

## A design method for optical fiber filter of lattice structure

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### 격자형 광파이버필터의 설계에 관한 연구

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#### 요 약

디지털 신호처리기술 및 종래의 아날로그 신호처리기술로는 저주파수의 신호를 처리하는데는 아주 효과적이지만, 주파수 대역폭이 GHz단위일 경우에는 많은 손실이 있기 때문에 遲延媒體로서 광파이버가 많이 사용된다. 광파이버는 저손실 그리고 극히 작은 지연도 정확하게 얻을 수 있기 때문에 채널분리 필터 등과 같은 고주파, 광대역신호의 고속처리 특히 수십GHz 단위의 채널을 광FDM다중화하는 분야등에 많은 주목을 받고 있다.

본 논문에서는 광파이버를 이용한 지연소자와 방향성결합기(directional coupler)를 구성 단위로 하고 코히런트 광원을 이용한 격자형 광파이버필터에 대하여 논한다. 코히런트 광원을 이용한 광파이버 필터는, 그 기본 구성요소인 방향성결합기의 특성 때문에

1) 입력신호는 광강도에 의해 처리되기 때문에 방향성결합기의 결합계수(=a)는 0과 1 사이의 값만 취할 수 있다.

2) 광신호의 전계강도를 E라 했을때 그 분기점에서 신호광은  $j\sqrt{a}E$ 와  $j\sqrt{1-a}E$ 로 분배된다.

본 논문에서는 방향성 결합기의 제약조건을 고려하고, 간단한 구성 및 설계를 위하여 가산소자와 분기소자가 동일한 결합계수를 가지는 격자형 광파이버필터를 제안하였다. 설계방침으로는 주어진 전달함수에 대해 광신호의 Energy를 최대한 유효하게 사용하는 설계법으로 그때의 설계공식 및 실험조건등을 유도하였다.

#### Abstract

The propagation and delay properties in optical fiber are particularly attractive because digital signal processing and conventional analog signal processing techniques such as those using surface acoustic wave devices are limited in their usefulness for signal bandwidth exceeding one or two GHz, although they are very effective at lower frequencies.

Since an accurate, low loss and short time delay elements can be obtained by using such an optical fiber, optical signal processing has attracted much attention for high speed and broad-band signal processing in particular channel separation filtering for optical FDM signals.

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In this paper, we consider a coherent optical lattice filter, which uses coherent light sources and consists of directional couplers and optical fiber delay elements. The optical fiber filters are more restricted than the usual digital filters. The reasons are as follows.

- 1) the coupling coefficients of directional couplers are restricted to the number between 0 and 1.
- 2) optical signal  $E$  (complex amplitude) is divided into  $j\sqrt{a}E$  and  $j\sqrt{1-a}E$  at the directional coupler.

Considering these restrictions and in this case all the coupling coefficients of summing and branching elements are set to be equal, we have given design formulae for optical lattice filter, which make the best use of optical signal energy.

## I. INTRODUCTION

The low loss and large time-bandwidth product of optical fibers, together with the advances in technologies manufacturing various kinds of optical devices, provide attractive tools for high speed and broadband signal processing [1]. Since the optical fiber has characteristics of an accurate and short-time delay element, it is expected that the high frequency signal processing can be realized. Otherwise, it has been hard to realize by utilizing conventional techniques. One of the major applications of such optical fiber signal processing may be channel separating filters used in the optical FDM system [2], in which the channel spacing will be of several GHz order. Conventional filters such as SAW filters are sure to fail to work in such high frequency.

The author has reported a basic design method for optical fiber filter with incoherent operation [3]. Although the incoherent system is easily tractable, the phase of optical carrier is discarded and only its intensity is retained. Therefore, the transfer function of incoherent optical fiber filter is strongly restricted since multiplier coefficients can take only positive values. In a coherent system, we can retain the phase of optical carrier and can use both positive and negative values as the multiplier coefficients. Although technical problems such that stability of source, control techniques of narrow spectrum and development of heterodyne or homodyne detection techniques etc. remain to be solved, they are almost over-

come now with the advances in device manufacturing technology. Recently, NTT laboratory in JAPAN reported coherent optical transversal filter with tapped delay line structure which uses coherent interference [4]. This filter operates as an HPF (high pass filter) in the frequency range between  $f_0$  and  $f_0 + 2.5\text{GHz}$ . And it operates on as an LPF (low pass filter) in the frequency between  $f_0$  and  $f_0 + 10\text{GHz}$  by changing the coefficients of directional couplers.

In this paper, we consider coherent optical fiber filter, which use coherent light sources and consist of directional couplers and optical fiber delay elements. This optical fiber filter is still different from the usual digital filter, since in the optical fiber filter [5].

- 1) the coupling coefficients of directional couplers are restricted to the number between 0 and 1,
- 2) optical signal  $E$  (complex amplitude) is divided into  $j\sqrt{a}E$  and  $\sqrt{1-a}E$  at the directional coupler, where  $a$  represents the coupling coefficient of directional coupler.

Considering these restrictions, we have reported design method of optical fiber filters in direct form [6],[7]. In this paper, we propose an optical fiber filter of lattice structure, which makes best use of optical signal energy and whose design is an optimal one for all the coupling coefficients of summing and branching elements to be set as 1/2. As summing and branching elements are set to be equal, we can easily realize the given transfer function by determining the coefficients of multipliers. Besides in this case we

can gain effective and more simple design method.

## II. PRELIMINARIES

In this section, we briefly describe the basic concept of optical signal processing and introduce some fundamental elements, such as optical fiber delay, summing, branching and multiplier elements, that are needed to construct optical fiber filter.

### 1. Coherent optical signal processing

Coherent light is a light whose frequency and phase as well as its amplitude are very stable and can be used to convey phase information and amplitude information. These informations are detected by optical heterodyne or homodyne detector that is very sensible. An example of optical fiber filtering system making use of coherent light is shown in Fig.1 [8]. In Fig.1, a frequency-stabilised HoNe laser is used as a source and produces a coherent optical carrier for the AM modulator. The AM modulated optical carrier, which is described in section II.2 in detail, is used for the input to optical fiber filter. After the processing by

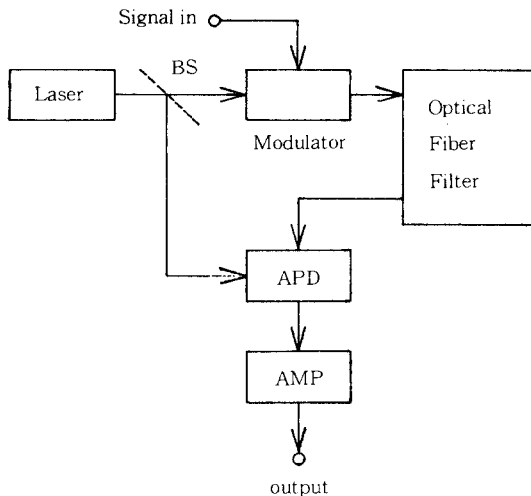


Fig 1. Filtering system making use of coherent light

optical fiber filter, the output signal is obtained by homodyne detector followed by an avalanche photodetector. The local carrier for homodyne detection is obtained by splitting off a portion of the laser signal prior to modulation.

### 2. Input signal to optical fiber filter

Let  $s(t)$  be the signal to be processed and  $\cos\omega t$  the coherent optical carrier. Then

$$E(t) = s(t)\cos\omega t$$

is used as the input to the optical fiber filter. One method to get  $E(t)$  in principle is shown in Fig.2. As is seen in Fig.2, the AM modulated signal

$$\hat{E}(t) = [1 + s(t)]\cos\omega t$$

is used to produce  $E(t)$  by removing the optical carrier  $\cos\omega t$  from  $\hat{E}(t)$ . This is done by adding the reversed phase optical carrier  $-\cos\omega t$ , obtained through variable phase shifter, to the AM modulated signal  $\hat{E}(t)$ .

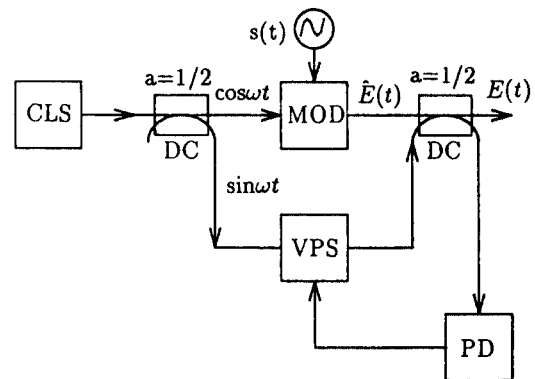


Fig 2. Input signal to optical fiber filter

### 3. Basic elements for optical fiber filter

#### 3.1 Unit delay element

Optical fiber provides a precise time delay that can be employed as a unit time delay element. For example, optical fiber with refractive index 1.5 provides a propagation delay of 5 ns/m. Unit

delay element using optical fiber is shown in Fig.3, where  $z^{-1}$  represents a unit time delay. It is noted that the actual delay time is decided depending on the bandwidth of the signal  $s(t)$  to be processed. In this paper, we concentrate ourselves to the case where the time delays are the same for all sections, but this restriction would not be necessary in general. In Fig.3, the relationship between input and output is given by

$$E_2 = E_1 z^{-1} \quad (1)$$

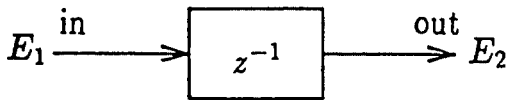


Fig 3. Unit delay element

### 3.2 Summing and branching element

A directional coupler is shown in Fig.4. The input-output relationship of a directional coupler is described by a  $2 \times 2$  transfer matrix [5] as

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} \sqrt{1-a} & j\sqrt{a} \\ j\sqrt{a} & \sqrt{1-a} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \quad (2)$$

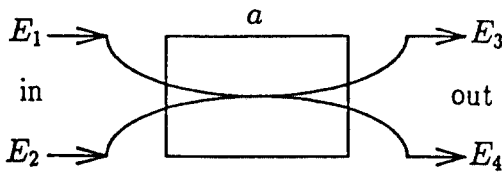


Fig 4. Directional coupler

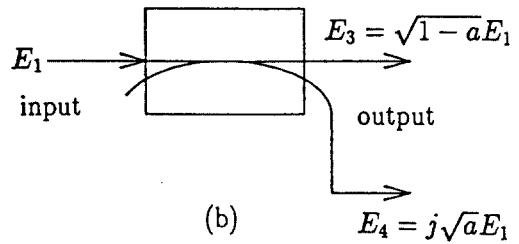
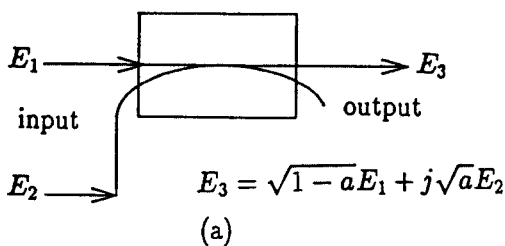
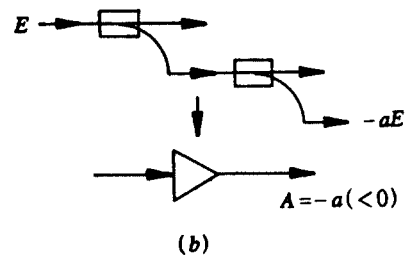
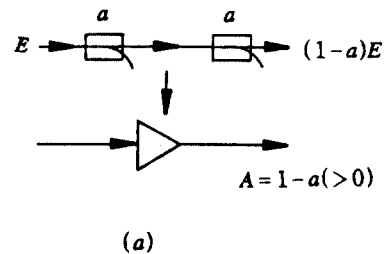


Fig 5. Summing and branching element

where  $a$  is the coupling coefficient, and  $(E_1, E_2)$  and  $(E_3, E_4)$  are input and output complex amplitude, respectively. Using this directional coupler, we can construct summing and branching element as shown in Fig.5.

### 3.3 Multiplier

We can construct a multiplier by cascading two directional couplers in tandem as shown in Fig.6. Note that multiplier coefficients are restricted to the number between  $-1$  and  $1$ . For simplicity, we use the symbol shown in Fig.6(c) for the multiplier of Fig.6(a) or (b). In the subsequent sections, we shall discuss the optical fiber filter of lattice structure, which is constructed using the basic elements described in this section.



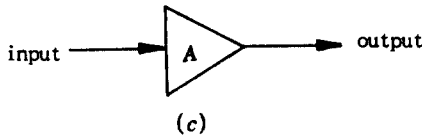


Fig 6. Multiplier element

### III. DESIGN METHOD FOR OTICAL FIBER OF LATTICE STRUCTURE

Compared to the other structures, lattice structures have some advantages, such as modularity, regularity, ease of implementation and good sensitivity [5]. In this section we propose a lattice structure and show how to design this optical fiber filter.

In Fig.7 is shown the basic building block for optical fiber filter of lattice structure. It is noted that all the coupling coefficients of summing and branching elements in Fig.7 are set to be equal (= 1/2) for simple design and structure. Then it is easy to see that the relationship between input and output in Fig.7 is given by

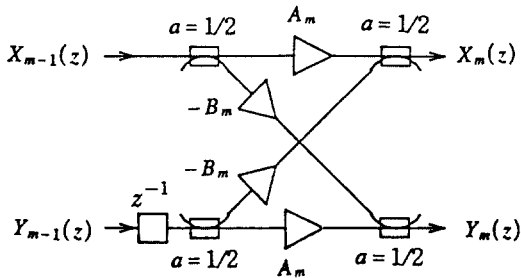


Fig 7. Basic building block

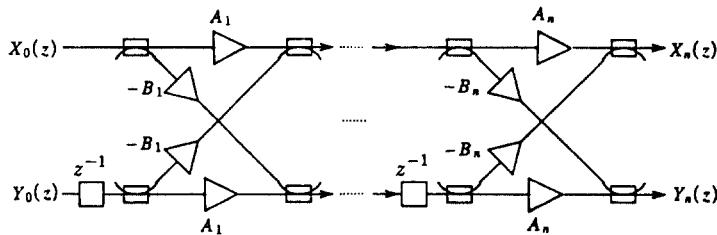


Fig 8. Optical fiber filter of lattice structure

$$\begin{bmatrix} X_m(z) \\ Y_m(z) \end{bmatrix} = 2^{-1} B_m H_m \begin{bmatrix} X_{m-1}(z) \\ Y_{m-1}(z) \end{bmatrix} \quad (3)$$

where

$$H_m \equiv \begin{bmatrix} 1 & k_m \\ k_m & z^{-1} \end{bmatrix} \quad (4)$$

$$k_m = \frac{A_m}{B_m}$$

Notes that  $k_m (m=1,2,\dots,n)$  can take arbitrary real value by choosing  $A_m$  and  $B_m$  properly.

The optical fiber filter of lattice structure is shown in Fig.8. As is easily seen from Fig.8, the relationship between input and output is given by

$$\begin{bmatrix} X_n(z) \\ Y_n(z) \end{bmatrix} = \left\{ \prod_{i=1}^n \frac{B_i}{2^i} \right\} H_n \cdot H_{n-1} \cdots H_1 \begin{bmatrix} X_0(z) \\ Y_0(z) \end{bmatrix} \quad (5)$$

For convenience, we define

$$C_m(z) \equiv \frac{2^m}{\prod_{i=1}^m B_i} \frac{X_m(z)}{X_0(z)} \quad (6)$$

$$D_m(z) \equiv \frac{2^m}{\prod_{i=1}^m B_i} \frac{Y_m(z)}{Y_0(z)}$$

Noting that  $X_0(z) = Y_0(z)$  and Eq.(3), we can rewrite Eq.(5) as follows :

$$\begin{bmatrix} C_m(z) \\ D_m(z) \end{bmatrix} = H_m \begin{bmatrix} C_{m-1}(z) \\ D_{m-1}(z) \end{bmatrix} \quad (7)$$

$$C_0(z) = D_0(z) = 1 \quad (8)$$

Then we have three lemmas.

Lemma 1 : In general,  $C_m(z)$  and  $D_m(z)$  are m-th order polynomials in  $z^{-1}$  as given as

$$C_m(z) = \sum_{i=0}^n c_{m,i} z^{-i}, \quad c_{m,0} = 1 \quad (8)$$

$$= \sum_{i=0}^{m-1} c_{m-1,i} z^{-i} + k_m z^{-1} \sum_{i=0}^{m-1} c_{m-1,m-1-i} z^{-i}$$

$$D_m(z) = \sum_{i=0}^n d_{m,i} z^{-i}, \quad d_{m,m} = 1$$

$$= c_{m-1,0} + \sum_{i=1}^{m-1} (c_{m-1,i} + k_m c_{m-1,m-1-i}) z^{-i} + k_m c_{m-1,0} z^{-m}$$

Proof: Immediate consequence of Eq.(7).

Lemma 2: The relation between  $C_m(z)$  and  $D_m(z)$  is given by

$$C_m(z) = z^{-m} D_m(z^{-1}), \quad m = 0, 1, \dots, n \quad (9)$$

Proof: We prove Eq.(9) by induction.

1. Eq.(9) is obvious for  $m = 0$ .
2. Assume that Eq.(9) is true for  $m = p-1$ . For  $m = p$ , we have from Eq.(7) and Eq.(4) that

$$\begin{aligned} z^{-p} D_p(z^{-1}) &= z^{-p} \{k_p C_{p-1}(z^{-1}) + z D_{p-1}(z^{-1})\} \\ &= k_p z^{-p} C_{p-1}(z^{-1}) + z^{-(p-1)} D_{p-1}(z^{-1}) \quad (10) \end{aligned}$$

Apply Eq.(9) with  $m = p-1$  to  $C_{p-1}(z^{-1})$  and  $z^{-(p-1)} D_{p-1}(z^{-1})$  in Eq.(10) to give:

$$\begin{aligned} z^{-p} D_p(z^{-1}) &= k_p z^{-1} z^{-(p-1)} C_{p-1}(z^{-1}) + C_{p-1}(z) \\ &= k_p z^{-1} D_{p-1}(z) + C_{p-1}(z) \\ &= C_p(z) \end{aligned}$$

Hence Eq.(9) is also true for  $m = p$ .

Lemma 3.: The relation between  $C_m(z)$  and  $C_{m-1}(z)$  is given by

$$\begin{aligned} C_{m,0} &= C_{m-1,0} = 1 \\ C_{m,i} &= c_{m-1,i} + k_m c_{m-1,m-i} \\ C_{m,m} &= k_m c_{m-1,0} \end{aligned} \quad (11)$$

Proof: Form Eq.(7),  $C_m(z)$  is given by

$$C_m(z) = C_{m-1}(z) + k_m z^{-1} D_{m-1}(z)$$

Applying Eq.(9) to  $D_{m-1}(z)$  in the above expression, we get

$$\sum_{i=0}^m c_{m,i} z^{-i} = C_{m-1}(z) + k_m z^{-1} z^{-(m-1)} C_{m-1}(z^{-1})$$

Noting that  $c_{m,0} = 1$  and comparing each coefficient of both hand sides, we immediately get Eq.(11).

From above lemmas, we have following theorem.

Theorem 1: For the transfer function  $C_m(z)$  given by Eq.(8),  $k_m (m = n, n-1, \dots, 1)$  is given by the following recursive formulae.

$$k_m = c_{m,m} \quad (12)$$

where

$$\begin{aligned} c_{m-1,0} &= c_{m,0} = 1 \\ c_{m-1,i} &= \frac{c_{m,i} - k_m c_{m,m-i}}{1 - k_m^2} \end{aligned}$$

Proof: Since the first and second equations in Eq.(12) are obvious from Eq.(11), we show the third. By replacing  $i$  by  $m-i$  in the second equation of Eq.(11), we get

$$c_{m,m-i} = c_{m-1,m-i} + k_m c_{m-1,i}$$

Combining this equation with the second one of Eq.(11) and solving them for  $c_{m-1,i}$ , we immediately get the third equation of Eq.(12).

Now let us assume that a desired  $n$ th-order transfer function is given as follows:

$$H_d(z) = h_0 + h_1 z^{-1} + \dots + h_n z^{-n} \quad (13)$$

It is clear from Theorem 1 that if and only if  $h_0 \equiv 1$ , we can set  $C_n(z) = H_d(z)$  and determine  $k_m = \frac{h_m}{B_m} (m = n, n-1, \dots, 1)$  as far as  $k_m \neq \pm 1$  or equivalently  $c_{m,m} \neq \pm 1 (m = n, n-1, \dots, 1)$ . So, hereafter

we assume  $h_0 \equiv 1$  in Eq.(13).

We first consider the case in which  $c_{m,m} \neq \pm 1$  ( $m=n, n-1, \dots, 1$ ) and investigate how to realize a constant ( $=\alpha$ ) times of a given transfer function by an optical fiber filter of lattice structure with  $\alpha$  as large as possible in order to make good use of the optical signal energy. In this case

$$C_n(z) = H_d(z)$$

and from Eq.(5), the whole transfer function  $X_n(z)/X_0(z)$  of the optical lattice filter shown in Fig.8 is given by

$$\frac{X_n(z)}{X_0(z)} = \frac{\prod_{i=1}^n B_i}{2^n} C_n(z) = \frac{\prod_{i=1}^n B_i}{2^n} H_d(z) \quad (14)$$

It is obvious from Eq.(14) that  $\alpha (= \prod_{i=1}^n (B_i/2))$  is maximized by maximizing each  $|B_i|$ . Thus the multiplier coefficients  $A_m$  and  $B_m$  ( $m=n, n-1, \dots, 1$ ) of lattice structure that maximize  $\alpha$  are given as follows :

[Decision of multiplier coefficients]

$$A_m = c_{m,m}, B_m = 1, \text{ for } |c_{m,m}| < 1 \quad (15)$$

$$A_m = \frac{c_{m,m}}{|c_{m,m}|}, B_m = \frac{1}{|c_{m,m}|}, \text{ for } |c_{m,m}| > 1$$

Eq.(15) is easily obtained by finding  $B_m$  that has maximum magnitude under the condition that

$$-1 \leq A_m, B_m \leq 1 \text{ and } A_m/B_m = c_{m,m} (=k_m \neq \pm 1)$$

Apparently, the maximum value of  $\alpha$  is given by

$$\alpha_{\max} = \prod_{i=1}^n (B_i/2)$$

Thus the design procedure for optical fiber filter of lattice structure in case of  $c_{m,m} \neq \pm 1$  ( $m=n, n-1, \dots, 1$ ) is summarized as follows :

Design procedure :

step 1 : Let  $c_{n,i} = h_i, i = 1, 2, \dots, n,$

Set  $m := n.$

step 2 : Determine  $k_m$  by Eq.(12).

Also determine  $c_{m-1,i} (i=1, 2, \dots, m-1)$  by Eq.(12).

Set  $m := m-1.$

Step 3 : Repeat step 2 until  $m=1.$

step 4 : Determine  $A_i$  and  $B_i (i=1, 2, \dots, n)$  by Eq.(15).

Example : Let the desired transfer function be

$$H_d(z) = 1 - z^{-1} - 3z^{-2}.$$

From Theorem 1, we have

$$k_2 = -3, c_{1,0} = 1, c_{1,1} = 1/2.$$

And from step 2 of the design procedure, we also have

$$k_1 = 1/2.$$

Thus from step 4, multiplier coefficients are obtained as follows :

$$A_2 = -3, B_2 = 1/3, A_1 = 1/2, B_1 = 1.$$

In this example the maximum value of  $\alpha$  is given by

$$\alpha_{\max} = 1/12.$$

We shall conclude this subsection by mentioning little the case in which  $c_{m,m} = \pm 1$  for some  $m$ . In this case, as is easily seen from Eq.(11), either

$$c_{m,i} = c_{m, m-1} (i=0, 1, \dots, m), \text{ if } c_{m,m} = 1$$

$$c_{m,i} = -c_{m, m-1} (i=0, 1, \dots, m), \text{ if } c_{m,m} = -1$$

must be satisfied. Otherwise, it is concluded that the given transfer function  $\alpha H_d(z)$  cannot be realized by the optical fiber filter of lattice structure.

#### IV. CONCLUSION

The demands for very high speed and broadband signal processings, such as radar, moving picture signal processing and optical FDM signal processings, have been increasing continuously. In this paper, considering these applications, we have shown an optimal design method for coherent optical fiber filter of lattice structure, which

use coherent light sources, directional couplers and optical fiber delay elements.

Differing from the digital filters in which there is no essential restriction on the filter coefficient, the optical fiber filter considered in this paper have some physical restrictions such that the coupling coefficient  $\alpha$  of directional coupler is restricted to the value between 0 and 1, optical signal  $E$  (complex amplitude) is divided into  $j\sqrt{\alpha}E$  and  $\sqrt{1-\alpha}E$  at the directional coupler, etc. Considering these restrictions, we have given the realizability condition and shown how to realize a constant ( $=\alpha$ ) times of a given transfer function with  $\alpha$  as large as possible in order make good use of the optical signal energy.

As for the topics related to this paper, we have been investigating : (1) optimal designs for other types of optical fiber filters such as lattice which the coefficient of coupler is not to be set (2) comparison among the various structures (3) experimental results using this lattice model.

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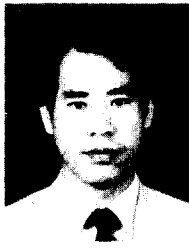
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