論 文

Digital Filter Design using the Symbol Pulse Invariant Transformation

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ABSTRACT

In general, when IIR digital filters are designed from analog filters, the bilinear transformation and the impulse invariant transformation are commonly used. It is known, however, that high frequency response of digital filters designed by these transformations can not be well approximated to the sampled analog signals.

In this paper, the symbol pulse invariant transformation is analyzed theoretically so that the symbol pulse invariant transformation which was originally applicated to a rectangular pulse is newly applied to double rate pulse signals and generic shape pulse signals. Also, the relation of spectra between a transfer function of digital filter and one of analog filter is considered.

Further, we apply to design the digital high pass filters using the symbol pulse invariant transformation method.

key words : *symbol pulse invariant transformation, impulse invariant transformation, digital filter, analog filter*

Ⅰ. Introduction

Digital signal processing has become an important tool in the multitude of diverse fields of electronic engineering.

In this paper, the problem of the transform for digital filter design from a given analog transfer function is considered. In that case, a digital filter is put in correspondence with an original analog filter. This procedure will be achieved by the transformation methods. The impulse invariant transformation and the bilinear transformation are most widely used[2].

In this paper, we first review the traditional transformations and point up their advantages and disadvantages. Next, we prove that instead of this rectangular simple pulse signal, we can

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adopt the same transformation to a double rate pulse signal with a sum of unit step functions. Furthermore, we show that we can apply the same transformation to any generic waveform signal and in particular this transformation can be applied to a signal approximated with straight lines.

Finally, we show that the symbol pulse invariant transformation can be used to active the desired high pass digital filter.

II. Traditional transformations

The impulse invariant transformation and the bilinear transformation have their own advantages and limitations.

The properties of the impulse invariant transformation is pointed up in the following. This method shows that a digital filter with an impulse response identical to the sampled impulse response of a given continuous filter can be derived via

$$H(s) = \sum_{i=1}^{m} \frac{A_i}{s+s_i} \rightarrow \sum_{i=1}^{m} \frac{A_i}{1-e^{-s_i T} z^{-1}} = H(z)$$

where T is the sampling interval.
Attractive feature includes the fact that the frequency scale is not warped. Therefore waveforms tend to be preserved.

Unattractive features include the following ; Mapping is not algebraic. Owing to overlapped mapping, the frequency response is aliased. If T is small, the gain becomes large.

Next, the properties of the bilinear transformation are pointed up in the following. This method is defined by

$$s \rightarrow \frac{2(1-z^{-1})}{T(1+z^{-1})} . \tag{1}$$

This method provides a simple mapping between analog and digital filters which is algebraic in nature. It also has the advantage that stable analog filters are mapped to stable digital filters. Further, analog filters can be mapped to digital filters with out aliasing.

The disadvantage of this method is that the frequency scale in the frequency domain is warped. Also, neither the impulse response nor the phase response of the analog filters are well preserved.

Next, the properties of the symbol pulse invariant transformation include the following.

In order to obtain better waveform preservation and more less aliasing property, we considered the symbol pulse invariant transformation. This technique is a transformation using the relationship between the output $y(n)$ of the digital filter which is earned from the sampled output $y(nT)$ of the analog filter and input signal $x(n)$. In this method, the advantages include that ; The rectangular pulse signal used in this method is practically used in communication system. Therefore this method is realistic. And also, the stable analog filters are mapped to stable digital filters. Further, the signal waveform is well preserved. One disadvantage of this technique is that the pole position can be aliased unless T is selected small enough, because the rectangular pulse signal is not a bandlimited signal.

In general, the design of the digital filters by using the transformations is accomplished, when the transfer function for the analog filter is known.

If we represent the analog filter by a partial fraction expansion with only first order poles,

$$H_a(s) = K\delta_{MN} + \sum_{l=1}^{L} \frac{C_l}{s-s_l} \tag{2}$$

where C_l is the residue of the lth pole s_l, K is a constant multiplying coefficient and

$$\delta_{MN} = \begin{cases} 0 , & M \neq N \\ 1 , & M = N \end{cases}$$

where M is a degree of denominator and N is one of numerator. Using the symbol pulse invariant transformation, the transfer function of a digital filter is obtained as like Eq. (3)

$$H(z) = K + \sum_{l=1}^{L} \frac{C_l}{-s_l} \frac{z^{-1}(1-e^{s_l T})}{1-e^{-s_l T} z^{-1}}. \quad (3)$$

From above results, the transformation from $H_a(s)$ to $H(z)$ is expressed as follows,

$$\frac{C_l}{s-s_l} \Rightarrow \frac{C_l}{-s_l} \frac{z^{-1}(1-e^{s_l T})}{1-e^{-s_l T} z^{-1}}. \quad (4)$$

To be completely general, Eq. (4) should also include some poles of multiple order. If s_i are the simple poles of $H_a(s)$, and s_j are some poles of multiple order, $H_a(s)$ is expressed with the followings :

$$H_a(s) = K\delta_{MN} + \sum_{i=1}^{I} \frac{C_i}{s-s_i} + \sum_{j=1}^{J} \left(\sum_{n=1}^{r} \frac{C_j}{(s-s_j)^n}\right). \quad (5)$$

When M is equal to N, $H_a(s)$ represents a high pass filter ; otherwise, a low pass filter. Similarly, we can derive the transfer function of a digital filter in this case.

This is expressed with[3]

$$H(z) = K + \sum_{i=1}^{I} \frac{C_i}{-s_i} \frac{z^{-1}(1-e^{s_i T})}{1-e^{-s_i T} z^{-1}} + \sum_{j=1}^{J} \{ \sum_{n=1}^{r} [\frac{1}{(r-n)!} \frac{\partial^{r-n}}{\partial a^{r-n}} \frac{C_j z^{-1}(1-e^{aT})}{(-a)(1-e^{aT} z^{-1})}] |_{a=s_j} \}. \quad (6)$$

Therefore, when $H_a(s)$ has a pole of multiple order, the transformation can be expressed as follows ;

$$\frac{C_j}{(s-s_j)} \Rightarrow [\frac{1}{(r-n)!} \frac{\partial^{r-1}}{\partial a^{r-1}} \cdot \frac{C_j z^{-1}(1-e^{aT})}{(-a)(1-e^{aT} z^{-1})}] |_{a=s_j} \}$$

When a transfer function has poles of multiple order, the transformation can be derived easily. Hereafter, we assume that all pole of a transfer function are of order one.

Ⅲ. Transformation Relationship

The symbol pulse invariant transformation[2] was derived by using a rectangular pulse signal. Here, instead of this rectangular simple pulse signal, we adopt the same transformation to a double rate pulse signal with a sum of unit step functions. Furthermore, we apply the same transformation to any generic waveform signal.

3.1. Double Rate Pulse Signal

In this section, we derive the transformation for approximating an output signal with a sampled signal when an input signal is the double rate pulse signal.

The double rate pulse signal is shown in Fig. 1.

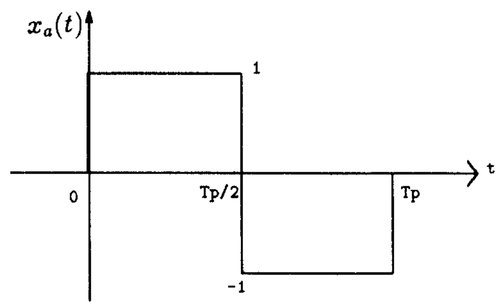


Fig. 1. Double rate pulse signal

$$x_a(t) = \begin{cases} 1, & 0 \le t < T_p/2 \\ -1, & T_p/2 \le t < T_p \\ 0, & t < 0,\ t \ge T_p, \end{cases} \quad (7)$$

where T_p is the signal length. The output response $y_a(t)$ of the analog filter for a double rate pulse signal is derived by a convolution calculation.

$$y_a(t) = \begin{cases} \int_0^t h(t-\tau)d\tau, & 0 \le t < T_p/2 \\ \int_0^{T_p/2} h(t-\tau) - \int_{T_p/2}^{t} h(t-\tau)d\tau, & T_p/2 \le t < T_p \\ \int_0^{T_p/2} h(t-\tau)d\tau - \int_{T_p/2}^{T_p} h(t-\tau)d\tau, & t \ge T_p \\ 0, & t < 0. \end{cases} \quad (8)$$

Here, the impulse response $h(t)$ of the analog filter is obtained by the inverse Laplace transform from Eq. (1) and can be expressed with

$$h(t) = K\delta(t) + \sum_{l=1}^{L} C_l e^{s_l t} u(t). \tag{9}$$

Next, we sample $x_a(t)$ with a period T. We assume that T is T_p/N and N is a positive integer. Therefore, a sampled signal $x_a(nT) = x(n)$ can be written as

$$x(n) = \begin{cases} 1, & 0 \le n < N/2 \\ -1, & N/2 \le n < N \\ 0, & n < 0,\ n \ge N. \end{cases} \tag{10}$$

The sequence $y_a(nT) = y(n)$ is derived by sampling a continuous signal $y_a(t)$ every T seconds. Therefore $y(n)$ is expressed with[1]

$$y(t) = \begin{cases} Ku(n) + \sum_{l=1}^{L} \frac{C_l}{-s_l}(1 - e^{-s_l nT}) \\ \qquad , \quad 0 \le n < N/2 \\ \sum_{l=1}^{L} \frac{C_l}{-s_l} e^{s_l nT}(e^{-s_l NT/2} - 1) - Ku(n) \\ \quad - \sum_{l=1}^{L} \frac{C_l}{-s_l}(1 - e^{s_l nT} e^{-s_l NT/2}) \\ \qquad , \quad N/2 \le n < N \\ \sum_{l=1}^{L} \frac{C_l}{-s_l}(e^{-s_l NT/2} - 1) e^{s_l nT} \\ \quad - \sum_{l=1}^{L} \frac{C_l}{-s_l} e^{s_l nT}(e^{-s_l NT} - e^{-s_l NT/2}) \\ \qquad , \quad n \ge N. \end{cases} \tag{11}$$

Then, taking the z-transform of Eq. (11) gives

$$Y(z) = \Phi(z) + \sum_{l=1}^{L} \frac{C_l}{-s_l} \frac{-(1 - z^{-N/2})^2}{1 - e^{-s_l T} z^{-1}} \tag{12}$$

where $\Phi(z) = \frac{(1 - z^{-N/2})^2}{1 - z^{-1}} \left(K + \sum_{l=1}^{L} \frac{C_l}{-s_l}\right)$

Further taking the z-transform of Eq. (10) gives

$$X(z) = \frac{(1 - z^{-N/2})^2}{1 - z^{-1}} \tag{13}$$

From Eq. (12), Eq. (13) and the relation $H(z) = Y(z)/X(z)$, the transfer function $H(z)$ is obtained.

$$H(z) = K + \sum_{l=1}^{L} \frac{C_l}{-s_l} \frac{z^{-1}(1 - e^{s_l T})}{1 - e^{-s_l T} z^{-1}}. \tag{14}$$

As we can see that Eq. (14) and Eq. (3) are the same, we can say that the transformation expressed with Eq. (4) can be employed ; that is, the output signal in the digital system expressed with the transfer function, Eq. (14), is equal to the sampled signal obtained from the output signal of the system expressed with Eq. (2) when an input signal is the double rate pulse signal.

3.2. A Generic Waveform

In this section, we derive the transformation for approximating an output signal with a sampled signal when an input signal is represented with a sum of step functions and ramp functions. Firstly, we consider the case of dividing a analog input signal $x_a(t)$ with length T_β as shown in Fig. 2. When an input signal is approximated with straight lines, the signal is expressed with

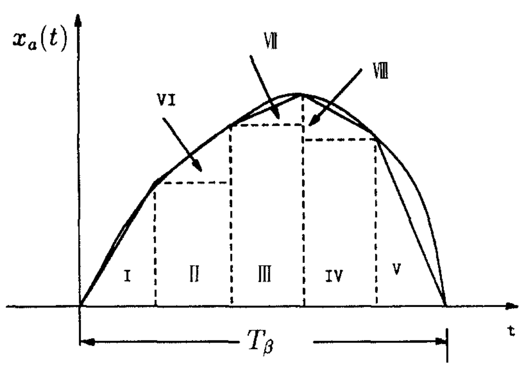


Fig. 2. Approximated straight lines of a generic wave form

$$x(\tau) \simeq \left[\frac{x\left(\frac{kT_\beta}{M} + \frac{T_\beta}{M}\right) - x\left(\frac{kT_\beta}{M}\right)}{\frac{T_\beta}{M}} \right] \left(\tau - \frac{kT_\beta}{M}\right)$$

$$+x\left(\frac{kT_\beta}{M}\right) \tag{15}$$

where $\frac{kT_\beta}{M} \le \tau \le \frac{(k+1)T_\beta}{M}$, T_β is the length of the signal, $k=1, 2, \cdots, M$ and M is an integer. Then, the output response is calculated by a convolution of the input signal $x(\tau)$ and the system impulse response $h(t)$, that is to say

$$y_a(t) = \int_{-\infty}^{t} x(\tau)\,h(t-\tau)\,d\tau. \tag{16}$$

Here, the signal consists of the sum of step functions as Ⅱ, Ⅲ, Ⅳ, regions and ramp functions as Ⅰ, Ⅴ, Ⅵ, Ⅶ, Ⅷ regions in Fig. 2 approximately; That is, we can approximate the output signal with a sum of step functions and ramp functions. When a signal is represented with a sum of step functions, the same transfer function as Eq. (3) can be derived similarly. Therefore, we need to calculate the transfer function for Ⅰ, Ⅴ, Ⅵ, Ⅶ, Ⅷ regions. Next, we prove that the same relationship as Eq. (3) for a ramp function is derived.

The ramp function in I region is defined as

$$P(t) = \begin{cases} t, & 0 \le t < T_p \\ 0, & elsewhere \end{cases} \tag{17}$$

where T_p is a width of a ramp signal. From Eq. (9), Eq. (16) and a ramp signal $P(t)$, we can obtain $y_a(t)$ as like Eq. (18)

$$y_a(t) = \begin{cases} Kt + \sum_{l=1}^{L} C_l \int_0^t \tau e^{s_l(t-\tau)} \\ \quad = Kt + \sum_{l=1}^{L} \frac{C_l}{-s_l} t - \sum_{l=1}^{L} \frac{C_l}{s_l^2}(1-e^{s_l t}) \\ \qquad , \quad 0 \le t \le T_p \\ \sum_{l=1}^{L} \frac{C_l}{-s_l} e^{s_l t}\{T_p e^{-s_l T_p} + \frac{1}{s_l}(e^{-s_l T_p} - 1)\} \\ \qquad , \quad T_p < t \end{cases} \tag{18}$$

where the sampling period T is T_β/M, the output $y_a(nT)$ is derived by sampling $y_a(t)$. Then taking z-transform, $Y(z)$ is represented with

$$\begin{aligned} Y(z) = {} & (K + \sum_{l=1}^{L} \frac{C_l}{-s_l}) \sum_{n=0}^{M-1} nTz^{-n} \\ & + \sum_{l=1}^{L} \{ \frac{C_l}{s_l^2} [\frac{(1-MZ^{-M+1})s_l TZ^{-1}}{(1-z^{-1})(1-e^{-s_l T}z^{-1})} \\ & + \frac{s_l(M-1)Tz^{-M-1}}{(1-z^{-1})(1-e^{-s_l T}z^{-1})} \\ & + \frac{\Theta(z)(z^{-1}-z^{-M-1})}{(1-z^{-1})(1-e^{-s_l T}z^{-1})}]\} \end{aligned} \tag{19}$$

where $\Theta(z) = \frac{(s_l T)^2}{2!} + \frac{(s_l T)^3}{3!} + \cdots$.

Assuming $|s_l T| \ll 1$, $Y(z)$ is given as

$$\begin{aligned} Y(z) = {} & (K + \sum_{l=1}^{L} \frac{C_l}{-s_l}) \sum_{n=0}^{M-1} nTz^{-n} \\ & + \sum_{l=1}^{L} \{ \frac{C_l}{s_l^2} [\frac{s_l Tz^{-1}(1-Mz^{-M+1})}{(1-z^{-1})(1-e^{-s_l T}z^{-1})} \\ & + \frac{s_l Tz^{-1}(M-1)z^{-M}}{(1-z^{-1})(1-e^{-s_l T}z^{-1})}]\}. \end{aligned} \tag{20}$$

Taking z-transform of a ramp function expressed with Eq. (17), $X(z)$ is given by

$$X(z) = T[\frac{z^{-1}(1-Mz^{-(M-1)})}{(1-z^{-1})^2} + \frac{(M-1)z^{-(M+1)}}{(1-z^{-1})^2}]. \tag{21}$$

From Eq. (20) and Eq. (21), the obtained transfer function $H(z)$ is shown to be equivalent to Eq. (3). Therefore, the validity of the transformation expressed with Eq. (4) is proved to be proper. The output signal of the digital system, is equal to the sampled signal obtained from the output signal of the analog system, under the condition $|s_l T| \ll 1$ when the input analog signal is approximated signal with a sum of step functions and ramp functions.

3.3. The Relationship of Spectra

In this section, the relation between the spectrum of an analog signal and the spectrum of a sampled signal is expressed with

$$X(e^{j\Omega T})=\frac{1}{T}\sum_{r=-\infty}^{\infty} X_a(j\Omega+j\frac{2\pi r}{T}) \tag{22}$$

$$Y(e^{j\Omega T})=\frac{1}{T}\sum_{r=-\infty}^{\infty} Y_a(j\Omega+j\frac{2\pi r}{T}) \tag{23}$$

where $X_a(j\Omega)$ and $Y_a(j\Omega)$ are spectra of input signal and an output signal, respectively. $X(e^{j\Omega T})$ and $Y(e^{j\Omega T})$ are spectra of a sampled input signal and a sampled output signal, respectively. From above equations, the transfer function of a digital system designed by using a symbol pulse invariant transformation is obtained.

$$H(e^{j\Omega T})=\frac{\sum_{r=-\infty}^{\infty} Y_a(j\Omega+j\frac{2\pi r}{T})}{\sum_{r=-\infty}^{\infty} X_a(j\Omega+j\frac{2\pi r}{T})} \tag{24}$$

In this case, the transfer function is expressed with

$$H(z)|_{z=e^{sT}}=\frac{Y_a(s)}{X_a(s)}=H_a(s). \tag{27}$$

Ⅳ. Design of high pass digital filters

The impulse invariant transformation and the bilinear transformation are useful transfer functions for designing a digital filter from analog filters. However, in the problem of the design of high pass filters, the impulse invariant transformation can not be used[11]. Therefore, we will not discuss for designing a digital high pass filter from an analog filter with the impulse invariant transformation. The case of the bilinear transformation, to overcome the problem of the frequency scale, the prewarping method are adopted usually. However, we do not adopt the method in this paper, since the impulse response of the time domain and the phase response of the frequency domain are not equal between in analog and in digital representations[9].

4.1. Symbol pulse invariant transformation

The design of a high pass filter often begins with a requirement to translate a frequency, such as $s \rightarrow 1/s$.

Specifically, when the relation is applied to an analog Butterworth filter which is a low pass filter, a high pass filter is expressed with

$$H_h(s)=\prod_{k=1}^{N/2}\left[1-\frac{2\cos\phi_k s+1}{s^2+2\cos\phi_k s+1}\right] \tag{28}$$

where, $\phi=(2k-1)\pi/2N$, $k=1, 2, \cdots, N/2$.

Application of the partial fraction expansion in Eq.(28) yields

$$H_h(s)=1-\prod_{k=1}^{N/2}\left[\frac{\Psi_k}{s+e^{j\phi_k}}+\frac{\bar{\Psi}_k}{s+e^{-j\phi_k}}\right] \tag{29}$$

where

$$\Psi_k=Z_k(2^{N-1}A)^{-1}(\sin\alpha+j\cos\alpha),$$

$$\bar{\Psi}_k=\bar{Z}_k(2^{N-1}A)^{-1}(\sin\alpha-j\cos\alpha).$$

Parameters Z_k, $\bar{Z}_k$, A and α are represented respectively :

$$Z_k=\{\prod_{k=1}^{N/2}[s^2+2\cos\phi_k s+1]-s^N\}|_{s=-e^{j\phi_k}},$$

$$\bar{Z}_k=\{\prod_{k=1}^{N/2}[s^2+2\cos\phi_k s+1]-s^N\}|_{s=-e^{-j\phi_k}},$$

$$A=\prod_{\substack{i=1\\ i\neq k}}^{N/2}[\sin\frac{(i-k)\pi}{2N}]\prod_{i=1}^{N/2}[\sin\frac{(i+k-1)\pi}{2N}],$$

$$\alpha=[(2Nk-N-4k+2)\pi]/4N$$

Applying the symbol pulse invariant transformation to Eq.(29) yields

$$H_h(z)=1-\prod_{k=1}^{N/2}\left[\Psi_k e^{j\phi_k}\frac{z^{-1}(1-e^{-e^{-j\phi_k T}})}{1-e^{-e^{-j\phi_k T}}z^{-1}}+\bar{\Psi}_k e^{-j\phi_k}\frac{z^{-1}(1-e^{-e^{j\phi_k T}})}{1-e^{-e^{j\phi_k T}}z^{-1}}\right]. \tag{30}$$

For convenience, let the real and imaginary part of the complex be Ψ_R and Ψ_I, respectively.

$R_e\{\Psi_k\} = \Psi_R,\ I_m\{\Psi_k\} = \Psi_I.$

Then, the transfer function for the high pass digital filter becomes $H_h(z)$

$$H_h(z) = 1 - \prod_{k=1}^{N/2} \frac{A_{1k} z^{-1} + A_{2k} z^{-2}}{1 - B_{1k} z^{-1} - B_{2k} z^{-2}} \quad (31)$$

where

$$A_{1k} = 2[1 - e^{-\cos\phi_k T}\cos(\sin\phi_k T)]\,\gamma_{1k} + 2e^{-\cos\phi_k T}\sin(\sin\phi_k T)\,\gamma_{2k},$$

$$A_{2k} = -2e^{-\cos\phi_k T}[\cos(\sin\phi_k T) - e^{-\cos\phi_k T}]\,\gamma_{1k} - 2e^{-\cos\phi_k T}\sin(\sin\phi_k T)]\,\gamma_{2k},$$

$$B_{1k} = 2e^{-\cos\phi_k T}\cos\phi_k T, \qquad B_{2k} = -e^{-2\cos\phi_k T},$$

$$\gamma_{1k} = (\Psi_R\cos\phi_k - \Psi_I\sin\phi_k),$$

$$\gamma_{2k} = (\Psi_I\cos\phi_k + \Psi_R\sin\phi_k).$$

4.2. Bilinear transformation

A transfer function of a digital filter is obtained from this method by making the algebraic substitution of Eq.(1).

$$H(z) = H(s)|_{s \to 2(1-z^{-1})/T(1+z^{-1})}$$

when this transformation is applied to the transfer function of an analog filter Eq.(28), the transfer function of the digital filter is readily attained.

$$H_h(z) = 1 - \prod_{k=1}^{N/2} \frac{A_{0k} + A_{1k} z^{-1} + A_{2k} z^{-2}}{1 - B_{1k} z^{-1} - B_{2k} z^{-2}} \quad (32)$$

where, T is sampling period and

$$A_{0k} = [1 + (T/2)^2 + T\cos(\frac{(2k-1)\pi}{2N})]^{-1},$$

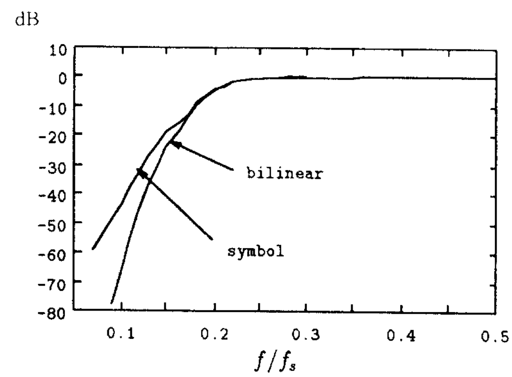
$$A_{1k} = -2[1 + (T/2)^2 + T\cos(\frac{(2k-1)\pi}{2N})]^{-1},$$

$$A_{2k} = [1 + (T/2)^2 + T\cos(\frac{(2k-1)\pi}{2N})]^{-1},$$

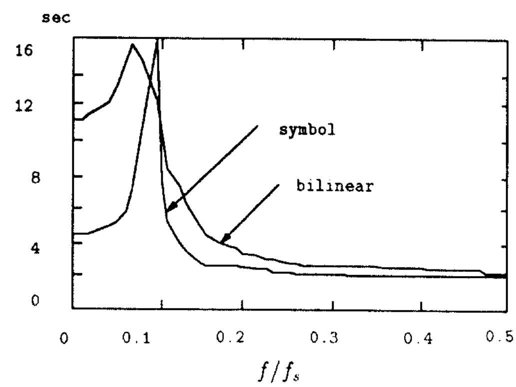
$$B_{1k} = [-2 + 2(T/2)^2]\,[1 + (T/2)^2 + T\cos(\frac{(2k-1)\pi}{2N})]^{-1},$$

$$B_{2k} = [1 + (T/2)^2 - T\cos(\frac{(2k-1)\pi}{2N})]\,[1 + (T/2)^2 + T\cos(\frac{(2k-1)\pi}{2N})]^{-1}.$$

The high pass filter obtained by the bilinear transformation is designed for comparisons with one by the symbol pulse invariant transformation. The specifications of HPF are the followings : a pass band cutoff frequency $w_p = 0.5\pi$, stop band edge frequency $w_c = 0.34\pi$, pass band attenuation $B_p = 1.5dB$, minimum stop band attenuation $B_s = 15dB$ and $f_s = 10kHz$. Fig.3(a) shows the magni-



(a) Gain[dB]



(b) Group delay[s]

Fig. 3. Designed characteristics of HPF

tude property and Fig.3(b) shows the group delay. The group delay is defined by

$$\tau(e^{jw}) = -Re\left[\, z\frac{dH(z)/dz}{H(z)} \,\right]_{z=e^{jw}}$$

The two transformations have very similar magnitude property in pass band. For the filter designed by the symbol pulse invariant transformation, the group delay shows 3.5 secs in the pass band, and also for the filter obtained by bilinear transformation, the group delay shows from 3 secs to 4 secs, which is not a constant delay characteristic.

Further, we consider LPF, BPF, and BSF. The specifications of BSF are the followings : the sampling period is 0.45[s], the stop band edges are $\pi/2$ and $\pi/5$, and the order is 2. The specifications of LPF and BPF are made by the values given in [7] and [8], respectively. Also, the gain characteristics of LPF, BPF, and BSF are given as Fig. 4, Fig. 5, and Fig.' 6, respectively. From above results, we can classify the optimal transformation, according to purpose of designing as follows : for only gain properties the bilinear transformation is optimal ; for group delay properties the symbol pulse invariant transformation is optimal.

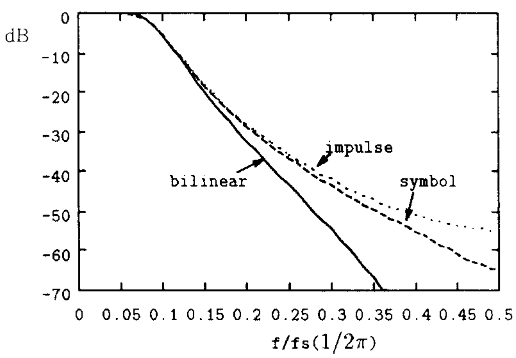


Fig. 4. Gain characteristic of LPF

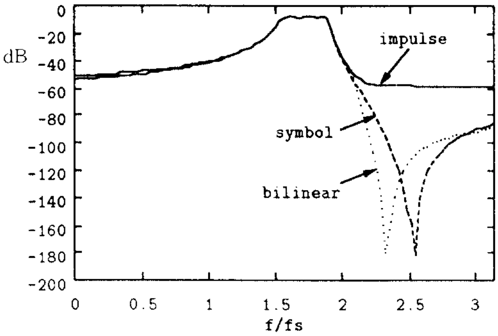


Fig. 5. Gain characteristic of BPF

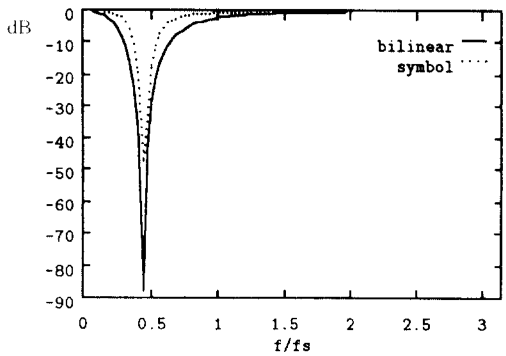


Fig. 6. Gain characteristic of BSF

V. Conclusions

In this paper, the symbol pulse invariant transformation is analyzed so that the symbol pulse invariant transformation which was applied to a rectangular pulse is newly applied to double rate pulse signals and generic shape pulse signals. It is also expanded to the general case of the analog filter.

Further we express the relationship of spectra on the transfer functions between digital filter and analog filter for the symbol pulse invariant transformation. Finally, we prove that the symbol pulse invariant transformation can be used to design a high pass digital filter. Also we show the characteristic of frequency response for HPF including LPF, BPF, and BSF. As a result, we can classify the optimal transformation, according to purpose of designing as follows : for only gain properties the bilinear transformation is optimal ; for group delay properties the symbol pulse invariant transformation is optimal.

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