論 文

Design of Optimal FIR Filters for Data Transmission

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데이타 전송을 위한 최적 FIR 필터 설계

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ABSTRACT

For data transmission over strictly band-limited non-ideal channels, different types of filters with arbitrary responses are needed. In this paper, we proposed two efficient techniques for the design of such FIR filters whose response is specified in either the time or the frequency domain. In particular, when a fractionally-spaced structure is used for the transceiver, these filters can be efficiently designed by making use of characteristics of oversampling. By using a minimum mean-squared error criterion, we design a fractionally-spaced FIR filter whose frequency response can be controlled without affecting the output error. With proper specification of the shape of the additive noise signals, for example, the design results in a receiver filter that can perform compromise equalization as well as phase splitting filtering for QAM demodulation. The second method addresses the design of an FIR filter whose desired response can be arbitrarily specified in the frequency domain. For optimum design, we use an iterative optimization technique based on a weighted least mean square algorithm. A new adaptation algorithm for updating the weighting function is proposed for fast and stable convergence. It is shown that these two independent methods can be efficiently combined together for more complex applications.

要 約

제한된 주파수 대역폭을 이용하여 신호를 전송하기 위해서는 여러종류의 특성을 갖는 필터들이 필요하다. 이 논문에서는 이러한 필터들을 효율적으로 설계하기위한 두가지 방식을 제시하였다. 특히 fractionally-spaced(FS) 구조가 사용될때 더욱 효율적으로 필터를 설계할 수 있다. FS 구조의 특성을 최소자승오차 방식과 결합하여, 출력오차에 영향을 주지않고, 적절한 주파수 특성을 갖는 SF 필터 설계 방식을 제시하였다. 예로, noise 신호들을 적절히 이용하면, 한개의 SF 필터가, QAM 복조에 필요한 phase splitter, 수신 필터 그리고 등화기 기능까지 갖도록 설계할 수 있다. 두번째로 임의의 주파수 특성이 요구되는 필터의 설계 방식을 제시하였다. weighting factor를 이용한 최소자승법을 iterative하게 사용하여 최적

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설계를 얻는다. 이를위해 weighting factor를 효율적으로 update하기 위한 새로운 알고리듬을 이용하였다. 마지막으로, 더욱 복잡한 조건을 갖는 필터를, 이 두가지 방식을 같이 이용하여, 효율적으로 설계할 수 있는것을 보였다.

I. Introduction

Data transmission over strictly band-limited non-ideal channels requires different kinds of efficient signal processing techniques. For example, digital filters need to be optimally designed for applications such as pulse shaping, matched filtering, equalization, decimation and interpolation. The design of finite-impulse response (FIR) filters has long been subject to significant interest[1]-[7]. The McClellan-Parks (MP) algorithm (Remez exchange technique)[1] is one of the most widely used algorithms. However, many algorithms including the MP algorithm are limited to the design of linear phase FIR filters. Therefore, they cannot be used in the applications where a complex response with nonlinear phase is required, such as the group delay equalizer and chirp all-pass filters. In many cases, both the magnitude and phase response of the desired filter need to be arbitrary. Although some efforts have been reported to this ends[2][3], they don't seem to be easily applied to designing filters for data transmission.

Since their introduction by Lucky[8], fractionally-spaced FIR (FSF) filters (equalizers) have been widely used in data transmission devices such as voiceband modems. Lucky proposed the use of a $T/2$-spaced transversal equalizer with alternating fixed and adaptive tap gains, in order to simultaeously obtain spectral shaping and minimization of intersymbol interference (ISI). Here, T denotes the symbol interval. It has been shown that a fractionally-spaced structure has many advantages, such as insensitivity to the choice of sampling or timing phase, as compared to a conventional T-spaced linear receiver[9][10].

Despite extensive research on the design of digital filters and the application of FSF structures to adaptive equalization (*e.g.*, references in [11]), not much attention has been given to the design of FSF transmitter (Tx) and receiver (Rx) filters. Chevillat and Ungerboeck[12] considered the design of Tx/Rx FSF filters which can achieve optimum spectral concentration in the given bandwidth and lead to zero ISI when cascaded. They used an iterative projected gradient algorithm to obtain the filter coefficients, however, they did not include any channel distortion (*i.e.*, no equalization). Qureshi[11] discussed the use of an infinite-length FSF filter to design a linear optimum receiver.

When data is transmitted using conventional combined amplitude and phase modulation the use of an FSF passband equalizer can eliminate the need for a Rx filter (fixed compromise equalizer) and phase-splitter. When the channel is severely dispersed, however, it may still be desirable to have a Rx filter to reduce the number of taps of the required adaptive equalizer. In particular, when other interfering signals are present on the channel, the use of a receiver filter is indispensable. When the carrier frequency and/or passband is variable, for example in a V.Fast modem[13], implementing a passband equalizer becomes involved because of its dependency on the above parameters. Hence, using an adaptive baseband equalizer is preferred in many practical applications. In these cases, an FSF Rx filter can replace the combined function of the phase splitter, the matched filter and the compromise equalizer, in a conventional T-spaced receiver. Similarly, the use of an FSF filter can also be applied to a Tx filter which performs compromise pre-equalization in addition to spectral shaping. Sometimes, interpolation or decimation filtering needs to be added to the Tx or Rx filter, respectively. We will consider the design of such complex filters in this paper.

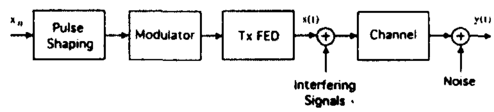
We assume data transmission using quadrature amplitude modulation (QAM) over a band-limited linear channel with a fair amount of amplitude and phase (or group delay) distortion. We also assume that the channel may have other signals. We first consider the design of an optimum FSF filter in the sense that the filter when used as a Rx filter minimizes the MSE between its output and the desired output. At the same time, it eliminates the need for a phase splitter. By properly modeling the unwanted signals as the additive noise components which are independent of each other, we can efficiently design such an FSF filter for a given tap size and delay constraints.

In many cases, the filter needs to strictly control the spectrum both outside and inside the passband. To meet this kind of specification, it may be desirable to optimize the filter coefficients in the frequency domain. For this purpose, we devise another method based on a weighted least mean squares (LMS) criterion. The weighted least squares optimization method is well-known and seems practical for the design of such filters [4]-[7]. It has been shown that the weighted least squares technique will produce an equiripple design, if a suitable weighting function is used[6][7]. However, there is no analytical method to obtain the least squares weighting function which will produce a minimax optimum solution. Therefore, we will use an iterative optimization technique to obtain a set of optimum filter coefficients. Here, we devise a new adaptation algorithm that leads to fast convergence and stability. Although most of the reported weighted LMS methods are limited to the design of a filter specified with either amplitude or phase constraint, this method can be applied to the design of both FSF and T-spaced FIR filters with complex constraints. Note that this design method can be sequentially combined to the first method for more complicated applications.

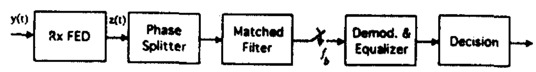
In Section 2, the linear transmission system and channel models used for designing Tx/Rx filters are briefly described. We explain the first design technique in Section 3. As an example, the proposed method is applied to the design of a Rx filter for a V.32 type of voiceband modem. Section 4 describes the second filter design algorithm. As an example, we design a decimation filter for a sigma-delta modulation ($\Sigma\Delta$M) A/D converter. Finally, we address the combined use of these two methods to obtain better results.

Ⅱ. System modeling

Let's consider a linear bandpass data transmission system which uses conventional QAM. A simple block diagram of the transmission system is shown in Figure 1.



(a)Transmitter and channel



(b)Receiver

Figure 1. A block diagram of linear data transmission system

The complex sequence of discrete input data symbols $\{x_n\}$ is converted into a baseband pulse waveform $p(t)$. The spectral shaping is performed so that the shaped signal can satisfy the Nyquist criterion. Then, this baseband signal is modulated using QAM, using a carrier frequency f_c Hz. An additional interpolation filter may be required before D/A conversion. The Tx analog front end device (FED) may further control the frequency spectrum of the D/A converted signal using analog filters before transmission. For $t>0$, the output signal $s(t)$ of the Tx can be expressed as

$$s(t) = \mathrm{Re}\{\sum_n x_n h_t(t-nT) e^{j2\pi f_c t}\} \tag{1}$$

where T is the symbol interval of the Tx input and $h_t(t)$ is the impulse response of the overall Tx expressed in terms of a complex baseband equivalent form. $h_t(t)$ can also be calculated using the inverse Fourier transform

$$h_t(t) = \mathcal{F}^{-1}\{P(f)\,B_t(f)\} \tag{2}$$

where $P(f)$ and $B_t(f)$ are the baseband frequency responses of the shaping pulse $p(t)$ and the Tx FED, respectively.

The Tx output is delivered to the Rx FED through a band-limited non-ideal channel. Although many kinds of impairments including nonlinear distortion exist on the channel, a simple linear channel model is assumed. That is, the channel characteristics are specified by the amplitude distortion and the group delay (or phase), $H_c(f)$, in the frequency domain. Other signals which can interfere with the transmitted signal $s(t)$ can also exist on the same channel. For example, the received signal in a V.22*bis* modem can interfere with the transmitted signal, and vice versa. A separate secondary channel can also interfere with the main channel. Assuming these interfering signals are uncorrelated with $s(t)$, they can be modeled as the sum of additive noise terms like white Gaussian noise. The total additive noise term $q(t)$ at time t is

$$q(t) = w(t) + \sum_i q_i(t) \tag{3}$$

where $w(t)$ is the additive white Gaussian noise (AWGN) and $q_i(t)$ denotes the ith interference signal term. Thus, the input signal $y(t)$ to the Rx can be expressed by

$$y(t) = s(t) \otimes h_c(t) + q(t), \tag{4}$$

where $h_c(t)$ is the impulse response of the channel and $\otimes$ denotes the convolution operator.

The received signal $y(t)$ is converted to digital samples at the rate of f_s Hz after being processed by the Rx FED. For ease of implementation, the sampling frequency f_s is usually set to a value which is an integer multiple of the symbol rate f_b, *i. e.*, $f_s = Mf_b$, where M is an integer gerater than or equal to 1. Additional processing, for example decimation filtering, may be required depending upon the A/D conversion scheme. Let $h_r(t)$ be the impulse response of the Rx FED including the A/D converter.

Assuming that the sampling time interval $T_s(=1/f_s)$ is normalized to 1, the output signal $z(k)$ of the Rx FED at time $t = kT_s$, $k = 0, 1, \cdots$, can be expressed by

$$z(k) = \sum_i x_i h_{eq}(k - iM) + n_{eq}(k). \tag{5}$$

Here, $h_{eq}(\cdot)$ and $n_{eq}(\cdot)$ are the impulse responses of the equivalent channel and noise calculated by

$$h_{eq}(k) = h_t(k) \otimes h_c(k) \otimes h_r(k) \tag{6}$$

$$n_{eq}(k) = q(k) \otimes h_r(k), \tag{7}$$

and $h_r(\cdot)$ is the impulse response of the Rx FED module.

In a conventional (T-spaced) linear receiver operating at the symbol rate f_b, the real passband FED output $z(k)$ should be converted to an analytic signal before being demodulated. Then, a matched filter is used to maximize the SNR at the symbol-rate sampling instant. For an ideal AWGN channel, in fact, a matched Rx filter, a symbol rate sampler with the correct sampling phase, and a memoryless detector together make the optimum linear receiver. Even though these symbol rate sampled sequences provide the sufficient statistics for estimation of the transmitted sequences $\{x_n\}$, the performance of the memoryless decoder deteriorates due to the presence of ISI on the channel. Therefore, an additional equalizer is required to combat the ISI before decoding.

Before demodulation, a phase splitting filter is used to generate an analytic signal. For simplicity of implementation, however, a simple low-pass filter (LPF) is often used for removing the $2\omega_c$ carrier modulation term after the demodulation process. However, if the carrier frequency is not sufficiently higher than the effective signal bandwidth, it may not be easy to implement such an LPF. When the channel causes severe distortion, compromise equalization may be desirable before adaptive equalization. Since the channel response is not symmetric with respect to the carrier frequency, the use of an LPF with real coefficients may not be appropriate for compensating such asymmetric attenuation at the band edges. Therefore, we consider the use of a Rx filter with complex coefficients.

Ⅲ. The MMSE FSF filter

By using the advantages of oversampling characteristics, the combined functionality of the phase splitter, the matched filter and the compromise (finite-length) equalizer in the T-spaced receiver can be realized by a single FSF filter with complex coefficients. For efficient implementation, we assume that the tap spacing T_o of an FSF filter is restricted to $T_o = \frac{L}{M} T = LT_s$, where L and M are relatively prime integers, and $L < M$. Let c $= \{c_0, c_1, c_{N-1}\}$ be the coefficients of an FSF filter with a finite tap size N. Then, for the given input signal $z(n)$, the output $\hat{x}_n$ of the finite-length FSF at time $t = nT (= nMT_s)$ is

$$\hat{x}_n = \hat{x}(nM) = \sum_{i=0}^{N-1} c_i z(nM - iL). \qquad (8)$$

The system model for such a Rx filter design can be simplified to an equivalent model as shown in Figure 2.

Now, for a given tap size N, we find a set of optimum filter coefficients which minimize the mean-squared error (MSE) defined by

$$\varepsilon(D) = \varepsilon\{|x_{n,D} - \hat{x}_n|^2\}$$
$$= \varepsilon\{|x_{n,D} - \sum_{i=0}^{N-1} c_i z(nM - iL)|^2\} \qquad (9)$$

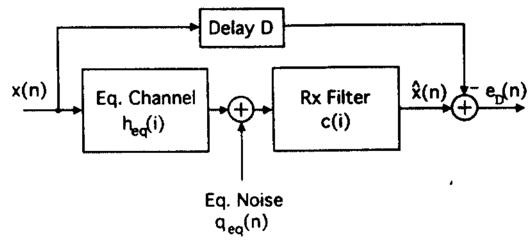


Figure 2. An equivalent model for the design of an MMSE Rx filter

where ε denotes the expectation operator, $0 \le D \le N-1$ represents the amount of the processing delay by the filter, *i.e.*, the main tap location, and $x_{n,D}$ is the input reference signal delayed by DT_o. By setting $\frac{\partial \varepsilon(D)}{\partial c_j} = 0$, for $j = 0, 1, \cdots, N-1$, we get

$$\varepsilon\{[x_{n,D} - \sum_{i=0}^{N-1} c_i z(nM - iL)] z^*(nM - jL)\} = 0 \qquad (10)$$

where * denotes the complex conjugate. Assuming that the input data sequences $\{x_n\}$ are uncorrelated and have unity power, (*i.e.*, $\varepsilon\{x_i x_j^*\} = \delta_{ij}$), the optimum MMSE FSF filter coefficients can be obtained by solving, for $j = 0, 1, \cdots, N-1$,

$$\sum_{i=0}^{N-1} c_i \{\sum_n h_{eq}(nM - iL) h_{eq}^*(nM - jL) + \varphi_n(iL - jL)\}$$
$$= h_{eq}^*(DL - jL) \qquad (11)$$

where $\varphi_n(\cdot)$ is the auto-correlation of the equivalent noise $n_{eq}(\cdot)$ given by

$$\varphi_n(jL) = \varepsilon\{n_{eq}(iL) n_{eq}^*(iL - jL)\}. \qquad (12)$$

Using matrix notation, a set of the optimum coefficients is calculated by

$$\mathrm{c} = \mathbf{R}^{-1}\mathbf{h}, \qquad (13)$$

where $\mathbf{R}$ is an $N \times N$ covariance matrix and $\mathbf{h}$ is an N-element cross correlation vector whose elements are

$$r_{ij} = \sum_n h_{eq}(nM - iL) h^*_{eq}(nM - jL) + \varphi_n(iL - jL),$$

$$h_i = h^*_{eq}(DL - iL), \qquad (14)$$

for $0 \le i, j \le N-1$.

It should be noted that, since the input signal has no spectral components in $f_b/2 < |f| \le M f_b/2$, many solutions for **c** can result in the same MSE. This implies that there is flexibility in determining the filter response in this out-of-passband frequency range without affecting the output signal or MSE. This propoerty of a fractionally-spaced structure can be used for other additional purposes.

The phase splitting filter requires a Hilbert transformer which may be very complicated for a wideband signal. In practice, however, the analytic signal can be obtained from the real passband signal by sufficiently attenuating the folded image components. As seen in the above equation (11), the attenuation characteristics of the MMSE FSF filter outside the passband can be controlled by the power and shape of the additive noise components : The larger the noise power, the greater the resulting attenuation. Therefore, the FSF filter can do the job of a phase splitter by properly characterizing the folded image signal as an additive noise signal term which needs to be suppressed. In this way, a single FSF can replace the tasks which can be fulfilled by three separate processing modules in the conventional receiver.

To illustrate the application of this idea, consider the design of a Rx filter for a V.32 type of voiceband modem. The transmitter modulates the input signal at a baud rate f_b of 2400 Hz using QAM with a carrier frequency f_c of 1800 Hz. Assume that the received signal is sampled at the rate f_s of 7200 Hz and is filtered by the Rx FSF filter at the same rate $f_o(=1/T_o)$ of 7200 Hz, *i.e.*, a $T/3$-spaced filter. Assume that the baseband pulse $p(t)$ is generated by a square-root raised cosine shaping filter with a roll-off factor $\alpha = 0.15$ and also both the Tx and Rx FED modules do not affect the signal passband characteristics. We model a channel in terms of the amplitude and the group delay in the frequency domain as shown in Figure 3. The additive noise terms are modeled by three signal components : A separate secondary channel signal acting as an interfering signal, the folded image signal used for phase splitting and the additive channel noise. Figure 4 depicts the frequency spectra of these three noise terms as well as their total equivalent noise shape. Here, the equivalent noise signal is shown when the power levels of the interfering signal, the channel noise and the image signal relative to the signal power are assumed to be -30 dB, -17 dB and -13 dB, respectively. Note that the fluctuation in the equivalent noise spectrum is due to truncation of the impulse response to a filter tap size.. For these given conditions, an FSF Rx filter with a tap size of $N = 24$ was designed. Figure 5 shows a plot of the frequency response of the designed filter $C(f)$ using the MMSE FSF algorithm, when the main tap location (*i.e.*, the output delay D) is set to 17. The real and imaginary impulse responses of this complex filter are shown as the solid and dotted lines in Figure 6. It can be seen that the image signal can be suppressed by nearly 40 dB. Notice that it may be required to place the main tap location in the second half in order to sufficiently compensate for the given group delay. The designed filter output has a total MSE of -35.8 dB, whereas the input has the ISI of -13.3 dB in addition to the channel noise.

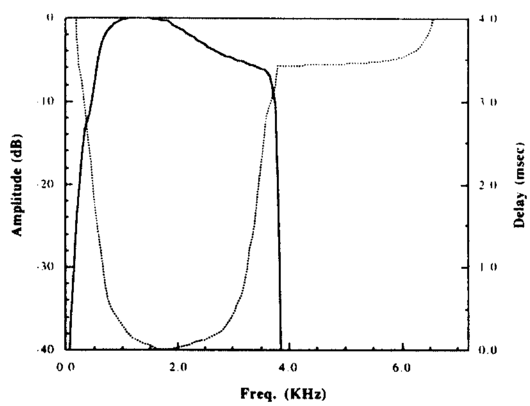


Figure 3. Frequency response of the channel $H_c(f)$

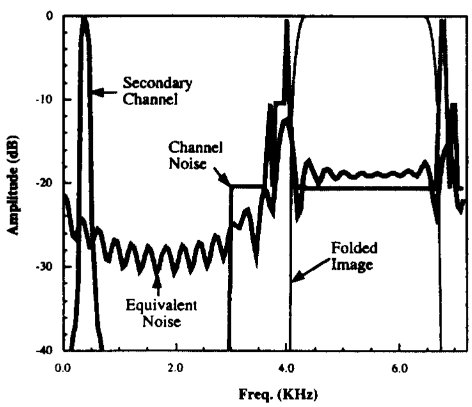


Figure 4. Equivalent noise characteristics

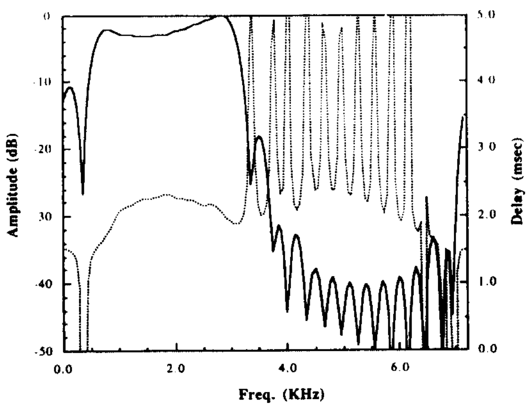


Figure 5. Frequency response of the filter $C(f)$

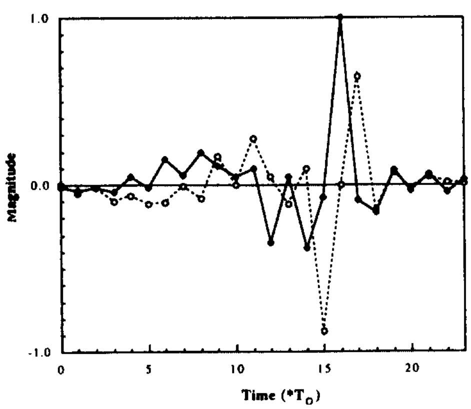


Figure 6. Impulse response of the filter $C(f)$

Ⅳ. The spectrum control filter : A weighted LMS filter

In practice, it is often required to explicitly control the spectrum outside the passband, including the side lobes. The MMSE FSF filter can be designed to control the leakage (attenuation) of the signal power outside the passband by specifying the noise shape. However, there is no predictable relationship between the attenuation characteristics and the input noise control at each frequency. Therefore, the above method may not be efficient for designing filters that guarantee the desired spectrum attenuation for each frequency range. To circumvent such an inefficiency, we propose another filter design method. This method performs an optimization in the frequency domain. We call this kind of a filter the spectrum control filter.

Let $\hat{H}(f)$ be the desired frequency response of the filter, which specifies the required minimum attenuation of the filtered output outside the passband as well as the desired passband characteristics. Now, consider the design of a filter which is optimal in the sense that the stopband(including the both side lobes) attenuation is maximized, while satisfying the minimum attenuation requirement of $\hat{H}(f)$ for all frequencies. In addition, the passband characteristics should be preserved with minimum degradation. To get the best results, we use an iterative algorithm based on an adaptively weighted LMS criterion in the frequency domain. To make the design error uniformly controlled, the weighting factor $W(f)$ for each frequency f needs to be adaptively adjusted depending on the design error for the next iteration. We devise a new adaptation algorithm for updating the weight factor, which can provide a stable iteration process with fast convergence.

For a given required filter response $\hat{H}(f)$, we first set up the design model in terms of a target frequency response $\tilde{H}(f)$ and an admissible maximum error bound $E(f)$. $\tilde{H}(f)$ and $E(f)$ outside the passband can be initialized such that, if the difference between the response of the designed filter and $\tilde{H}(f)$ is less than the bound $E(f)$, the designed filter can guarantee the required attenuation specified dy $\hat{H}(f)$. To give the filter design process the maximum flexibility, the target amplitude response $|\tilde{H}(f)|$ outside the passband is simply set to sero (*i.e.*, infinite attenuation). Since the group delay (or phase) characteristics outside the passband do not affect the filter attenuation performance, it can be set to any arbitrary value. The maximum error bound $E(f)$ is set to a value equal to the amplitude of $\hat{H}(f)$. This implies that, if the response error between the designed filter and the target response $|\tilde{H}(f)|$ is less than $E(f)$ for each frequency f, the amplitude of the designed filter will be less than $|\hat{H}(f)|$.

On the other hand, it is desirable for the characteristics of the passband to be preserved as close as to those of the desired response. The target response $\tilde{H}(f)$ in the passband is set to that of $\hat{H}(f)$. The error bound is set to a value corresponding to the maximum acceptable variation (or ripple) δh. In some cases (*e.g.*, a raised cosine shaping filter with a roll-off factor larger than 0.15), the side lobe (or transition) characteristics may need to be preserved as well. In this case, $\tilde{H}(f)$ and $E(f)$ in the corresponding frequency range can be specified similar to those in the passband.

We can therefore initialize the design specification in a simple way,

$$\tilde{H}(f) = \begin{cases} \hat{H}(f), & f_L \le f \le f_U \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

$$E(f) = \begin{cases} \delta h, & f_L \le f \le f_U \\ |\hat{H}(f)|, & \text{otherwise,} \end{cases} \tag{16}$$

where f_L and f_U are the lower and upper frequency of the effective passband. Therefore, provided that the absolute error between the newly designed filter and $\tilde{H}(f)$ is uniformly less than the error bound, the designed filter always guarantees the required amplitude attenuation outside the passband. At the same time, it preserves the desired passband characteristics with a maximum error δh.

The spectrum control filter can be designed by finding a set of the coefficients which minimize the weighted MSE J defined by

$$J = \int_0^{f_o} W(f)\,|\tilde{H}(f) - \hat{C}(f)|^2\, df, \tag{17}$$

where $\hat{C}(f)$ is the frequency response of the new filter to be designed and $W(f)$ is the weighting function to be updated for each time. Representing $\hat{C}(f)$ in terms of its coefficients $\{\hat{c}_i\}$, the weighted MSE J is

$$J = \int_0^{f_o} W(f)\,|\tilde{H}(f) - \sum_{i=0}^{N-1} \hat{c}_i e^{-j2\pi i f/f_o}|^2\, df. \tag{18}$$

Differentiating J with respective to $\hat{c}_j$, $j=0, 1, \cdots, N-1$, and setting it to zero, we have

$$h_w(j) = \sum_{i=0}^{N-1} \hat{c}_i\, w(j-i) \tag{19}$$

where $h_w(\cdot)$ and $w(\cdot)$ are the inverse discrete Fourier transform of $W(f)\tilde{H}(f)$ and $W(f)$, respectively. The optimum filter coefficients $\{\hat{c}_i\}$ are obtained by solving N simultaneous equations given by (19).

In order to get a fast converging solution, the weighting factor $W(f)$ should be properly updated for the next iteration depending upon the design error $e(f)$ between the designed filter and the target : If the error $e(f)$ at frequency f becomes larger, the weight $W(f)$ should be increased for the next iteration. Let $\gamma_k(f)$ be the updating factor used in the k-th iteration. For fast convergence, we use the multiplicative, rater than the additive form, for adapting W_k. The weighting factor is adjusted by

$$W_{k+1}(f) = \gamma_k(f)\, W_k(f) \tag{20}$$

for the $(k+1)$-th iteration. A practical and simple way can be to generate the updating factor in proportion to the power of the error normalized by the error bound. In order to prevent undesirable conditions, we generate the updating factor

$$\gamma_k(f) = 1 + \Delta \bar{e}_k^2(f) \tag{21}$$

where Δ is the step size to control the adaptation speed and $\bar{e}_k(f)$ denotes the design error normalized by the error bound $E(f)$ after the k-th iteration,

$$\bar{e}_k(f) = \frac{|\tilde{H}(f) - \hat{C}(f)|}{E(f)} \tag{22}$$

Since $\gamma(f)$ is always larger than 1, it is necessary to normalize the weighting factor with respect to its maximum weight for each time for stabile iteration. It may be desirable to limit the lower bound for $W(f)$ to avoid any ill conditions. Thus, the weighting factor for the $(k+1)$-th iteration is updated by

$$W_{k+1}(f) = \frac{W_{k+1}(f)}{\max\{W_{k+1}(f)\}}, \tag{23}$$

$$W_{k+1}(f) = W_{min}, \text{ if } W_{k+1}(f) < W_{min},$$

where W_{min} is the minimum weighting factor. The iteration can be started with any arbitrary initial value, say $W_0(f) = 1.0$, for all f. However, to reduce the number of iterations, the weighting factor $W_0(f)$ can be initialized by setting it equal to the inverse of the error bound $E(f)$. The iteration can be terminated if the maximum error of $\bar{e}_k(f)$ is less than a desired threshold value.

This method can be easily applied to the design of various kinds of filters with real or complex-coefficients, including multi-rate FIR filters such as interpolation and decimation filters. As an application of this method, we consider the design of a real-coefficient decimation filter for the Rx FED, where $\Sigma\Delta$M based oversampling A/D converters are widely used. Many $\Sigma\Delta$M A/D converters use simple comb filters for the decimation process, where the output needs to be generated at a sampling rate higher than at least 4 times the input signal bandwidth. If the $\Sigma\Delta$M coder is of order K, then the use of a comb filter with an order of $K+1$ would be sufficient[14]. The transfer function of a $(K+1)$-th order comb filter is

$$\left(\sum_{i=0}^{K-1} z^{-i}\right)^{K+1} = \left(\frac{1}{R}\ \frac{1-z^{-R}}{1-z^{-1}}\right)^{K+1}, \tag{24}$$

where R is the decimation factor. An additional decimation filter is required to generate the final output at a desired sampling rate. In this case, the decimation filter may be required to compensate the passband distortion caused by the comb filters.

Assume that the Rx FED generates the output samples at a rate of $6f_b$ using a second order $\Sigma\Delta$M A/D converter and $f_b = 3000$ Hz. Consider the design of an FIR decimation filter at a ratio of 2 to 1 to generate a $T/3$-spaced passband sample which is modulated by a carrier frequency of 1800 Hz. Assuming that the signal has a 15% excess bandwidth, the cutoff frequency of the passband will be 3.48 KHz. When the decimation ratio R of the third-order comb filter is 128, the attenuation of the passband will be up to 1.6 dB. Assume also that the decimation filter needs to have a minimum of 48 dB attenuation in the image alias band and to compensate the passband attenuation due to comb decimation filtering. The design model can be specified as a dotted line shown in Figure 7. Here, the transition band is a don't care region and is modeled by a third-order curve. Figure 7 and 8 respectively show a plot of the frequency and impulse response of the filter designed by using the weighted LMS algorithm, when the filter tap size is 22 and the main tap location is located at the 11th tap. The designed filter provides the desired passband response with a variation of less than 0.13 dB, while having a minimum stopband attenuation of 48 dB. Note that, although it is not explicitly described as in the first design method, this method may also result in the designed filters having significantly different responses depending on the main tap location.

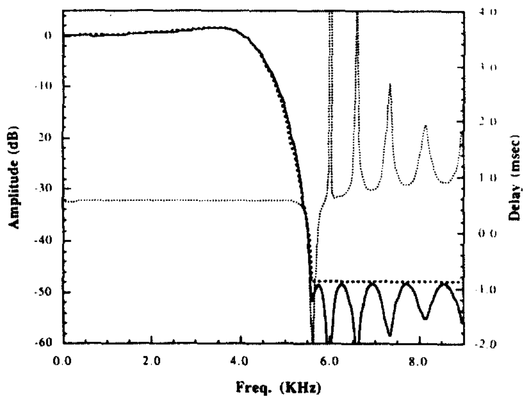


Figure 7. Frequency response of the decimation filter $\hat{C}(f)$

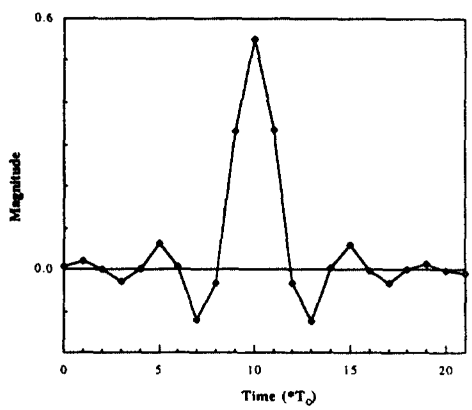


Figure 8. Impulse response of the decimation filter $\hat{C}(f)$

When this method is applied to design an FSF filter requiring complex functions such as compromise equalization, there may be difficulty in specifying $\hat{H}(f)$ in the passband. In this case, the two design methods presented in this paper can be nicely combined. The first MMSE method is applied to obtain a filter with a desirable passband characteristics. Then, the second weighting LMS method is applied to provide sufficient attenuation outside the passband, while retaining the passband characteristics obtained by the first method.

For example, consider the design of a Tx shaping filter. In some applications, a Tx filter may be desirable not only to control the stopband attenuation, but also to pre-compensate the presumed channel distortion. To design such a Tx filter, it is generally necessary to simultaneously optimize for the both constraints. This method, however, may not be practical, because there may not exist a solution satisfying the given constraints and the calculation may become extremely complicated. Instead, we sequentially apply the above two methods for the design of such an FSF Tx filter in a sub-optimum sense.

Assume that the frequency response of the passband filter $C(f)$ designed by the former MMSE method is given in Figure 9, where the required out-of-band spectrum control $R(f)$ is depicted by a dotted line. To obtain a filter satisfying this spectral attenuation outside the passband, the spectrum control filter is designed based on $C(f)$. The desired response $\hat{H}(f)$ is set equal to $C(f)$ in the passband and $R(f)$ outside the passband. Then, the target response $\tilde{H}(f)$ and the error bound $E(f)$ are specified according to (15) and (16), respectively. When δh is set to 0.15 dB, the amplitude response of the new filter $\hat{C}(f)$ designed using the second method is shown as a solid line in Figure 10, where $R(f)$ is replotted for verification. Note that the desired attenuation characteristics of the filter $\hat{C}(f)$ can be obtained by

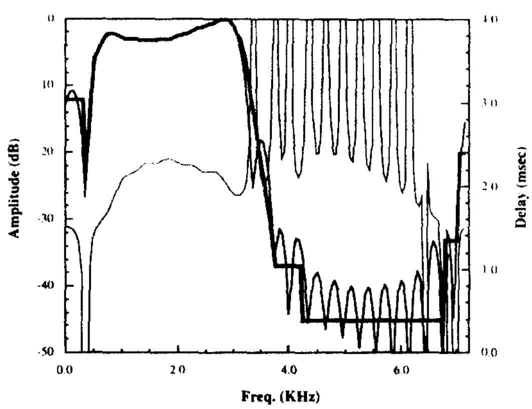


Figure 9. Specification for desired spectrum control

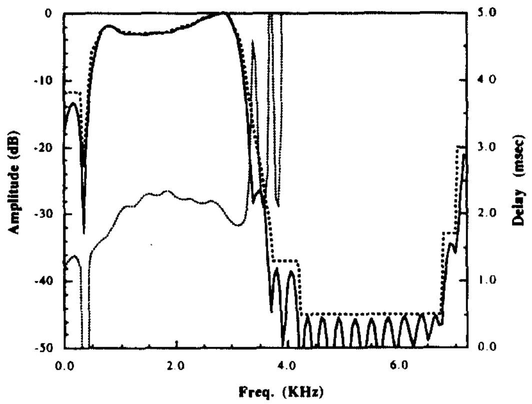


Figure 10. Frequency response of the filter $\hat{C}(f)$

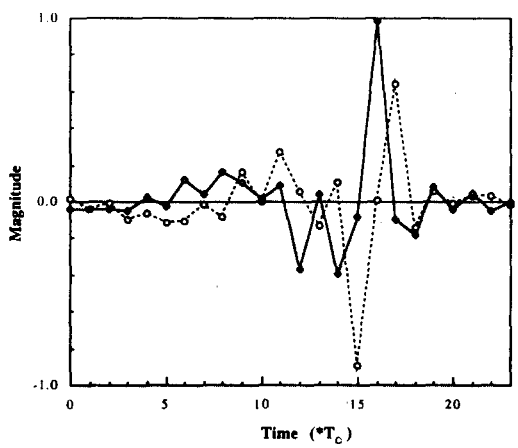


Figure 11. Impulse response of the filter $\hat{C}(f)$

sacrificing a fraction of a dB in the output MSE. The impulse response of new filter $\hat{C}(f)$ is depicted in Figure 11. This example illustrates that a combination of the two methods can be nicely applied to a filter design with complicated constraints.

V. Conclusion

Data transmission systems require many different kinds of digital FIR filters with complex responses. We have proposed two general-purpose design techniques for the design of such filters. The first method can be applied to design an FSF filter requiring a complex frequency response. The second method uses a weighted least squares technique to design an FIR filter whose response is explicitly specified in the frequency domain. We have devised a new fast adaptation algorithm for updating the weighting function. The proposed methods result in a significant saving in the filter tap size compared to conventional methods. It has been shown that the two methods can be effectively combined for better design results.

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