1-k^2}\left(\frac{R_1^2+R_2^2}{\Omega}\right)\right]\cdot I_{m-1}\left[\frac{2mk\ R_1\ R_2}{(1-k^2)\Omega}\right] \quad (6)$$

where, k^2 : power correlation coefficient between $R_1(t)$, $R_2(t)$,

$$k^2=\frac{\langle(R_1^2-\Omega)(R_2^2-\Omega)\rangle}{\sqrt{\langle(R_1^2-\Omega)^2\rangle\langle(R_2^2-\Omega)^2\rangle}} \quad 0\leq k^2\leq 1,$$

$I_u[\cdot]$: modified Bessel function of the first kind of order u.

2. The p.d.f. of Combined Envelope

A primary step to solve the problem of diversity reception is a derivation of the p.d.f. of the combined envelope. In the following $R_i(t)$ denotes the equivalent combined signal.

1) Selection Combining(SC)

The p.d.f. of $R_S(t)=\max[R_1(t),\ R_2(t)]$ is represented in ref.(9) as

$$p_S(R)=\int_0^1\int_0^{\pi}C_S\ W_S^{2m}\ M(R,2m,2\Omega W_S\sqrt{1-k^2})\ dx\ dy \quad (7)$$

where, $C_S=\frac{2^{2m}(2m-1)}{\pi}\ y^{2m-1}\ \sin^{2m-2}x$,

$$W_S=\frac{\sqrt{1-k^2}}{(1+y^2-2ky\ \cos x)}\ .$$

2) Equal Gain Combining(EGC)

The p.d.f. of $R_E(t)=[R_1(t)+R_2(t)]/\sqrt{2}$ is given in ref.(9)as

$$p_E(R)=\int_0^1\int_0^{\pi}C_E\ W_E^{2m}\ M(R,2m,2\Omega W_E\sqrt{1-k^2})\ dx\ dy \quad (8)$$

where, $C_E=\frac{2(2m-1)}{\pi}\ (1-y^2)^{2m-1}\ \sin^{2m-2}x$,

$$W_E=\frac{\sqrt{1-y^2}}{(1+y^2)-(1-y^2)k\ \cos x}$$

3) Maximal Ratio Combining(MRC)

The p.d.f. of $R_M(t)=\sqrt{R_1^2(t)+R_2^2(t)}$ is given in ref.(9)as

$$p_M(R)=\int_0^1\int_0^{\pi}C_M\ W_M^{2m}\ M(R,2m,2\Omega W_M\sqrt{1-k^2})\ dx\ dy \quad (9)$$

where, $C_M\ \frac{\Gamma(m+1/2)}{\sqrt{\pi}\ \Gamma(m)}\ \sin^{2m-1}x$,

$$W_M=\frac{\sqrt{1-k^2}}{(1-k\ \cos x)}\ .$$

3. Average Error Rate

In diversity reception, the average error rate can be obtained by ref.(15) as

$$\langle P_{EF}\rangle_D=\int_0^{\infty}P_E\cdot p_i(R)\ dR$$

$$=\int_0^1\int_0^{\pi}C_i\ W_i^{2m}q(R,2m,2\rho_G W_i\sqrt{1-k^2})\ dx\ dy, \quad (10)$$

where, $p_i(R)$: the p.d.f. of combined envelope,

$$p_i(R)=\int_0^1\int_0^{\pi}C_i\ W_i^{2m}\ M(R,2m,2\Omega W_i\sqrt{1-k^2})\ dx\ dy,$$

- i=S : Selection Combining ; eq. (7)
- i=E : Equal Gain Combining ; eq. (8)
- i=M : Maximal Ratio Combining ; eq. (9).

Solving the above quation,

$$\langle P_{EF}\rangle_D=\frac{\alpha}{\sqrt{\pi}}\ \frac{\Gamma(2m+1/2)}{\Gamma(2m+1)}\cdot\left[\frac{1}{\rho_{G,m}(\beta)^2}\right]^{2m}$$

$$\cdot \frac{D_i}{(1-k^2)^m} \qquad (11)$$

$$\text{where, } D_i = \int_0^1 \int_0^\pi C_i \, dx \, dy$$

$$= \begin{cases} \dfrac{2^{2m}\Gamma(m+1/2)}{\sqrt{\pi}\Gamma(m+1)}, & \text{in } i=S \\[2ex] \dfrac{\Gamma(2m+1)\Gamma(m+1/2)}{\Gamma(2m+1/2)\,\Gamma(m+1)}, & \text{in } i=E \\[2ex] 1, & \text{in } i=M. \end{cases}$$

V. Discussion and Conclusion

Based on the results in section II, III and IV, the error rate performance has been evaluated and then presented in Fig.3 through Fig. 11 as a function of carrier to noise ratio (CNR), fading figure(m), and the power correlation coefficient between two branches (k^2).

Fig.3 presents the error rate performance of digital radio signals(CPSK, DPSK, FSK, MSK) in Gaussian noise only. From this figure we can confirm the better error performance is obtained in the order of MSK, FSK, 4CPSK, 4DPSK. Fig.4 shows the error rate performance of digital signals in nonfading(noise only) and Rayleigh fading environments, which indicates the degree of performance deterioration by Rayleigh fading. For example, at $P_E=10^{-4}$, MSK signal is degraded about 25[dB] by Rayleigh fading. Fig.5 represents the effect of fading on digital radio signals according to the fading depth. At this figure, m=1 is the case of Rayleigh fading(deep fading) and the degree of fading becomes shallower as m becomes larger. The performance of radio signal is found to be more deteriorated in deep fading. Fig.6-Fig.9 shows the diversity reception improvement in each digital radio signal with the variation of CNR, m,and combining method. Diversity improvement means the reduction of the average CNR in diversity reception compared with the average CNR in nondiversity reception under the assumption of the same system performance. For example, when m=2, faded PSK signal is improved about 10[dB] for fixed error rate $P_E=10^{-5}$ by adopting MRC. In mobile environments the space diversity is adopted when the correlation coefficient is greater than 0.7. So we assume the correlation coefficient to be 0.7 in Fig.6-Fig.9. The effect of diversity combining methods on MSK signal according to the correlation coefficient is represented in Fig.10. From this figure it is verified that in each combining method, the smaller is the correlation coefficient between two branches, the better error rate performance is obtained. And among the combining methods MRC gives the best improvement. The degree of performance improvement with the branch number(M) is shown in Fig.11. M=1 corresponds to nondiversity case and M=2,3,4 means the case of reception diversity branch is 2,3,4 respectively. The error rate performance becomes more improved as M becomes larger. For example, at CNR=25[dB] in 4CPSK, 4 branches reception case is improved by 6×10^{-4} than 2 branch case in terms of error rate.

From these we conclude that :

(1) Generally the order of better error rate performance is MSK, FSK, 4CPSK, and 4DPSK.

(2) Digital radio signals are degraded by fading and the degree of deterioration is more serious in deep fading than shallow fading environments.

(3) The error rate performance of digital radio signals degraded by fading can be highly

compensated by diversity reception. That is, the use of diversity permits reliable reception at lower transmitter powers than would be required if diversity was not used.

(4) In each combining method, the smaller is the correlation coefficient between two branches, the better error rate performance can be obtained in diversity reception.

(5) Among the combining methods, Maximal Ratio Combining method gives the best improvement.

(6) The better error rate performance can be obtained as the diversity branches increase. But the best value of M taking into account the system complexity and the amount of improvement is 2 or 3.

These results will be used to estimate the system performance and to design the digital radio communication system opearting in fading channel.

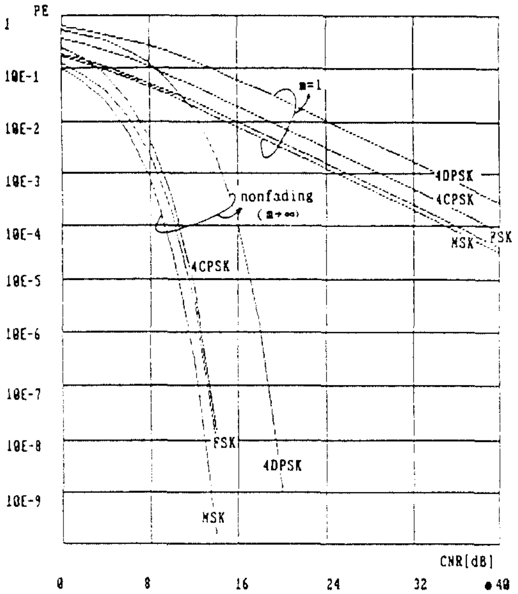

Fig 4. The error rate performance of digital radio signals in nonfading and Rayleigh fading environments.

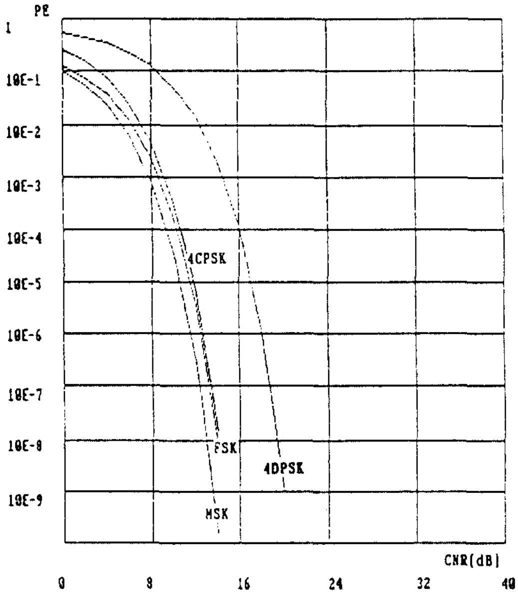

Fig 3. The error rate performance of digital radio signals in Gaussian noise.

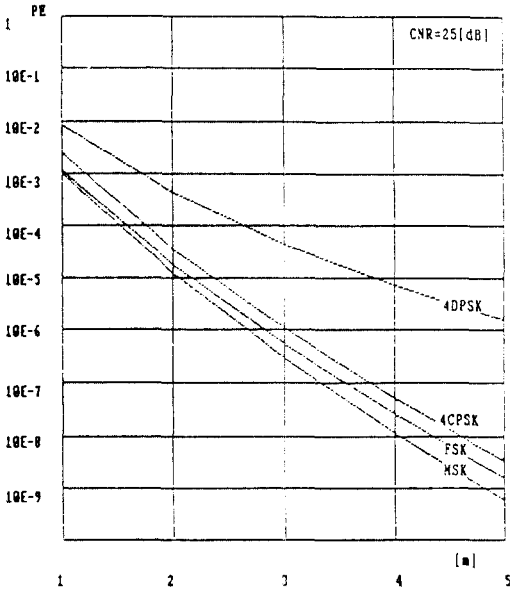

Fig 5. The error rate performance of digital radio signals in fading environments(CNR=25[dB]).

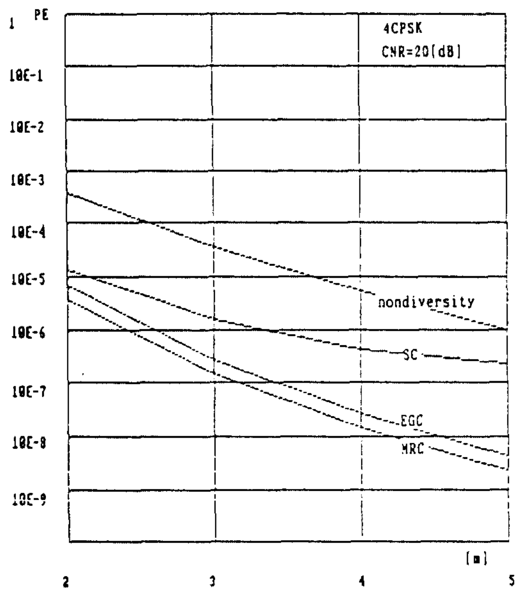

Fig 6. The error rate performance of 4CPSK signal in diversity reception(CNR=20[dB], $k^2=0.7$).

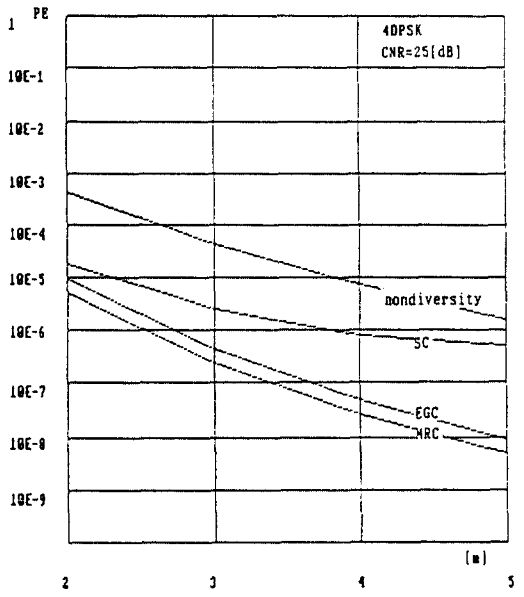

Fig 7. The error rate performance of 4DPSK signal in diversity reception(CNR=25[dB] $k^2=0.7$).

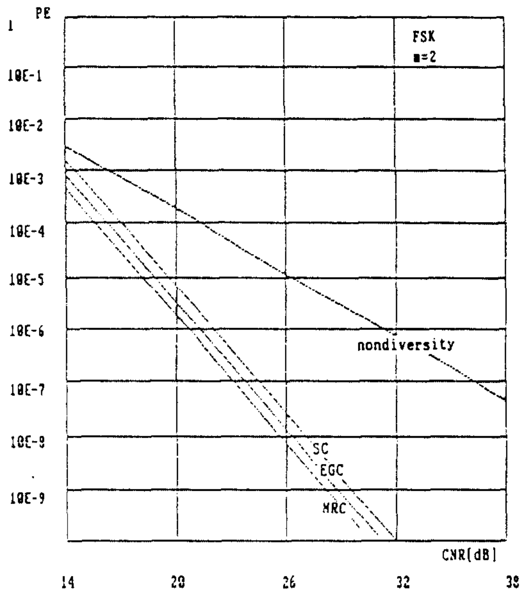

Fig 8. The error rate performance of FSK signal in diversity reception(m=2, $k^2=0.7$).

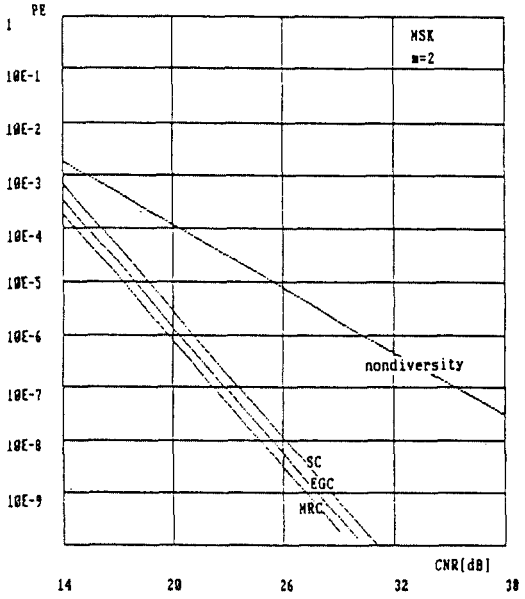

Fig 9. The error rate performance of MSK signal in diversity reception(m=2, $k^2=0.7$).

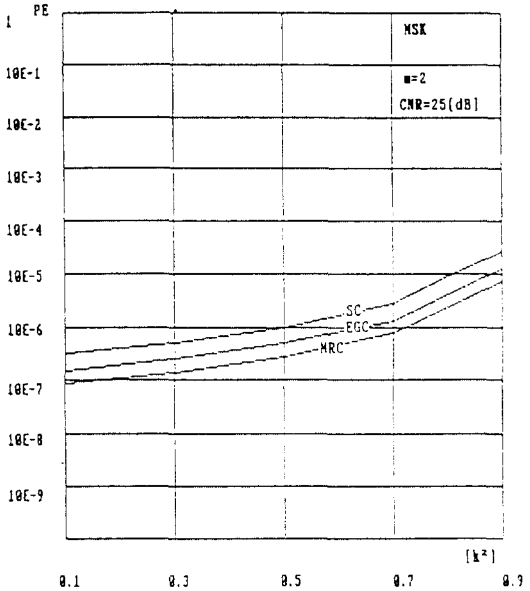

Fig 10. The error rate performance of combining methods with the variation of k^2.

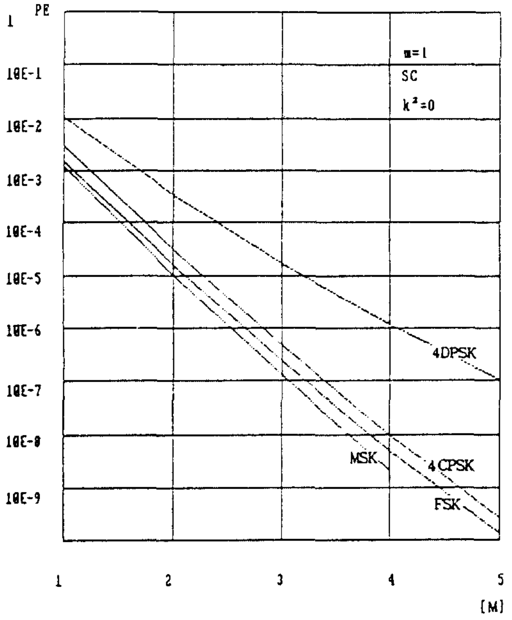

Fig 11. The error rate performance of digital radio signals in diversity reception with the variation of M (CNR=25[dB], m=1, SC, $k^2=0$).

APPENDIX(FORMULAS)

(A.1) $\mathrm{erfc}(x)=1-2x\frac{2x}{\sqrt{\pi}}F_1(1/2:3/2:-x^2)$

(A.2) $_1F_1(k+1:3/2:-x^2)=\exp(-x^2)\ _1F_1(1/2-k:3/2:x^2)$

(A.3) $g(p)=\int_0^\infty e^{-pt}f(t)\,dt$

$f(t)=t^{i-1}\ _mF_n(a_1,\cdots a_m:b_1\cdots b_n:\lambda t)$

$g(p)=\Gamma(i)\ p^{-i}\ _{m+1}F_n(a_1\cdots a_m,\ i:b_1\cdots b_n:\lambda/p)$

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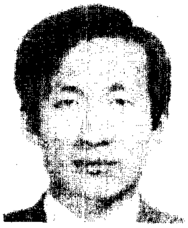

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