論　文

A Routing Algorithm for Minimizing Packet Loss Rate in High-Speed Packet-Switched Networks

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고속의 패킷 교환망에서 패킷 손실율을 최소화하기 위한 경로 제어 알고리즘

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ABSTRACT

Gradient projection (GP) technique is applied for solving the optimal routing problem (ORP) in high-speed packet-switched networks. The ORP minimizing average network packet loss probability is non-convex due to packet losses at intermediate switching nodes and its routing solution cannot be directly sought by the GP algorithm. Thus the non-convex ORP is transformed into a convex problem called the reduced-ORP (R-ORP) for which the GP algorithm can be used to obtain a routing solution. Through simulations, the routing solution of the R-ORP is shown to be a good approximation to that of the original ORP. Theoretical upper bound of difference between two (ORP and R-ORP) routing solutions is also derived.

要 約

고속의 패킷 교환망에서 최적 경로 제어 문제(ORP : Optimal Routing Problem)를 해결하기 위해 gradient projection (GP) 기법을 적용하였다. 망 평균 패킷의 손실율을 최소화하는 ORP는 교환 노드들에서 패킷들이 손실됨으로써 non-convex가 되어, GP 알고리즘에 의해서는 그 경로해를 직접 구할 수가 없다. 따라서, 이 non-convex ORP를 reduced-ORP (R-ORP)라 불리는 convex 문제로 변형시킨 후, 그 경로해를 구하고자 GP 알고리즘을 이용하였다. 컴퓨터 모의 실험에 의해서는 R-ORP를 통해 얻어진 경로해가 원래 ORP의 최적 경로해에 아주 근사함을 보여준다. 또한, ORP와 R-ORP들을 통해 얻어진 두 경로해간의 차이에 있어서의 이론적인 상한이 유도된다.

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I. Introduction

The advent of fiber optics combined with VLSI technology enables communication networks to operate in a very high-speed (Gbps-Tbps) environment [1]. Higher communication bandwidths are essential to the applications such as broadband integrated services digital network (B-ISDN).

The transfer mode recommended by CCITT for the basis of B-ISDN is called the asynchronous transfer mode (ATM)[2]. In ATM networks, messages are split into short fixed-length (53 bytes) packets with address labels attached. While being delivered to their destinations, some packets are lost for several reasons, one of which is buffer overflow at intermediate switching nodes. When a packet arrives and finds no available buffer space at a switching node, it is simply discarded by the node. The tolerance for packet loss varies with the type of traffic carried. In ATM networks, end-to-end packet loss probability is normally required to be less than 10^{-6} to 10^{-9} and we are concerned about minimizing average network packet loss probability. In [3] and [4], an ATM network is modeled by the network of M/M/1 or *k*-M/M/1 queues – the objective functions are average network packet delay. Lee and Yee [5] proposed an algorithm minimizing the maximum of link packet loss probabilities in an ATM network. However, the average network packet delay does not precisely reflect the effects of lost packets and the minimax algorithm in [5] does not guarantee its routing solution to be optimal.

In the following discussions, we are not restricted only to ATM networks but focus on more generalized high-speed packet-switched networks. These high-speed networks networks allow variable sizes of packet length and can be modeled by the network of *k*-M/M/1/b queues. The optimal routing problem (ORP) in high-speed networks takes average network packet loss probability as an objective function to be minimized. Considering packet losses at switching nodes, this ORP becomes a non-convex problem that cannot be solved by the gradient projection (GP) algorithm. The GP algorithm is applicable only to convex problems [6]. To detour this difficulty, we assume that no packets are lost at intermediate switching nodes on each of active paths. With this assumption, the ORP can be transformed into a convex problem called the *reduced*-ORP (R-ORP). Then the obiective function (average network packet loss probability) is convex with respect to its variables (path flows) and the (routing) solution can be obtained by using the GP algorithm.

The routing solution of the R-ORP may be different from an exact optimal routing solution of the original ORP. The analytical results show that the difference of two (ORP and R-ORP) routing solutions is negligible when the network is not heavily congested. For example, the percentage difference of two routing solutions reduces to be less than 1% if the maximum link packet loss probability is in the order of less than 10^{-6} (when the buffer size of each queue is 30). Numerical examples support that the difference can be ignored even with higher maximum link packet loss probabilities.

This paper is organized as follows. Section 2 introduces a flow model of high-speed packet switches and the ORP for high-speed networks. In Section 3, the R-ORP for high-speed networks is proposed and the GP algorithm as a solution technique is discussed. Then the theoretical upper bound of differences in network flows and routing solutions between the ORP and the R-ORP is provided. In Section 4, numerical examples are presented to demonstrate the results obtained from Section 3. Finally, Section 5 summarizes this paper.

II. Problem Formulation

2.1 Flow Model

Many switch architectures have been proposed recently for high performance packet switching, e. g., Knockout switch [7] and Batcher-Banyan switch [8], etc. Switches for high-speed networks can be classified according to the location of buffers : input queueing, output queueing, and shared buffering. The switches based on output queueing principle are known to have the best performance among others and achieve optimal throughput-delay performance [9]. For a typical high-speed packet switch with output queueing, every out-going link can be modeled by k-M/M/1/b queue as shown in Figure 1.

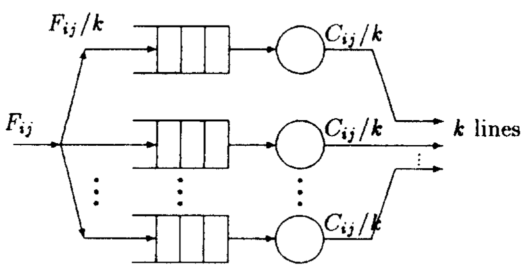

Figure 1. A k-M/M/1/b queueing model

For ATM networks, k-M/D/1/b queues seem to be more suitable than k-M/M/1/b queues since the ATM uses a fixed size of packet length. With the k-M/D/1/b queueing model, however, it is difficult to construct and analyze the network of queues since packet arrival after the first queue at their entry points of the network can be no longer described by Poisson process. It is not known that Kleinrock independence approximation still holds for the network of M/D/1-like queues.

For the k-M/M/1/b queueing model, it is assumed that any packet bound for an out-going link is routed with equal probability to any output port associated with the link. Each channel in an out-going link has an equal capacity C_{ij}/k where C_{ij} is the total capacity of link (i, j). Let us denote input flow on an out-going link (i, j) by F_{ij}. Then packet loss probability on link (i, j), denoted by $B_{ij}(F_{ij})$, is given by

$$B_{ij}(F_{ij}) = \frac{(1-\rho_{ij})\rho_{ij}^{b}}{1-\rho_{ij}^{b+1}}$$

where $\rho_{ij} = F_{ij}/C_{ij}$ is the utilization factor of link (i, j). Note that $B_{ij}(F_{ij})$ is a function of F_{ij}(also ρ_{ij}) and is not convex with respect to F_{ij}. It seems to be convex when $\rho_{ij} < 1$, but levels off approaching to 1 as ρ_{ij} increases beyond 1. Packet loss rate on link (i, j), denoted by $L_{ij}(F_{ij})$, is given by

$$L_{ij}(F_{ij}) = C_{ij} \cdot \frac{(1-\rho_{ij})\rho_{ij}^{b+1}}{1-\rho_{ij}^{b+1}}$$

and now becomes convex with respect to F_{ij}. Since $L_{ij}(F_{ij})$ is twice continuously differentiable, it can be used as a link cost function for the ORP based on the GP algorithm.

The first and second derivatives of $L_{ij}(F_{ij})$ with respect to F_{ij}, which are essential for the GP algorithm, are also given by

$$L'_{ij}(F_{ij}) = \frac{(b+1)\rho_{ij}^{b} - (b+2)\rho_{ij}^{b+1} + \rho_{ij}^{2b+2}}{(1-\rho_{ij}^{b+1})^2}$$

and

$$L''_{ij}(F_{ij}) = \frac{1}{C_{ij}} \cdot \frac{b+1}{(1-\rho_{ij}^{b+1})^3} \cdot [b\rho_{ij}^{b-1} - (b+2)(1-\rho_{ij}^{b})\rho_{ij}^{b} - b\rho_{ij}^{2b+1}].$$

Let $\mathcal{L}$ denote the set of links in the network. In [10], it is proved that $0 \le L'_{ij}(F_{ij}) \le 1$ and $L''_{ij}(F_{ij}) \ge 0$, $\forall (i, j) \in \mathcal{L}$ for a more general case (M/G/n/n queueing model).

2.2 The Optimal Routing Problem (ORP)

In high-speed networks, packets are not preserved at intermediate switching nodes. Thus for each path p and each link (i, j), path flow x_p and link flow F_{ij} have the following non-linear relationship :

$$F_{ij} = \sum_{p \in P_{ij}} (x_p \cdot \prod_{(k,l) \in U_{ij}(p)} (1 - B_{kl}(F_{kl}))), \; p \in P_w, \; \forall w \in W \quad (1)$$

where $U_{ij}(p)$ is the set of upstream links of link (i, j) on path p, P_{ij} is the set of all the paths traversing link $(i, j) \in \mathcal{L}$, P_w is the set of admissible paths for origin-destination (OD) pair w, and W is the set of OD pairs. Let us denote average network packet loss probability by $B(F)$ where $F=\{F_{ij}:(i,j)\in\mathcal{L}\}$. Then $B(F)$ adopted as an objective function in the ORP is defined by the sum of link packet loss rates divided by the total traffic demands over the network :

$$B(F)=\frac{1}{\gamma}\sum_{(i,j)\in\mathcal{L}} L_{ij}(F_{ij}) \tag{2}$$

where $\gamma=\sum_{w\in W} r_w$. The link-formulated ORP for high-speed networks is that of minimizing the objective function (2) subject to the traffic balance equation (1).

By eliminating the constraint (1), the link-formulated ORP can be transformed as follows :

$$\begin{aligned} &\text{minimize} && B(x) \\ &\text{subject to} && \textstyle\sum_{p\in P_w} x_p = r_w, \quad \forall w\in W, \\ &&& x_p \geq 0, \quad \forall p\in P_w,\ \forall w\in W. \end{aligned}$$

This path-formulated ORP for high-speed networks becomes a non-convex problem due to the non-linear traffic balance equation (1). Thus, there may exist several local minima that make it impossible to use the ordinary GP algorithm for solving the ORP. The routing solution of the ORP, when solved by the GP algorithm, may not be globally optimal.

Ⅲ. The Reduced Optimal Routing Problem (R-ORP)

In the ORP mentioned in the previous section, path flows are correlated with each other. Therefore, packet losses on a path affect those of other path flows, causing an objective function (average network packet loss probability) to be non-convex with respect to its variables (path flows). The idea behind the R-ORP is to get rid of this correlation by ignoring the effects of packet losses at intermediate switching nodes. That is, we assume that no packets are lost at upstream switching nodes of link (i,j). Then F_{ij} is expressed by

$$F_{ij}=\sum_{p\in P_{ij}} x_p. \tag{3}$$

This is a simple linear traffic balance equation and replaces the constraint (1) for the R-ORP. The convexity of $L_{ij}(F_{ij})$ with respect to F_{ij} implies that $B(x)=\frac{1}{\gamma}\sum_{(i,j)\in\mathcal{L}} L_{ij}(F_{ij})$ is also convex with respect to $x=\{x_p\}$, $p\in P_w$, $w\in W$ due to the linear property of the equation (3). The R-ORP now becomes a convex problem and a unique global minimum exists. Thus a routing solution for the R-ORP can now be obtained by applying the ordinary GP algorithm.

3.1 Flow Difference between the ORP and the R-ORP

To disting uish input link flows between the ORP and the R-ORP, let input flow on link $(i,j)\in\mathcal{L}$ for the ORP be denoted by F_{ij}^{O} and for the R-ORP by F_{ij}^{R}. Denote also the portion of packet loss rate on link (i,j) due to path flow x_p by $L_{ij}^{p}(F_{ij}^{O})$ (path-link packet loss rate). Rewriting the traffic balance equation (1) for the ORP,

$$F_{ij}^{O}=\sum_{p\in P_{ij}}\Big(x_p-\sum_{(k,l)\in U_{ij}(p)} L_{kl}^{p}(F_{kl}^{O})\Big)=F_{ij}^{R}-\Delta F_{ij}$$

where

$$F_{ij}^{R}=\sum_{p\in P_{ij}} x_p,$$

and the input link flow difference on link (i,j), denoted by ΔF_{ij}, between the ORP and the R-ORP is given by

$$\Delta F_{ij}=\sum_{p\in P_{ij}}\sum_{(k,l)\in U_{ij}(p)} L_{kl}^{p}(F_{kl}^{O}).$$

In Section 2 it was shows that ΔF_{ij} causes $B(x)$ for the ORP to be non-convex with respect to $x=\{x_p\}$. Overall network flow difference ΔF is defined to be the link flow differences summed over the network :

$$\Delta F=\sum_{(i,j)\in\mathcal{L}}\Delta F_{ij}.$$

Since $F_{ij}^R \geq F_{ij}^O$ for each link $(i, j) \in \mathcal{L}$, ΔF is always greater then or equal to zero.

Let us denote $L_n^p(F_n^O)$ by path-link packet loss rate due to input flow F_n^O at the n-th link on path p. The overall path packet loss rate on path p, denoted by $L^p(x_p)$, is as follows ;

$$L^p(x_p) = \sum_{n=1}^{h_p} L_n^p(F_n^O)$$

where h_p is the number of hops on path p. Recall also that the average network packet loss probability is given by

$$B(x) = \frac{L(x)}{\gamma}$$

where the overall network packet loss rate is given by

$$L(x) = \sum_{(i,j) \in \mathcal{L}} L_{ij}(F_{ij}^O)$$

$$= \sum_{w \in W} \sum_{p \in P_w} L^p(x_p).$$

If we define network diameter to be $h = \max_{p \in P_w, w \in W} h_p$, then ΔF is upper-bounded by the following theorem.

Theorem 1

$$\Delta F \leq hL(x).$$

Proof :

$$\Delta F = \sum_{(i,j) \in \mathcal{L}} \Delta F_{ij}$$

$$= \sum_{(i,j) \in \mathcal{L}} \sum_{p \in P_{ij}} \sum_{(k,l) \in U_{ij}(p)} L_{kl}^p(F_{kl}^O))$$

$$= \sum_{w \in W} \sum_{p \in P_w} \sum_{(i,j) \in p} \sum_{(k,l) \in U_{ij}(p)} L_{kl}^p(F_{kl}^O))$$

$$= \sum_{w \in W} \sum_{p \in P_w} \sum_{m=1}^{h_p} \sum_{n=1}^{m} L_n^p(F_n)$$

$$\leq \sum_{w \in W} \sum_{p \in P_w} h_p L^p(x_p)$$

$$\leq h \sum_{w \in W} \sum_{p \in P_w} L^p(x_p)$$

$$\leq hL(x)$$ □

This can be quite easily understood by intuition. For each link (i, j) on path p, packet losses at upstream links are ignored. Then the flow difference on path p, denoted by ΔF_p, will be at most

$$\Delta F_p = h_p L^p(x_p).$$

Summing up ΔF_p over all paths $p \in P_w$, $w \in W$, it is obvious that ΔF is upper-bounded by $hL(x)$ since $h \geq h_p$, $\forall p \in P_w$, $\forall w \in W$.

3.2 The GP Method for the R-ORP

The GP algorithm can be applied for solving the R-ORP since the objective function $B(x)$ is convex with respect to path flows $x = \{x_p\}$, $p \in P_w$, $w \in W$. Let $p_w \in P_w$ be the path having the minimum first derivative length among all admissable paths for $w \in W$. As described in [6], the original R-ORP is transformed to involve only non-negative constraints :

$$\begin{array}{ll} \text{minimize} & \tilde{B}(\tilde{x}) \\ \text{subject to} & x_p \geq 0, \\ & p \in P_w,\ p \neq p_w,\ \forall w \in W. \end{array}$$

Now GP iterations for solving the R-ORP are as follows :

$$x_p^{k+1} = \max\{0,\ x_p^k - \alpha^k (b_{pp_w}^k)^{-1} (b_p^k - b_{p_w}^k)\},$$
$$p \in P_w,\ p \neq p_w,\ \forall w \in W$$

where the first derivative length

$$b_p^k = \sum_{(i,j) \in l_p} L'_{ij}(F_{ij}^k), \qquad b_{p_w}^k = \min_{p \in P_w} b_p^k,$$

and the second derivative length

$$b_{pp_w}^k = \sum_{(i,j) \in L_p} L''_{ij}(F_{ij}^k).$$

Here, l_p is the set of links belonging to a path p and L_p is the set of links belonging to either path p or p_w, but not both. Step size α can be chosen a constant value or can be adjusted according to $b_{pp_w}^k$ with iteration step k. The optimality conditions for which the GP algorithm terminates require that all the non-zero path flows produce equally minimum first derivative length when an optimal routing solution is achieved.

3.3 Upper Bound of the R-ORP Solution

Let us denote average network packet loss rate for the ORP and the R-ORP by $L^O(x)$ and $L^R(x)$ for any x, respectively. Suppose that there exists an optimal routing solution x_O^* of the ORP for which the GP method cannot be used. Applying the GP iterations to the R-ORP, however, a routing solution x_R^* for the R-ORP can be obtained since $L^R(x)$ is convex with respect to x.

We are interested in how close $L^R(x)$ is to $L^O(x)$ around the optimal routing solution x_O^* : more precisely, the difference between $L^R(x_R^*)$ and $L^O(x_O^*)$. For any $x=\{x_p\}$, $p\in P_w$, $w\in W$, F_{ij}^R and F_{ij}^O can be computed in most cases [11] and $L_{ij}(F_{ij}^R) \geq L_{ij}(F_{ij}^O)$, $\forall(i,j)\in\mathcal{L}$ since $F_{ij}^R \geq F_{ij}^O$, $\forall(i,j)\in\mathcal{L}$. Thus, it is obvious that

$$L^R(x) \geq L^O(x), \quad \forall x=\{x_p\},\ p\in P_w,\ w\in W.$$

Let us define $l_O' = \max_{(i,j)\in\mathcal{L}} L_{ij}'(F_{ij}^O)$. Then the following theorem states the upper bound of $L^R(x)$.

Theorem 2

$$L^R(x) \leq L^O(x)(1+hl_O'), \quad \forall x=\{x_p\},\ p\in P_w,\ w\in W.$$

Prof :

$$\begin{aligned} L^R(x) &= \sum_{(i,j)\in\mathcal{L}} L_{ij}(F_{ij}^R) \\ &= \sum_{(i,j)\in\mathcal{L}} L_{ij}(F_{ij}^O+\Delta F_{ij}) \\ &= \sum_{(i,j)\in\mathcal{L}} L_{ij}(F_{ij}^O) + \sum_{(i,j)\in\mathcal{L}} \Delta F_{ij} L_{ij}'(F_{ij}^O) \\ &\leq L^O(x) + l_O' \Delta F \\ &\leq L^O(x) + hl_O' L^O(x) \end{aligned}$$

□

In the proof of Theorem 2, the third equality was obtained by ignoring higher order error terms of the first order Taylor expansion and the last inequality comes from Theorem 1.

At an optimum, define the percentage difference of two objective functions $L^R(x_R^*)$ and $L^O(x_O^*)$ between the ORP and the R-ORP by

$$\delta = \frac{L^R(x_R^*) - L^O(x_O^*)}{L^O(x_O^*)}$$

Then δ is again upper-bounded by the following corollary.

Corollary 2

$$\delta \leq hl_O'$$

Proof : At $x=x_O^*$, from Theorem 2,

$$L^R(x_O^*) - L^O(x_O^*) \leq hl_O' L^O(x_O^*).$$

Since $L^R(x_R^*) \leq L^R(x_O^*)$,

$$L^R(x_R^*) - L^O(x_O^*) \leq hl_O' L^O(x_O^*). \tag{4}$$

Divide (4) by $L^O(x_O^*)$, then we obtain

$$\delta \leq hl_O'.$$

□

Corollary 2 tells that δ can be arbitrarily made small depending on the value of l_O'.

Table 1 and 2 show, for a generic link (i,j), the values of B_{ij} and L_{ij}' as a function of ρ_{ij} for different buffer sizes ($b=30$ for Table 1 and $b=50$ for Table 2). Define the maximum link packet loss probability to be $b_o = \max_{(i,j)\in L} B_{ij}(F_{ij})$. Suppose that b_o at an optimal routing slution of the ORP is in the order of 10^{-6}. This order of packet loss probability is normally required by high-speed networks including ATM networks. Assume also that $h=10$. Then the upper bound of δ between the ORP and the R-ORP for the case of $b=30$ is $\delta \leq hl_O' = 1.94\cdot 10^{-3} \simeq 0.2\%$. When $b=50$, $\delta \leq hl_O' = 1.34\cdot 10^{-3} \simeq 0.1\%$. Even with $h=100$, δ becomes approximately 2% and 1%, respectively. With the smaller l_O', δ decreases more dramatically to almost 0%.

Table 1. Packet loss probability and the first derive of packet loss rate for a generic link $(i, j)(b = 30)$.

ρ	B_{ij}	L'_{ij}
0.3	$1.44 \cdot 10^{-16}$	$4.41 \cdot 10^{-15}$
0.4	$6.92 \cdot 10^{-13}$	$2.10 \cdot 10^{-11}$
0.5	$4.66 \cdot 10^{-10}$	$1.40 \cdot 10^{-8}$
0.6	$8.84 \cdot 10^{-8}$	$2.61 \cdot 10^{-6}$
0.7	$6.76 \cdot 10^{-6}$	$1.94 \cdot 10^{-4}$
0.8	$2.48 \cdot 10^{-4}$	$6.70 \cdot 10^{-3}$
0.9	$4.41 \cdot 10^{-3}$	$1.02 \cdot 10^{-1}$

Table 2. Packet loss probability and the first derive of packet loss rate for a generic link $(i, j)(b = 50)$.

ρ	B_{ij}	L'_{ij}
0.3	$5.03 \cdot 10^{-27}$	$2.54 \cdot 10^{-25}$
0.4	$7.61 \cdot 10^{-21}$	$3.83 \cdot 10^{-19}$
0.5	$4.44 \cdot 10^{-16}$	$2.22 \cdot 10^{-14}$
0.6	$3.23 \cdot 10^{-12}$	$1.60 \cdot 10^{-10}$
0.7	$5.40 \cdot 10^{-9}$	$2.63 \cdot 10^{-7}$
0.8	$2.85 \cdot 10^{-6}$	$1.34 \cdot 10^{-4}$
0.9	$5.81 \cdot 10^{-4}$	$2.19 \cdot 10^{-2}$

Ⅳ. Numerical Examples

In this section, the accuracy of the R-ORP routing solution is demonstrated by numerical examples. The source program is written in C and run on a SUN 4/370. Figure 2 shows the topology of a 14-node network to be simulated. Assume that every link between switching nodes has fixed 32 channels each of which is attached by a buffer of size $b = 30$.

For traffic demands, two sets of OD pairs are generated:80 and 95 OD pairs. Then the GP iterations are applied for solving the R-ORP. Each OD pair is randomly generated with its traffic demand ranging from 1 to 7. Total traffic demands generated are 265.96(for 80 OD pairs) and 321.54 (for 95 OD pairs), respectively. The unit of traffic demands would be packets/sec. For GP iterations, we use a constant step size which is carefully chosen to avoid any undesirable behavior

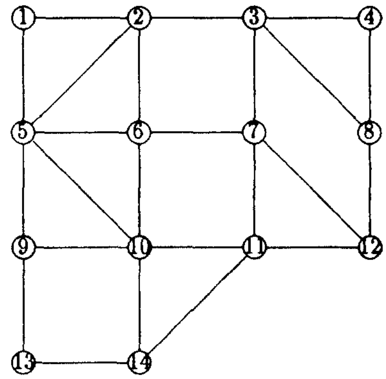

Figure 2. 14 node network.

(oscillation or divergence, etc.)

For the two cases (ORP and R-ORP), the objective functions and the maximum first derivatives of link packet loss rate l'_O are compared with different utilization factors. Let us define the maximum first derivative of link packet loss rate for the R-ORP to be $l'_K = \max_{(i,j)\in L} L'_{ij}(F^R_{ij})$. Table 3 through 6 show the values of objective function ($B^K(x)$ and $B^O(x)$) and the maximum first derivatives of link packet loss rate (l'_K and l'_O) at some iteration steps for each set of OD pairs. Denote the link utilization of mostly congested link at each iteration by ρ_K. As the GP iterations proceed, $B^K(x)$ as well as ρ_K decreases monotonically. The last row of the tables indicates $B^K(x)$ at the routing solution x^*_K of the R-ORP. To compute $B^O(x)$ of the ORP at each GP iteration, we use the method proposed by [11]. The iterative method in [11] recursively updates link flows in time (iterations) and space (network) until the following stopping criteria is satisfied :

$$\sum_{(i,j)\in L} |B^{O,k}_{ij}(F_{ij}) - B^{O,k-1}(F_{ij})| \le 0.0001 \cdot \sum_{(i,j)\in L} B^{O,k-1}(F_{ij})$$

where k indicates an iteration step.

Using the routing solution of the R-ORP, an actual optimal routing solution of the ORP can be estimated by Theorem 2. From Table 3 and 5, we observe that δ decreases as the GP iterations proceed. The reason is that, from Corollary 2, δ is proportional to l_O' multiplied by h. As ρ_R decreases, l_O' and l_R' also decrease monotonically. From Table 4 and 6, it is noted that the difference between l_O' and l_R' is less than 1% and reduces to 0% around x_R^* (when $\rho_R=0.66$ in Table 4 and $\rho_R=0.81$ in Table 6). It is obvious that $B^O(x_O^*) < B^R(x_O^*)$ since $F_{ij}^O < F_{ij}^R$, $(i,j) \in \mathcal{L}$ for all $x=\{x_p\}$, $p \in P_w$, $w \in W$. Assume that l_R' at x_R^* is always greater than or equal to l_O' at x_O^*. This assumption is reasonable since $B^R(x_R^*) \geq B^O(x_O^*)$ – but yet to be proved. Then, the upper bound of δ can be estimated : $\delta = hl_O' \leq hl_R' = 0.03\%$ (80 OD pairs) and $\delta = hl_O' \leq hl_R' = 7.49\%$ (95 OD pairs). For both cases (85 and 95 OD pairs), $h=9$ at the final routing solutions. The upper bound of δ can be reduced further by increasing the size of output buffer. If $b=50$, $\delta = 0.00\%$ (85 OD pairs) and $\delta=0.21\%$ (95 OD pairs). In practice, the above theoretical upper bound of δ is quite loose. From Table 3 and 5, when the routing solutions of the R-ORP are obtained, δ approaches to 0 for both cases.

Table 3. Average network packet loss probability and percentage difference (80 ODs)

ρ_R	$B^R(x)$	$B^O(x)$	$\delta(\%)$
0.95	$5.31 \cdot 10^{-3}$	$5.05 \cdot 10^{-3}$	5.1
0.90	$2.59 \cdot 10^{-3}$	$2.53 \cdot 10^{-3}$	2.4
0.85	$9.06 \cdot 10^{-4}$	$8.99 \cdot 10^{-4}$	0.8
0.80	$1.76 \cdot 10^{-4}$	$1.76 \cdot 10^{-4}$	0.0
0.75	$3.01 \cdot 10^{-5}$	$3.01 \cdot 10^{-5}$	0.0
0.70	$4.43 \cdot 10^{-6}$	$4.43 \cdot 10^{-6}$	0.0
0.66	$6.96 \cdot 10^{-7}$	$6.96 \cdot 10^{-7}$	0.0

Table 4. Maximum first derivative of link packet loss rate (80 ODs)

ρ_R	l_R'	l_O'
0.95	$2.85 \cdot 10^{-1}$	$2.84 \cdot 10^{-1}$
0.90	$1.09 \cdot 10^{-1}$	$1.09 \cdot 10^{-1}$
0.85	$3.33 \cdot 10^{-2}$	$3.33 \cdot 10^{-2}$
0.80	$7.76 \cdot 10^{-3}$	$7.76 \cdot 10^{-3}$
0.75	$1.48 \cdot 10^{-3}$	$1.48 \cdot 10^{-3}$
0.70	$2.35 \cdot 10^{-4}$	$2.35 \cdot 10^{-4}$
0.66	$3.78 \cdot 10^{-5}$	$3.78 \cdot 10^{-5}$

Table 5. Average network packet loss probability and percentage difference (95 ODs)

ρ_R	$B^R(x)$	$B^O(x)$	$\delta(\%)$
0.95	$8.32 \cdot 10^{-3}$	$7.91 \cdot 10^{-3}$	5.2
0.90	$2.71 \cdot 10^{-3}$	$2.66 \cdot 10^{-3}$	1.9
0.85	$6.80 \cdot 10^{-4}$	$6.77 \cdot 10^{-4}$	0.4
0.81	$1.48 \cdot 10^{-4}$	$1.48 \cdot 10^{-4}$	0.0

Table 6. Maximum first derivative of link packet loss rate (95 ODs)

ρ_R	l_R'	l_O'
0.95	$2.88 \cdot 10^{-1}$	$2.88 \cdot 10^{-1}$
0.90	$1.14 \cdot 10^{-1}$	$1.14 \cdot 10^{-1}$
0.85	$3.36 \cdot 10^{-2}$	$3.36 \cdot 10^{-2}$
0.81	$8.32 \cdot 10^{-3}$	$8.32 \cdot 10^{-3}$

V. Summary

This paper proposes that the GP algorithm can still be valid for solving the non-convex ORP in high-speed networks. For this purpose, the original ORP is transformed into the R-ORP by assuming no packet losses at intermediate switching nodes. The difference of input link flows between the ORP and the R-ORP, and its upper-bound is derived. The smaller the difference of input link flows is, the closer the routing solution of the R-ORP is to that of the ORP. The percentage difference of optimal routing solutions between the ORP and the R-ORP is also derived analytically. However, the upper bound of the percentage difference is quite loose. Numerical examples show that the actual differences are much smaller than the theoretical ones. In summary, applying the GP algorithm to the R-ORP in high-speed networks provides an approximation to the optimal routing solution while saving computation time. It turns out that this approximate routing solution is close enough to the actual optimal routing solution if the network is not heavily congested.

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