

## Improvement and Application of Knutsson Filter for Texture Image Segmentation

Tae Hong Ahn, Ik Su Choy, Jae Seon Seo, Ja Min Kim, Jong An Park *Regular Members*

### 질감 영상의 영역분할을 위한 Knutsson 필터의 개선과 응용

正會員 安泰洪\*, 催益守\*\*\*\*, 徐載宣\*\*, 金在玟\*\*\*, 朴鍾安\*\*\*\*

본 논문은 1993년도 조선대학교 학술연구비의 지원을 받아 연구되었음

#### Abstract

This paper is to improve the 2-D Quadrature Polar Separable(QPS) filter, which uses an exponential attenuation function as the orientational function and the algebraically shifted radial weighting function. It is easier to control the frequency characteristics of the improved filter compared with Knutsson's.

In order to estimate the orientation and the frequency component of local texture in the frequency domain, a series of experiments have been carried out. The maximum error of orientational angle estimates in Knutsson filter is about 7 degrees, whereas the maximum error in the improved filter is 4 degree and the RMSE is only 2. 67 degree. And segmentation results show that it can be used as an efficient tool in texture processing.

#### 요약

본 논문은 필터의 방향각함수로 지수감쇠함수를 이용하며, 레디얼 가중함수의 대수적 편이를 용이하도록 개선한 2-D 직교극분리형(quadrature polar separable) 필터를 제안한다. 이것은 Knutsson 필터와 비교하여 주파수특성의 제어가 용이하다.

주파수영역에서 국부 질감영상의 방향과 주파수 성분을 평가하기 위하여 일련의 실험을 수행하였다. Knutsson 필터에서의 방향각 평가에 대한 최대오차는 7 °인 반면 개선된 필터의 최대오차는 4 °이고, RMSE는 단지 2. 67 °에 지나지 않는다. 또한 영역분할 결과는 개선된 필터가 질감영상 처리에 효과적 도구로 이용될 수 있음을 보였다.

\* 전남대학교 전자계산학과  
 \*\* (주)건화 통신  
 \*\*\* 동신대학교 전자공학과  
 \*\*\*\* 조선대학교 전자공학과  
 論文番號 : 94324  
 接受日字 : 1994年 11月 19日

## I. Introduction

Texture image analysis has been considered using various approaches. One approach performs processing directly upon the grey levels(e. g. , in spatial domain) in texture image. The second approach is based on the spatial frequency spectrum. The prime aim of the process is to smooth or restrict the frequency components of the data.

Various methods designing the digital filters have been reported and applied to a range of aspects in texture image processing.<sup>1,2)</sup> Among them Gabor filter is one of the most commonly used. Gabor filter can be easily implemented and exhibits the interesting characteristics for achieving optimal joint resolution in space and spatial frequency.<sup>2)</sup> However, since the contour of the frequency response of a Gabor filter is in elliptical shape, it cannot effectively tessellate the frequency plane when strongly directional textures are considered. For example, it requires many Gabor filters occupying different radial frequency bands in the same orientation in order to achieve effective discrimination of directional textures.

Knutsson has suggested a filter to tackle this problem, which means that directional textures can be discriminated using a small number of wider bandwidth filters.<sup>4)</sup> Knutsson proceeds the design procedures in Fourier domain. In his method a quadrature kernel pair is generated which are separable in polar coordinates in the Fourier domain. The filter response is characterised by two functions: a radial weighting function and an orientational weighting function.

However Knutsson filter is not optimal in terms of energy loss. Zhao et al. have developed a quadrature polar separable(QPS) filter with near optimal characteristics in terms of energy loss.<sup>5-7)</sup>

The purpose of this work is to improve Knutsson filter using an exponential attenuation function as the orientational function, and shifting algebraically the central frequency. In Section II, we briefly describe Knutsson's method. In Section III, we derive the exponential attenuation function for the orientation tuning and then introduce the improved Knutsson filter. In Section IV, we demonstrate that the new filter can be used as an efficient tool for the estimation of the frequency and orientation of local image textures. Finally we conclude with Section V.

## II. Knutsson Filter

The frequency response of a Knutsson filter is characterised by two independent kernel functions. The kernel functions which suggested by Knutsson are separable in polar coordinates in the Fourier domain. The kernel functions are expressed as in equation(1) and (2).

$$F_e(\rho, \varphi) = V_k(\rho) \cdot V_e(\varphi) \quad (1)$$

$$F_o(\rho, \varphi) = V_k(\rho) \cdot V_o(\varphi) \quad (2)$$

where,

$\rho$  : the polar variation of frequency,

$\varphi$  : the angle variation of frequency,

$V_k(\rho)$  : a radial weighting function and

$V_e(\varphi)$  and  $V_o(\varphi)$  : even and odd symmetrical orientational weighting functions

The radial weighting function  $V_k(\rho)$  is given as

$$V_k(\rho) = \exp\left[1 - \frac{4}{\log 2} \cdot B^{-2} \cdot \text{Ln}^2\left(\frac{\rho}{\rho_i}\right)\right] \quad (3)$$

where  $\rho_i$ , the central frequency, denotes the peak value of the function along the radial axis, and B is the frequency bandwidth. And  $V_e(\varphi)$ ,  $V_o(\varphi)$ , the orientational weighting functions, are expressed as

$$V_e(\varphi) = \text{COS}^{2A}(\varphi - \varphi_k) \quad (4)$$

$$V_o(\varphi) = V_e(\varphi) \cdot \text{Sin}[\cos(\varphi - \varphi_k)] \quad (5)$$

where  $A$  is the bandwidth of the orientational angle, and  $\varphi_k$  specifies the preference orientation the function has been tuned. Thus the frequency response of a Knutsson filter is given as

$$F(\rho, \varphi) = F_c(\rho, \varphi) + j \cdot F_o(\rho, \varphi) \quad (6)$$

The important feature on the filter function for such a kernel pair is that they should have a smooth variation and unimodal property, and that it is quadrature and polar separable in frequency domain. Therefore estimates of local orientation and frequency are produced by combining the output from quadrature filters with different frequency characteristics. These estimates are shown to provide good discrimination of different textures.

### III. Improved Knutsson(IK) Filter

We proceed to describe the suggested Improved Knutsson Filter. Our filter also has the property that its frequency response is polar separable. Equation(7) characterises the filter response.

$$F_{Ik}(\rho, \varphi) = F_{Ike}(\rho, \varphi) + j \cdot F_{Iko}(\rho, \varphi) \quad (7)$$

where the quadrature function pair  $F_{Ike}(\rho, \varphi)$  and  $F_{Iko}$  are given by

$$F_{Ike}(\rho, \varphi) = V_{Ik}(\rho) \cdot V_{Ike}(\varphi) \quad (8)$$

$$F_{Iko}(\rho, \varphi) = V_{Ik}(\rho) \cdot V_{Iko}(\varphi) \quad (9)$$

The first part of the IK filter, the radial weighting function  $V_{Ik}(\rho)$ , is similar to the one used in Knutsson filter(equation(3)). Recall equation(3), the radial bandwidth of the

Knutsson filter will be increased as increasing the central frequency  $\rho_i$  with a fixed value for parameter  $B$ . Then the Knutsson filter will not be appropriate to use in the case of discriminant textures with high dominant frequency contents. So we suggest an alternative method to obtain a radial function profile to deal with the situation mentioned above. We substitute some fixed value for the parameters  $\rho_i$  and  $B$  in equation(3) to obtain a radial function with the desirous radial bandwidth. Then we shift the radial profile to the required central frequency. The new radial weighting function is formulated as follows:

$$V_{Ik}(\rho) = \begin{cases} V_k(\rho - \rho_o), & \rho \geq \rho_o \\ 0 & \rho < \rho_o \end{cases} \quad (10)$$

where  $\rho_o$  is a radial frequency offset. Thus the real central frequency  $\rho_i$  is given by:

$$\rho_i = \rho_o + \rho_i \quad (11)$$

The second part of the IK filter is two orientational weighting functions,  $V_{Ike}(\varphi)$  and  $V_{Iko}(\varphi)$  and which form a quadrature pair. The orientational weighting function is obtained by combining two exponential attenuation functions  $V_1(\varphi)$  and  $V_2(\varphi)$  and given as

$$V_{Ike}(\varphi) = V_1(\varphi) + V_2(\varphi) \quad (12)$$

$$V_{Iko}(\varphi) = V_1(\varphi) - V_2(\varphi) \quad (13)$$

The two exponential attenuation functions, which approximate the Fourier transform of the first order Prolate Spheroidal function, are expressed as follows.

$$V_1(\varphi) = \begin{cases} e(-k_c(\varphi - \varphi_k)^2), & \varphi \leq 180 + \varphi_k \\ e(-k_c(360 - \varphi + \varphi_k)^2), & \varphi > 180 + \varphi_k \end{cases} \quad (14)$$

$$V_2(\varphi) = \begin{cases} e(-k_c \cdot (180 - \varphi_k + \varphi)^2), & \varphi \leq \varphi_k \\ e(-k_c(\varphi - \varphi_k - 180)^2), & \varphi > \varphi_k \end{cases} \quad (15)$$

where  $\varphi_k$  is the primary phase and  $kc$  is the attenuation coefficient which controls the orientational bandwidth of the filter. An asymptotic representation of the Fourier transform of the first order prolate spheroidal function obtained from Zhao et al.<sup>5)</sup> is expressed.

$$\Psi_1(c, \omega T/2\omega_c) = (c/2)^{3/4} (\omega T/\omega_c) \exp(-c\omega^2 T^2/8\omega_c^2) \quad (16)$$

The optimal value for the attenuation coefficient  $kc$  is evaluated using a least square method by approximating function  $V_1$  and  $V_2$  to equation(16).

The filter response can be considered by the kernel functions. Fig.1 shows the plots of the functions  $V_{IK}(\rho)$ ,  $V'_{IK}(\rho)$ ,  $V_{IK\phi}(\varphi)$ , and  $V_{IK\phi}(\varphi)$  with the filter parameters;  $NF=48$ (filter length).  $B=1.0$ ,  $kc=0.0026$ ,  $\varphi_k=22.5^\circ$ ,  $\rho_i=2^{-1.5}$ ,

and  $\rho_o=-5$ . Fig.2 shows the perspective view of the frequency response of an IK filter, which is composed of equation(7), and Fig.3 shows the contour plot of the filter response, with the above parameters.

The new filter has the following properties:

- ※ As the new filter is based on the Knutsson filter, it is quadrature and polar separable.
- ※ the central frequency can be shifted algebraically in contrast to the Knutsson filter.
- ※ As the exponential attenuation function is used as the orientational weighting function, it is more convenient to design the optimal filter.
- ※ As it uses the shift parameter of the central frequency and the exponential attenuation function of the angle, it is not only easy to design, but also easy to estimate local orientation and frequency components.

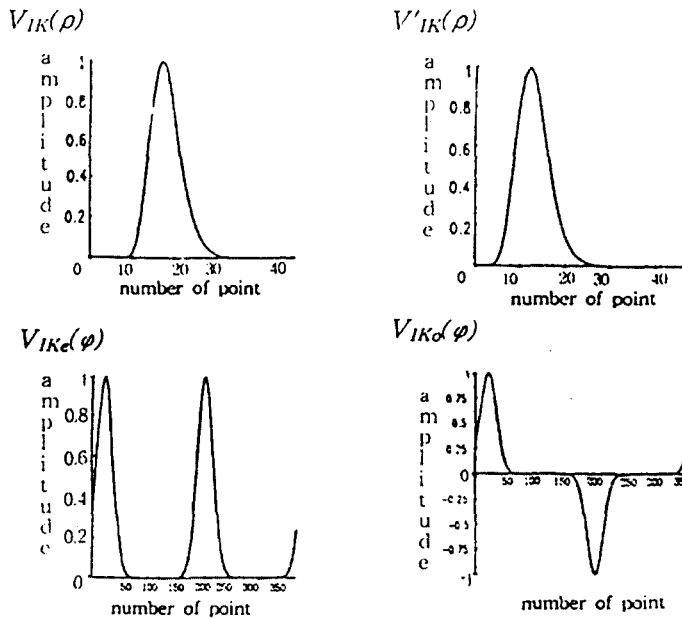


Fig. 1 Plots of the functions  $V_{IK}(\rho)$ ,  $V'_{IK}(\rho)$ ,  $V_{IK\phi}(\varphi)$  and  $V_{IK\phi}(\varphi)$

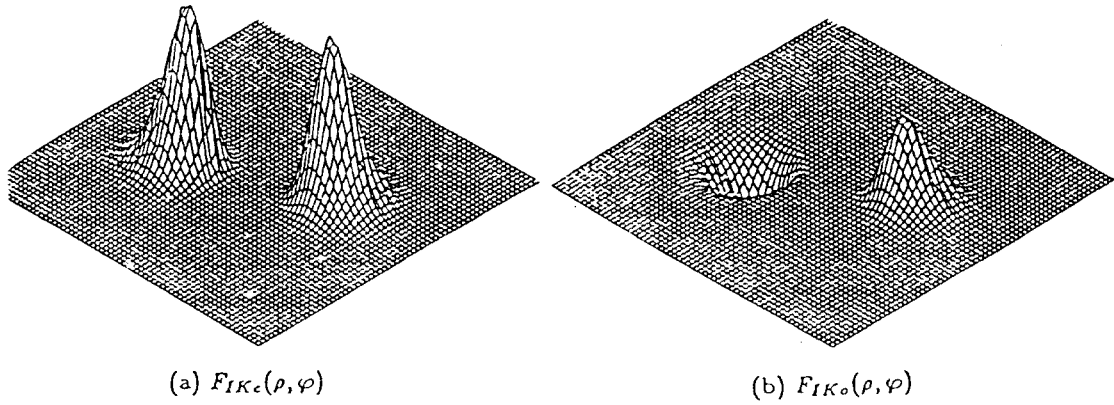


Fig.2 3-D frequency responses of the new filter function  $V_{IKc}(\rho, \varphi)$  and  $V_{IKo}(\rho, \varphi)$

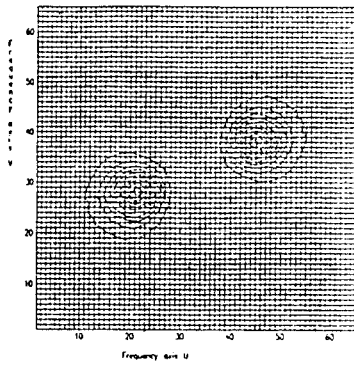


Fig. 3 Contour plot of  $F_{IKo}(\rho, \varphi)$

※ The frequency response is similar to the Knutsson filter's.

#### IV. Applications

In order to estimate the orientation and the frequency components of a local texture in the Fourier domain, we can form a set of the narrow bandwidth filters. When a random noisy image is passed through the filter, the synthetic texture with the corresponding texture orientation is

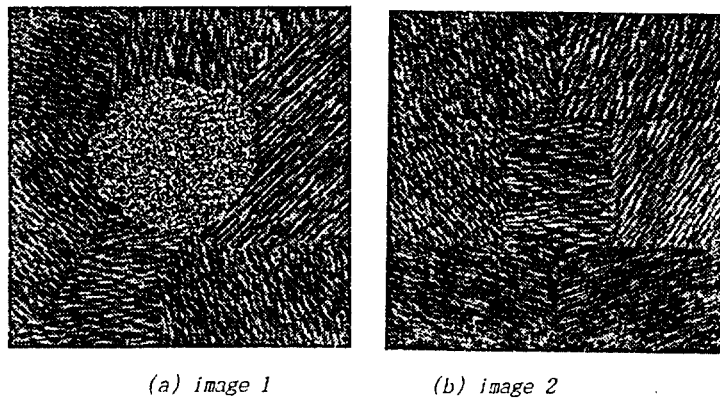


Fig.4 Synthetic texture images

produced. In our experiments we have driven a set of the narrow bandwidth orientational filters which have orientation angle  $\varphi_k=0^\circ, 22.5^\circ, 45^\circ, 67.5^\circ, 90^\circ, 112.5^\circ, 135^\circ, \text{ and } 157.5^\circ$ .

Two synthetic texture images which consist of seven different local textures are given in Fig.4

As the QPS filter can extract information on texture orientation and frequency, it may be used for feature description and segmentation as the following steps.

1. Choose four bandpass QPS filter with the same bandwidth but different orientations. In our experiment we have used the following parameters:  $NF=10, kc=0.0026, \rho_i=2^{-1.5}, \rho_o=-1, B=1.2$  and four orientation angles  $\varphi_k=0^\circ, 45^\circ, 90^\circ, \text{ and } 135^\circ$ . The frequency domain contour plot of the orientational filter based on four bandpass filters is shown in Fig 5. These filters are much wider than the filters used to created the test texture images.

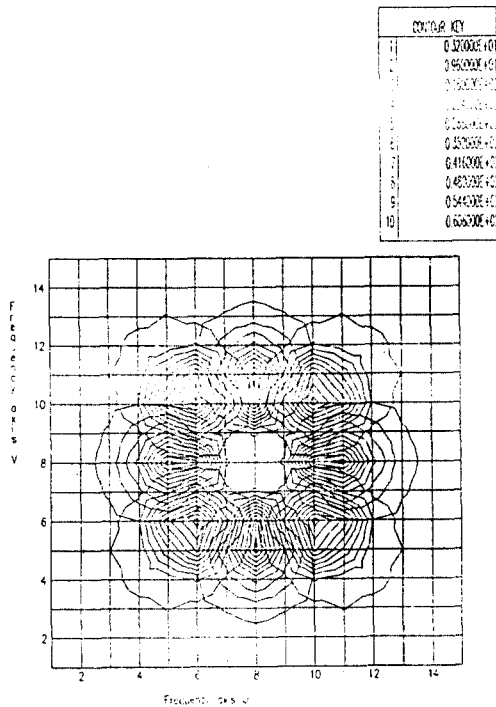


Fig 5. contour plot of four bandpass filters

Table 1 shows the estimated orientation angles to these 1-D synthetic textures after they have been analyzed by these orientational filters. the equation used to estimate the RMSE is as follows.

$$RMSE = \sqrt{1/k \sum_{i=1}^k (\varphi_{is} - \varphi_{ie})^2} \quad (17)$$

The maximum error is  $6.23^\circ$  and the minimum error is only  $2.67^\circ$

Table 1: Results of estimated local texture orientation

para. \ True ang.	B=1.4 $\rho_o=1$ $K_c=$ 0.0026	B=1.2 $\rho_o=-1$ $K_c=$ 0.0026	B=1.0 $\rho_o=-1$ $K_c=$ 0.05	B=1.4 $\rho_o=0$ $K_c=$ 0.05	B=1.0 $\rho_o=0$ $K_c=$ 0.003	B=1.0 $\rho_o=-1$ $K_c=$ 0.008
0	176(4)	176(4)	178(2)	2	2	177(3)
22.5	20	19	13	21	28	16
45	45	45	44	44	44	45
67.5	66	65	68	71	64	70
90	94	93	92	88	87	94
112.5	110	113	112	98	99	118
135	134	134	135	140	135	135
157.5	154	154	160	168	166	157
RMS err	2.74	2.67	3.64	4.73	6.23	3.60

2. Filter the synthetic texture image using the four filter pairs.
3. For each pixel form a vector whose components are energy values computed from the filter outputs.

$$f(x, y) = [f_0(x, y), f_1(x, y), f_2(x, y), f_3(x, y)]^T \quad (18)$$

$$f_i(x, y) = ((g(x, y) * F_{iK_{ei}}(x, y))^2 + (g(x, y) * F_{iK_{oi}}(x, y))^2)^{0.5} \quad (19)$$

where  $i=0, 1, 2, 3$

where  $g(x,y)$  is the input image, the asterisk(\*) denotes convolution.  $F_{iK}$  denotes impulse re-

sponse of the new filter after changing the Cartesian coordinates.

4. Segment the synthetic image using the non parametric classification method proposed by Spann and Wilson. This method includes quadtree smoothing or Gaussian smoothing.<sup>8,9)</sup> This includes quadtree smoothing, local centroid clustering and boundary estimation, the segmentation flowchart followed by the above process is shown in Fig.7.

Results of texture segmentation and classification of the Fig.4 based on four QPS filter pairs are shown in Fig.6

## V. Conclusions

An improved 2-D QPS filter has been proposed in this paper. As the filter uses the exponential attenuation function of the angle and the central frequency can be shifted algebraic-

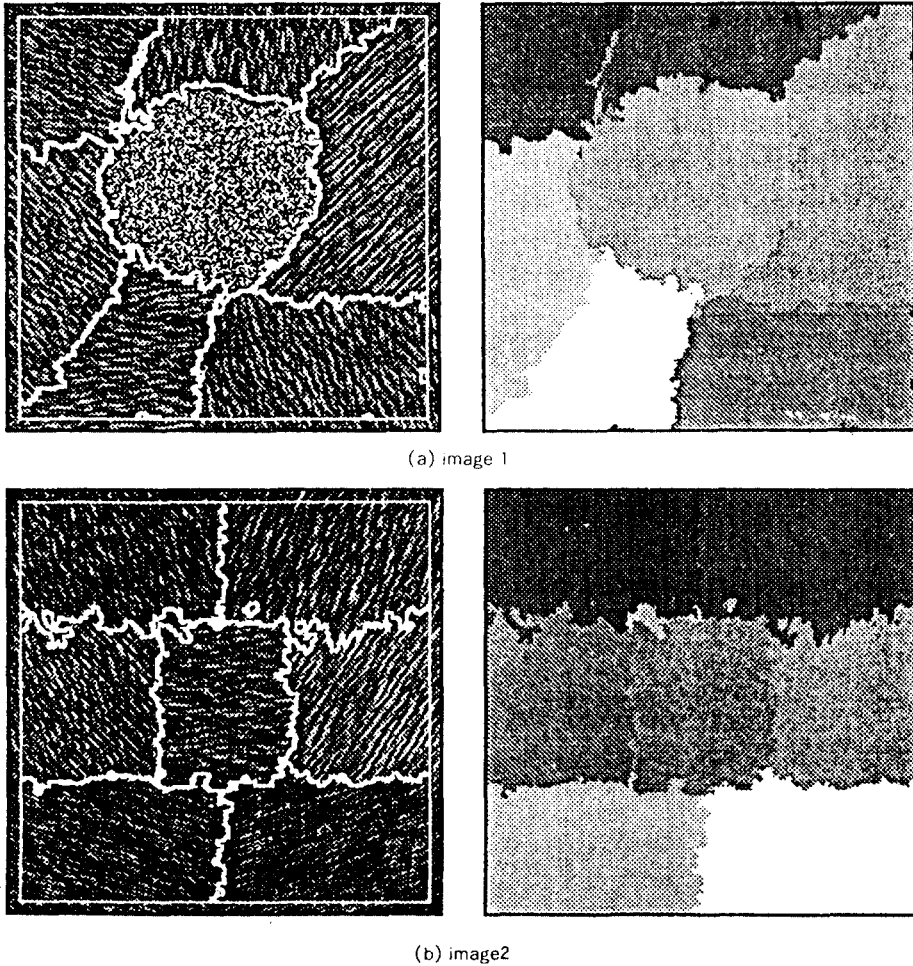


Fig6. Segmentation results of synthetic texture images

ally in contrast to the Knutsson filter, the filter characteristics is simple to control. In our experiments using the eight orientation angles, the estimated maximum error is  $4.0^\circ$  and the RMS error is only  $2.67^\circ$ . It appears the accurate estimation. And the segmentation results to synthetic texture images describe that it is extremely good. Further work is in progress to test the filter on real image data.

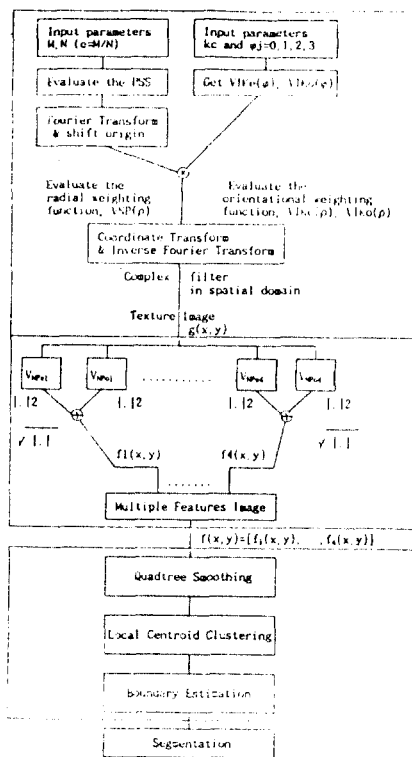


Fig.7 Feature description and segmentation step

References

1. D. W. Tufts and J. T. Francis, "Designing Digital Low-pass Filters-Comparison of some methods and criteria," IEEE Trans. on Audio, Electroacoust, vol. AU-18, pp. 487-494, 1970.

2. K. S. Shanmugam, F. M. Dickey, and J. A. Green, "An Optimal Frequency Domain Filter for Edge Detection in Digital Pictures," IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. PAMI-1, no. 1, pp. 37-49, January 1979.

3. I. Fogel and D. Sagi, "Gabor Filter as Texture Discriminator," Biol. Cyb. , vol. 61, pp. 103-113, 1989.

4. H. Knutsson, "Filtering and Reconstruction in Image Processing," Ph. D. dissertation, Linkoping University, 1982.

5. R. C. Zhao, J. Kittler, J. Illingworth and I. Ng, "An Improved 2-D Polar Separable Filter for Texture Analysis," Proc. of V European Signal Processing Conf. , Barcelona, Sept. 18-21, 1990.

6. R. C. Zhao, J. Kittler, J. Illingworth and I. Ng, "A New Quadrature Polar Separable Filter and its Application to Texture Analysis," IEEE Proc. of the Int. Symposium Circuit and System, New Orleans, Louisiana, May1-3, 1990.

7. R. C. Zhao, J. Kittler, J. Illingworth and I. Ng, "A 2-D Asymptotically Optimal Orientational Filter for Texture Analysis," Proc. of Int. Conf. on New Trends in Communication, Control and Signal Processing, Ankara, July 2-5, 1990.

8. R. Wilson and M. Spann, "Finite Prolate Spheroidal Sequences and Their ApplicationII:Image Feature Description and Segmentation," IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 10, no. 2, pp. 193-203, March 1988.

9. M. Spann and R. Wilson, "A Quadtree Approach to Image Segmentation that Combines Statistical and Spatial Information," Pattern Recognition- vol. 18, pp. 257-269, 1985.





安泰洪(Tae Hong Ahn), 정회원

1987년 : 조선대학교 전자공학과, 학사  
1992년 : 조선대학교 전자공학과, 석사

1993년~현재 : 조선대학교 전자공학과 박사과정  
1993년~현재 : 전남전문대학 전자계산기과 전임강사  
※주요관심분야: 디지털 신호처리, 영상처리



催益守(Ik Su Choy), 정회원

1985년 : 전남대학교 전기공학과, 학사  
1989년 : 미국 퍼시피대 전자공학과, 석사

1993년~현재 : 조선대학교 전자공학과 박사과정  
주요관심분야: 디지털 신호처리, 영상처리



徐載宣(Jae Seon Seo), 정회원

1981년 : 조선대학교 전자공학과, 학사  
1983년 : 조선대학교 전자공학과, 석사

1980년~1988년 : 건창통신공사 근무  
1988년~현재 : (주-전화통신 근무  
※주요관심분야: 디지털 신호처리, 패턴인식, 광통신



金在玟(Jae Min Kim), 정회원

1961년 : 전남대학교 전기공학과, 학사  
1970년 : 조선대학교 전기공학과, 석사

1990년 : 조선대학교 전기공학과 박사  
1967년~1972년 : 조선대학교 공업전문대학 전임강사  
1972년~1976년 : 경호건설 전기담당 이사  
1976년~1987년 : 전주 공업전문대학 조교수  
1987년~현재 : 동신대학교 전자공학과 부교수  
주요관심분야: 디지털 신호처리, 패턴인식



朴鍾安(Jong An Park), 정회원

1975년 : 조선대학교 공과대학 전자공학과, 공학사  
1986년 : 조선대학교 대학원 전기공학과, 공학박사

1983년~1984년: 미국 Massachusetts 주립대학 전기 & 전자공학과 객원교수  
1990년~1991년: 영국 Surrey 대학교 전기 & 전자공학과 객원교수  
1975년~현재: 조선대학교 공과대학 전자공학과 교수  
주요관심분야: 디지털 신호처리, 영상처리, 패턴인식