論 文

An Easy Way to Derive the Fourier Transforms of the Truncated Raised-Cosine Function and the n-th Order Powers of it Using Partial-Response System Concept : A Recursive Formula

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상승 Cosine 함수와 그 n-제곱 함수의 Fourier 변환을 구하기 위한 용이한 방법 : 부분응답 시스템 개념을 이용한 순환 공식

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ABSTRACT

In this paper, a new and easy analytical method to get the Fourier transforms of a popular type of truncated raised-cosine function and its powers (n=1,2,3,⋯ : positive integers) is proposed. This new method is based on the concept of the $(1+D)^2$-type partial response system, and the procedure is more compact than the conventional method using differentiations. Especially, the results are obtained as a sum of three functions which are easily manageable for each power. And they are recursively related to their powers. Therefore, they can be excellently applied to the computer-aided numerical solutions.

要 約

본 논문에서는 일반적 형태의 상승 Cosine함수와 그 n-제곱 함수에 대한 Fourier변환을 해석적으로 구하는 새롭고 용이한 방법을 제안하였다. 이는 $(1+D)^2$형 부분응답 시스템의 개념에 근거를 두고 있으며, 도함수에 의하여 변환되는 기존의 방법에 비하여 매우 간결함을 보인다. 특히, 이러한 방법에 의하여 유도되는 해는 각각의 차수에 대하여 다루기 용이한 세 함수의 합으로 주어지며, 이들은 차수에 따라 순환적 관계를 유지한다. 따라서 이러한 과정은 컴퓨터에 의한 수치적 변환에 대해서도 매우 잘 적용된다.

I. Introduction

The Fourier transform is one of the most popular analysis tools for the communication systems. One can easily find some derivations in the literatures on communication theory [1-3]. And also, some numerical solutions for the transform have been developed[4,5].

But there exist many functions used in theor-

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etical areas which have no such transforms. And there are some functions used in practical systems which have their unique transforms that are very complex and not easily derived from the conventional methods. The truncated raised-cosine function, so-called, is one of such functions and it is more difficult to derive the Fourier transforms of its powers.

A method by consecutive differentiations can be used for them[2]. But the procedures are tedious, and the higher the orders of them are, the more tedious and difficult the procedures must be carried out. We present an easy iterative method to derive the Fourier transforms of the truncated raised-cosine function and the n-th order powers of it using the $(1+D)^2$ type PRS system structure.

The PRS(partial response signaling) is a transmission method for digital data using the correlation concepts between the input samples. A unified study of the PRS systems has been presented by Kabal and Pasupathy[6]. Their generalized PRS system model is well applied to the various baseband and modulation schemes. In particular, the combination of the correlation polynomial $(1+D)^2$ and MSK(minimum shift keying) scheme, known as TFM(tamed frequency modulation), seems to stand out as the most promising bandwidth efficient signaling scheme in a sense[2,5].

Throughout this paper, we define the n-th order powers of the truncated raised-cosine functions with uniform amplitudes. And we derive a recursive formula for the Fourier transforms of them using the method of constructing appropriate bandlimiting filters for the $(1+D)^2$-type PRS system structure.

For Comparison, the procedure of the conventional method using consecutive differentiations is also presented briefly for the 1-st and 2-nd order truncated raised-cosine functions.

II. Definitions and a Brief Review of the Conventional Methods

At first, we define the Fourier transform pair as following, which has been used in many literatures on communication theory[1,3].

(Definition 1 : The Fourier transform pait)

The Fourier transform of a time-domain function x(t) is defined as a frequency-domain function

$$X(f) \equiv \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \tag{1}$$

and the inverse Fourier transform of the function X(f) is given by

$$x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df$$

And we define the n-th order powers of the truncated raised-cosine function as followings, which present the system functions of some useful communication systems[6,7].

(Definition 2 : The n-th order powers of the truncated raised cosine function)

$$g_n(t) \equiv \frac{A}{2^n}\left[1+\cos\frac{\pi t}{\tau}\right]^n \prod\left(\frac{t}{2\tau}\right),\ \text{for } n=1,2,3\cdots \tag{2}$$

From this definition, our functions have the same amplitude A for all n=1,2,3,···, and it can be shown graphically as Fig.1. The 1-st and 2-nd orders of (2) are given by

$$g_1(t) = \frac{A}{2}\left[1+\cos\frac{\pi t}{\tau}\right] \prod\left(\frac{t}{2\tau}\right) \tag{3}$$

and

$$g_2(t) = \frac{A}{4}\left[1+\cos\frac{\pi t}{\tau}\right]^2 \prod\left(\frac{t}{2\tau}\right) \tag{4}$$

respectively.

We can get the Fourier transforms of the

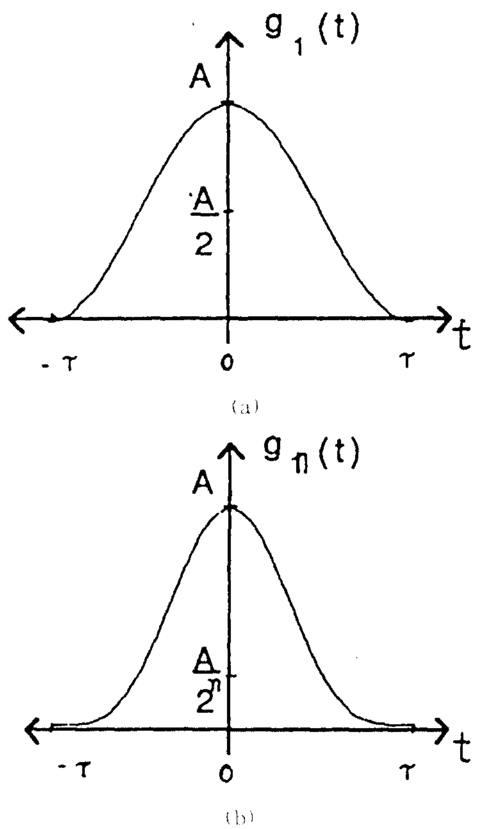

Fig. 1. The n-th order powers of the truncated raised cosine function.
(a) the 1-st order function
(b) the n-th order function

functions (3) and (4) using consecutive differentiations as in[2]. The first three derivatives of $g_1(t)$ are sketched in Fig.2 and we have

$$\frac{dg_1(t)}{dt} = -\left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)\sin\frac{\pi t}{\tau}\ \prod\left(\frac{t}{2\tau}\right) \tag{5}$$

$$\frac{d^2g_1(t)}{dt^2} = -\left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^2\cos\frac{\pi t}{\tau}\ \prod\left(\frac{t}{2\tau}\right) \tag{6}$$

$$\frac{d^3g_1(t)}{dt^3} = \left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^3\sin\frac{\pi t}{\tau}\ \prod\left(\frac{t}{2\tau}\right) + \left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^2\delta(t+\tau) - \left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^2\delta(t-\tau) \tag{7}$$

Fortunately, if we check the first and third derivatives, we can see that the first term of the third derivative can be written by a constant multiple of the first derivative and the remaining terms contain the shifted impulse function only. Therefore, (7) gives

$$(j2\pi f)^3 G_1(f) = -\left(\frac{\pi}{\tau}\right)^2(j2\pi f)G_1(f) + 2j\left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^2\sin(2\pi f\tau) \tag{8}$$

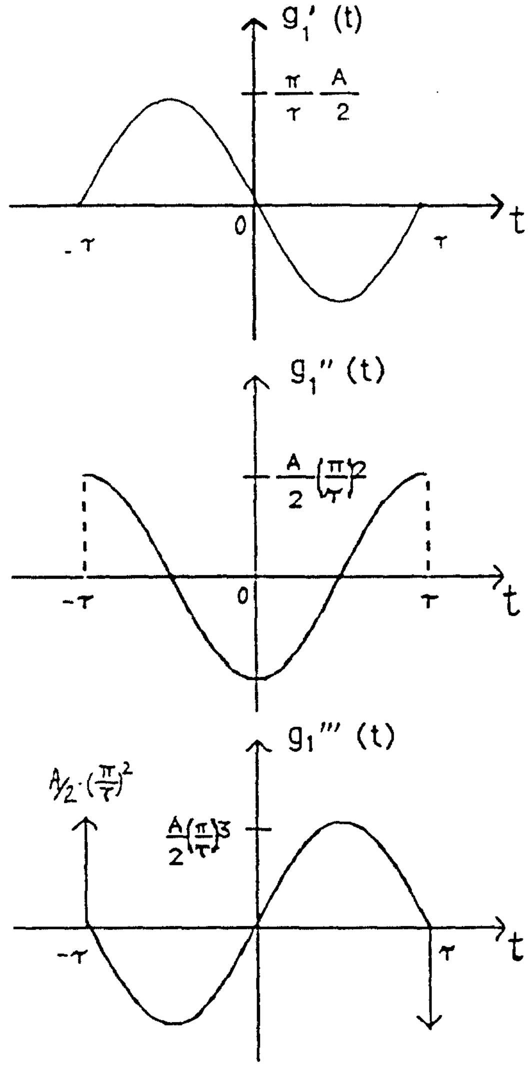

Fig. 2. Derivatives of the function $g_1(t)$.

and after a few steps, we have

$$G_1(f) = \frac{A\tau \,\mathrm{sinc}\,(2f\tau)}{1-(2f\tau)^2} \tag{9}$$

where

$$\mathrm{sinc}(a) = \frac{\sin \pi a}{\pi a} \tag{10}$$

The procedure to get the Fourier transform of the 1 st order function $g_1(t)$ is seemed to be relatively simple. But for the 2 nd order function $g_2(t)$, the scenario is quite different. The first three derivatives of $g_2(t)$ are given by

$$\frac{dg_2(t)}{dt} = -\left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)\left\{\sin\frac{\pi t}{\tau} + \left(\frac{1}{2}\right)\sin\frac{2\pi t}{\tau}\right\} \prod\left(\frac{t}{2\tau}\right) \tag{11}$$

$$\frac{d^2g_2(t)}{dt^2} = -\left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^2\left\{\cos\frac{\pi t}{\tau} + \cos\frac{2\pi t}{\tau}\right\} \prod\left(\frac{t}{2\tau}\right) \tag{12}$$

$$\frac{d^3g_2(t)}{dt^3} = \left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^3\left\{\sin\frac{\pi t}{\tau} + 2\sin\frac{2\pi t}{\tau}\right\} \prod\left(\frac{t}{2\tau}\right) \tag{13}$$

respectively and they are sketched in Fig.3. Although the consecutive differentiations are carried out upto any order, we have no forms more compact than (13). That is to say, a comparison of (11) and (13) reveals that the two terms in the braces will have more unmatched coefficients, even if the differentiations are performed more and more. Substituting (11) into (13), we obtain

$$\frac{d^3g_2(t)}{dt^3} = -4\left(\frac{\pi}{\tau}\right)^2 \frac{dg_2(t)}{dt} - 3\left(\frac{A}{2}\right)\left(\frac{\pi}{\tau}\right)^3 k(t) \tag{14}$$

where

$$k(t) = \sin\frac{\pi t}{\tau} \prod\left(\frac{t}{2\tau}\right) \tag{15}$$

and which is transformed as

$$K(f) = \frac{-j4f\tau^2}{1-(2f\tau)^2}\,\mathrm{sinc}(2f\tau) \tag{16}$$

We now have the transform of $g_2(t)$ after a tedious process of calculations with (14), (15) and (16). The result is obtained as

$$G_2(f) = \frac{3A\tau}{\{1-(2f\tau)^2\}\{4-(2f\tau)^2\}}\,\mathrm{sinc}(2f\tau) \tag{17}$$

As the order of powers of the function in (2) is

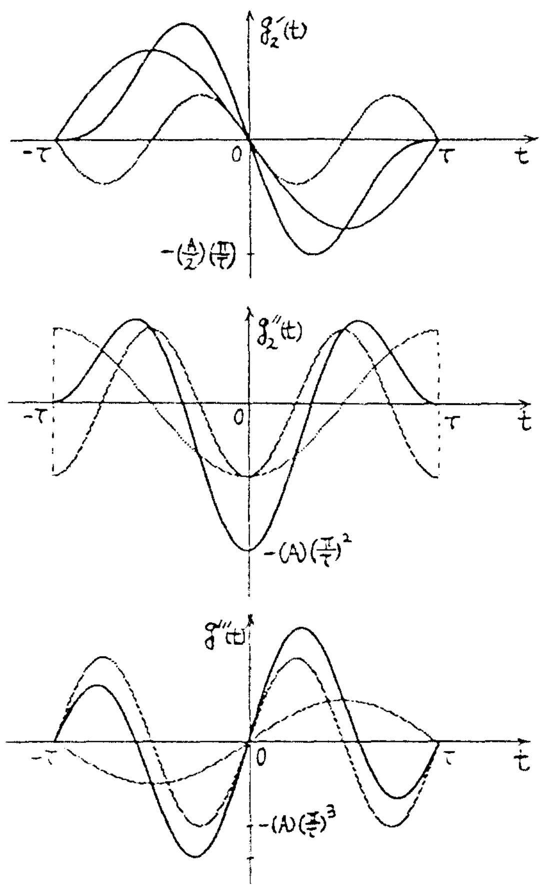

Fig. 3. Derivatives of the function $g_2(t)$.

increased, the forms of k(t) and K(f) will be more and more complicated to give terribly tedious and difficult procedutes.

Ⅲ. The $(1+D)^2$-type PRS System Models and Recursive Formula

1. The Models for the 1-st and 2-nd order functions

It is well known that the transfer function of the $(1+D)^2$-type PRS system has the form of truncated reised-cosine functions[6]. In this sub-section, we modify the standard $(1+D)^2$-type PRS system model and apply it to get the Fourier transforms of our functions with 1-st and 2-nd orders. The modification for the 1-st order function is shown in Fig.4.

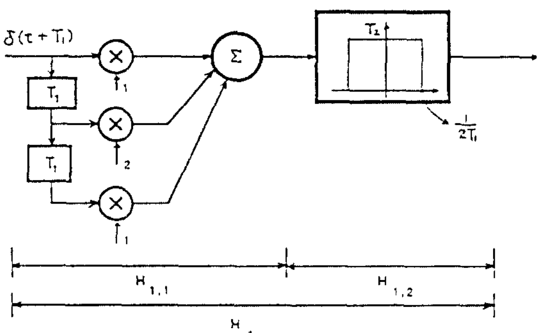

Fig. 4. Model for the 1-st order function.

From the structure of the transversal filter, we can easily obtain its impulse response as follows :

$$h_{1,1}(t)=\delta(t+T_1)+2\delta(t)+\delta(t-T_1) \tag{18}$$

and the Fourier transform of it is given by

$$H_{1,1}(f)=2(1+\cos 2\pi f T_1) \tag{19}$$

If we set the frequency response of the bandlimiting fillter as

$$H_{1,2}(f)=T_2\prod(T_1 f) \tag{20}$$

, then its inverse Fourier transform is given by

$$h_{1,2}(t)=\frac{T_2}{T_1}\,\text{sinc}\left(\frac{t}{T_1}\right) \tag{21}$$

From the System theory and the convolution theorem[3], the overall impulse reponse and the transfer function of the system shown in Fig.4 are given by

$$\begin{aligned} h_1(t) &= h_{1,1}(t)*h_{1,2}(t) \\ &= \frac{T_2}{T_1}\left[\text{sinc}\left(\frac{t+T_1}{T_1}\right)+2\,\text{sinc}\left(\frac{t}{T_1}\right)\right. \\ &\quad \left.+\text{sinc}\left(\frac{t-T_1}{T_1}\right)\right] \end{aligned} \tag{22}$$

and

$$\begin{aligned} H_1(f) &= H_{1,1}(f)\,H_{1,2}(f) \\ &= 2\,T_2\,(1+\cos 2\pi f T_1)\prod(T_1 f) \end{aligned} \tag{23}$$

respectively. In (22), the symbol '*' denotes the convolution of the two functions.

The functions $h_1(t)$ and $H_1(f)$ are even functions. Therefore, we have another pair from the duality of the Fourier transform[5] as follows.

$$\begin{aligned} g_1(t) &= 2\,T_2(1+\cos 2\pi T_1 t)\prod(T_1 t) \;<===>\; \\ G_1(f) &= \frac{T_2}{T_1}\left[\text{sinc}\left(\frac{f+T_1}{T_1}\right)+2\,\text{sinc}\left(\frac{f}{T_1}\right)\right. \\ &\quad \left.+\text{sinc}\left(\frac{f-T_1}{T_1}\right)\right] \end{aligned} \tag{24}$$

where the symbol '<===>' denotes that the two functions form a Fourier transform pair.

If we define the parameters as $T_1=(1/2\tau)$, $T_2=(A/4)$ respectively, then we obtain

$$g_1(t)=\frac{A}{2}\left[1+\cos\frac{\pi t}{\tau}\right]\prod\left(\frac{t}{2\tau}\right) \tag{25}$$

and its Fourier transform

$$G_1(f)=\frac{A\tau}{2}\left[\text{sinc}\left(\frac{f+1/2\tau}{1/2\tau}\right)+2\,\text{sinc}\left(\frac{f}{1/2\tau}\right)\right.$$

$$+\mathrm{sinc}\left(\frac{f-1/2\tau}{1/2\tau}\right)] \qquad (26)$$

The results (25) and (26) derived from the $(1+D)^2$ type PRS system model are equivalent to (3) and (9) respectively.

Fig. 5 shows the modified model for the 2 nd order function. In this model, the part of the transversal filter is the same as that of Fig.4, and the frequency response of the bandlimiting filter is given by the transfer function (23) of the previous model.

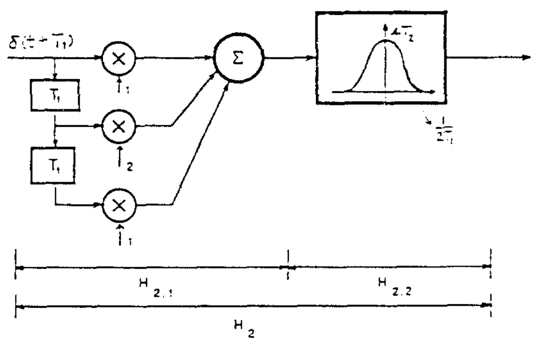

Fig. 5. Model for the 2 nd order function.

Repeating the previous procedure for the model in Fig.5, the system functions can be obtained as

$$h_{2,1}(t) = h_{1,1}(t) = \delta(t+T_1)+2\delta(t)+\delta(t-T_1)$$
$$\longleftrightarrow H_{2,1}(f) = H_{1,1}(f) = 2(1+\cos 2\pi f T_1) \qquad (27)$$

and

$$H_{2,2}(f) = H_1(f) = 2T_2(1+\cos 2\pi f T_1)\prod(T_2 f)$$
$$h_{2,2}(t) = h_1(t)$$
$$= \frac{T_2}{T_1}[\mathrm{sinc}\left(\frac{t+T_1}{T_2}\right)+2\mathrm{sinc}\left(\frac{t}{T_2}\right)$$
$$+\mathrm{sinc}\left(\frac{t-T_1}{T_1}\right)] \qquad (28)$$

The overall impulse response and transfer function can be represented from these results :

$$h_2(t) = h_{2,1}(t) * h_{2,2}(t)$$
$$= h_{2,2}(t+T_1)+2h_{2,2}(t)+h_{2,2}(t-T_1) \qquad (29)$$

and

$$H_2(f) = H_{2,1}(f)\,H_{2,2}(f)$$
$$= 4T_2(1+\cos 2\pi f T_1)^2\prod(T_2 f) \qquad (30)$$

where $h_{2,2}(t)$ is just $h_1(t)$ given in (28).

The functions in (29) and (30) are even. From the duality, we have another Fourier transform pair as followings :

$$g_2(t) = 4T_2(1+\cos 2\pi T_1 t)^2\prod(T_2 t)$$
$$G_2(f) = h_2(f+T_1)+2h_2(f)+h_2(f-T_1)$$
$$= G_1(f+T_1)+2G_1(f)+G_1(f-T_1) \qquad (31)$$

where $h_2(t) = h_1(t)$ from (28). And the fact that $h_1(f)$ is the same as $G_1(f)$ in equation (24) will be focused in the following sections.

Defining the parameters as $T_1 = 1/2\tau$, $T_2 = A/16$ respectively, we have

$$g_2(t) = \frac{A}{4}[1+\cos\frac{\pi t}{\tau}]^2\prod\left(\frac{t}{2\tau}\right) \qquad (32)$$

and its Fourier transform

$$G_2(f) = \left(\frac{1}{4}\right)[G_1\left(f+\frac{1}{2\tau}\right)+2G_1(f)+G_1\left(f-\frac{1}{2\tau}\right)] \qquad (33)$$

where $G_1(f)$ is just (26), and (32) is the same as (4). It can also be shown that this result (33) is equivalent to (17). The procedure shown here is much simpler than the conventional method using consecutive differentiations. And a clue for recursion is offered in this procedure, i.e. (31), (33).

2. Generalization and the Recursive formula

In this subsection, we generalize the idea presented in the previous subsection. We derive the Fourier transform of the n th order truncated raised cosine function given in (2) from the generalized model as in Fig.6. The overall system model consists of the same transversal filter as before and the bandlimiting filter whose frequency response is the overall transfer function of the model for the (n-1)-st order function.

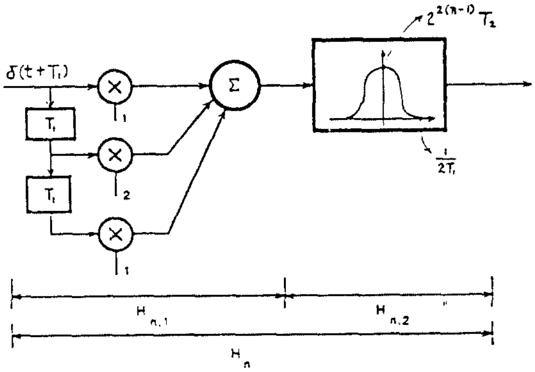

Fig. 6. Generalized model.

(Proposition 1)

For the model shown in Fig.6, defining the frequency response of the bandlimiting filter as

$$H_{n,2}(f)=2^{n-1}T_2(1+\cos 2\pi fT_1)^{n-1}\Pi(T_1f)$$
$$=H_{n-1}(f) \qquad \text{, for } n=1, 2, 3\cdots \qquad (34)$$

and

$$H_0(f)=T_2\Pi(T_1f) \qquad (35)$$

, the relations

$$P(n): H_n(f)=2^nT_2(1+\cos 2\pi fT_1)^n\Pi(T_1f) \qquad (36)$$

$$h_n(t)=h_{n-1}(t+T_1)+2h_{n-1}(t)+h_{n-1}(t-T_1) \qquad (37)$$

are true for all n=1, 2, 3, ⋯.

(Proof)

From the structure of the system model shown in Fig.6, we have the system functions for the transversal filter as

$$h_{n,1}(t)=\delta(t+T_1)+2\delta(t)+\delta(t-T_1)$$
$$\Longleftrightarrow H_{n,1}(f)=2(1+\cos 2\pi fT_1) \qquad (38)$$

, for all n=1, 2, 3,⋯, which is the same for any order. And from(35)

$$h_0(t)=\frac{T_2}{T_1}\text{sinc}\left(\frac{t}{T_1}\right) \qquad (39)$$

is obtained.

Next, we shall prove the proposition by mathematical induction as follows [9].

<Basis step> When n=1, from (34) and (35), we have

$$H_{1,2}(f)=H_0(f)=T_2\Pi(T_1f) \qquad (40)$$

And we get from this function with (38)

$$H_1(f)=H_{1,1}(f)H_{1,2}(f)=2T_2(1+\cos 2\pi fT_1)\Pi(T_1f) \qquad (41)$$

The inverse Fourier transform of $H_1(f)$ can be obtained from the relationship between (22) and (23), or from the convolution of $h_{1,1}(t)$ (from (38)) with $h_{1,2}(t)$ (from (34), (35), and (39)). We are given

$$h_1(t)=\frac{T_2}{T_1}\left\{\text{sinc}\left(\frac{t+T_1}{T_1}\right)+2\,\text{sinc}\left(\frac{t}{T_1}\right)+\text{sinc}\left(\frac{t-T_1}{T_1}\right)\right\}$$
$$=h_0(t+T_1)+2h_0(t)+h_0(t-T_1) \qquad (42)$$

Equations (41) and (42) indicate that P(1) is true.

<Induction step> Suppose that P(k) is true for some $k\geq 1$, we have

$$H_k(f)=2^kT_2(1+\cos 2\pi fT_1)^k\Pi(T_1f) \qquad (43)$$

and

$$h_k(t)=h_{k-1}(t+T_1)+2h_{k-1}(t)+h_{k-1}(t-T_1) \qquad (44)$$

Therefore, we get from (34)

$$H_{k+1,2}(f) = H_k(f) = 2^k T_2(1+\cos 2\pi f T_1)^k \Pi(T_3 f)$$
$$\Longleftrightarrow h_{k+1,2}(t) = h_k(t) = h_{k-1}(t+T_1)+2h_{k-1}(t)+h_{k-1}(t-T_1) \quad (45)$$

and from (38)

$$H_{k+1,1}(f) = 2(1+\cos 2\pi f T_1)$$
$$\Longleftrightarrow h_{k+1,1}(t) = \delta(t+T_1)+2\delta(t)+\delta(t-T_1) \quad (46)$$

Applying (45) and (46) to the generalized model in Fig.6, we ultimately have

$$H_{k+1}(f) = H_{k+1,1}(f)\,H_{k+1,2}(f) = 2^{k+1} T_2(1+\cos 2\pi f T_1)^{k+1} \Pi(T_3 f) \quad (47)$$

and

$$\begin{aligned} h_{k+1}(t) &= h_{k+1,2}(t) * h_{k+1,1}(t) \\ &= h_{k-1}(t+2T_1)+2h_{k-1}(t+T_1)+h_{k-1}(t) \\ &\quad +2h_{k-1}(t+T_1)+2h_{k-1}(t)+h_{k-1}(t-T_1) \\ &\quad +h_{k-1}(t)+2h_{k-1}(t-T_1)+h_{k-1}(t-2T_1) \\ &= h_k(t+T_1)+2h_k(t)+h_k(t-T_1) \end{aligned} \quad (48)$$

The equations (47) and (48) say that the statement P(k+1) is also true.

By induction, P(n) is true for all n = 1, 2, 3,…

(Q.E.D).

(Proposition 2)

$Q(n)$: The functions $H_n(f)$ and $h_n(t)$ which are presented in (Proposition 1) are even functions of f and t respectively for all n = 0, 1, 2,….

It can be easily shown that the $Q(n)$ is true by induction[7] used in the proof for the (Proposition 1).

From the duality of the Fourier transform and the fact that the functions are even, we have another Fourier transform pair as followings :

$$g_n(t) = 2^n T_2(1+\cos 2\pi T_1 t)^n \Pi(T_3 t) \Longleftrightarrow$$

$$G_n(f) = G_{n-1}(f+T_1)+2G_{n-1}(f)+G_{n-1}(f-T_1),$$
for all n = 1, 2, 3… (49)

with

$$G_0(f) = \frac{T_2}{T_3}\,\mathrm{sinc}\left(\frac{f}{T_3}\right) \quad (50)$$

If we set the parameters T and T as

$$T_1 = \frac{1}{2\tau},\ T_2 = \frac{A}{2} \quad (51)$$

, then we have

$$g_1(t) = \frac{A}{2}\left(1+\cos\frac{\pi t}{\tau}\right)\Pi\left(\frac{t}{2\tau}\right) \quad (52)$$

The equation (52) is just (2), and its Fourier transform is given by

$$G_n(f) = G_{n-1}\left(f+\frac{1}{2\tau}\right)+2G_{n-1}(f)+G_{n-1}\left(f-\frac{1}{2\tau}\right)$$
, for all n = 1, 2, 3,… (53)

where

$$G_0(f) = 2A\tau\,\mathrm{sinc}(2f\tau) \quad (54)$$

The resulting equation (53) with (54) is a complete recursion, and the form of the function (52) can be easily matched to any coefficient and any order of the truncated raised cosine impulse response. Therefore, the recursive formula can be easily solved by the computer aided numerical methods.

Ⅳ. Conclusions

A new and easy iterative method to get the Fourier transforms of the truncated raised cosine function and its n th order powers using the concept of the (1+D) type PRS system has been introduced. We have discovered that the proposed procedure is more compact and simpler than the conventional method based on consecutive differentiations. The results which have been obtained from this procedure constitute a completely recursive formula. And the recursive formula consists of a sum of three functions which are easily obtained from the last step.

We conclude that the formula introduced in this paper can be excellently applied to the analyses for many kinds of communication systems of which the system functions are the n th order powers of the truncated raised cosine. Especially, it is expected that the formula can be easily applied to the computerized numerical analyses by virtue of its inherent recursive characteristics.

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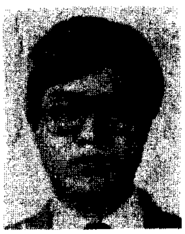

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