論文 95-5-5-9

Frequency Offset Effect On Error Probability Of Fast Frequency Hoping Spread Spectrum System With BFSK In Selective Rayleigh Fading Channel

Dong Ku Kim* Regular Members

주파수 선택적 페이딩채널에서 주파수에러에 대한 Fast Frequency Hoping 대역확산 시스템성능 평가

正會員 金 東 九*

ABSTRACT

Bit error probability of fast frequency hoping spread spectrum with BFSK(FFH/SS/BFSK) modulation in selective Rayleigh fading channel has been studied[1]. In this study, we show how the carrier frequency offset affects the performance of FFH/SS/BFSK system on fading multipath channel. The bit error probability is shown in a summation form using Laguerre polynomial decomposition technique, while it would be shown in joint double integration form without using Laguerre polynomial decomposition.

要 約

본 연구에서는 frequency tracking loop에서 발생할 수 있는 반송파 주파수의 오차로 인한 fast frequency hoping BFSK 확산 시스템성능의 영향을 연구하였다. AWGN과 다중경로 페이딩 채널에서 비트 오류 확률을 반송파 주파수의 오차의 함수로 나타내었고, 유도과정에서 Laguerre Polynomials Decomposition방법을 사용함으로서 비트 오류 확률을 컴퓨터로 쉽게 계산할 수 있는 수식으로 보였다.

* 연세대학교 공과대학 전파공학과
論文番號 : 94245-0909
接受日字 : 1994年 9月 9日

1. Introduction

Spread spectrum communication systems have been widely studied and mainly used for military applications. They provide a certain degree of protection against intentional or unintentional jamming. Synchronous carrier recovery is a difficult task in a fading multipath environment, which leads to the consideration of noncoherent demodulation techniques. Fast frequency hopping spread spectrum(FFH-SS) systems using frequency-shift keying modulation of a binary bit stream appear attactive. Bit error rate of FFH/SS/BFSK has been analyzed on selective Rayleigh and Rician fading channel[1]. However bit error rate can be degraded by carrier frequency offset in the frequency tracking loop.

This paper is composed as follows. The model of the FFH/SS/BFSK system is presented in Section 2. Detailed description of the statistics of the envelope detector outputs is given in Section 3. The probability density function of envelope detectors is decomposed by Laguerre polynomials in Section 4. In Section 5, the effect of frequency offset to system performance is evaluated using Laguerre polynomial decomposition technique. The bit error probability is shown in infinite summation form using Laguerre polynomial decomposition technique which is evaluated easily by computer, while it would be shown in joint double integration form without using Laguerre polynomial decomposition. In Section 6, numerical results are presented.

2. System Model and Definition

2.1 Transmitter Model

The data signal $a(t)$ at the input of the modulator is a sequence of nonoverlapping rectangular pulses of duration T, a_l, the amplitude of the lth pulse, $lT \leq t \leq (l+1)T$, gets value from $\{1, -1\}$ with equal probability. The signal, at the output of the FFH/SS/BFSK modulator performing one hop per data bit, is given by

$$s(t) = \sqrt{\frac{2E_b}{T}} \sum_{l=-\infty}^{\infty} P(t-lT)\cos\left[2\pi\left(f_c + f_l + a_l \frac{\beta}{2T}\right)t - \theta_l\right] \quad (1)$$

where $P(t)$ denotes a rectangular pulse of unit height and duration T, E_b is the signal energy per bit, fc is the carrier frequency and $\beta/2T$, $(\beta=1, 2, \cdots,)$ is one-half the spacing between the two FSK tones and the phase angle θ_l is a random variable uniformly distributed between 0 and 2π. The frequency f_l(l Modulo K) assumes values from the set $F=\{F_0, F_0+C/T, \cdots, F_0+(K-1)C/T\}$ where C is a positive integer and K denotes the number of frequencies used in frequency hoping and F0 is the fundamental hop frequency and T is the symbol duration. We assume that $(K-1)T \geq T_m$ in which T_m is the delay time spread of channel. This way of determining K can guarantee that the data bit al, transmitted with a given frequency f_l, is not perturbed by an echo using the same frequency emanating from another data bit.

2.2 Channel Model and Receiver Signal

The multipath characteristics and additive noise are assumed to be statistically independent. The multipath comprises M fading paths. Each path results from a different scattering channel. The mth path($m=0, 1, \cdots M-1$) is associated with three random processes $v_{m,l}(t)$, τ_m and $\theta_{m,l}(t)$ that respectively describe the strength coefficient, the time delay and the carrier phase shift introduced by the path. The index l is due to the fact that the transmitted frequency changes every T seconds. Since the Doppler shift is a function of the frequency of the transmitted signal, the strength coefficient and the carrier phase shift of each path are also functions of the frequency f_l transmitted during$[lT, (l-1)T]$. It can be assumed, however, that the path characteristics do not vary significantly within a transmission period of T so that these processes can be respectively described by the single random variables $v_{m,l}(t)$, τ_m and $\theta_{m,l}(t)$. Then the received signal can be written from Equation(1) as follows:

$$R(t) = \sqrt{\frac{2E_b}{T}} \sum_{m=0}^{M-1} \sum_{l=-\infty}^{\infty} v_{m,l} P(t - lT - \tau_m) \cos[2\pi(f_c + f_l + a_l\beta/2T)t - \theta_{m,l}] + N(t) \quad (2)$$

where $N(t)$ is AWGN with double-sided power density $N_0/2$, each $\theta_{m,l}$ is assumed to be uniformly distributed over$[0, 2\pi]$. All path strengths vm,l are i.i.d., in which pdf of $v_{m,l}$ is shown as

$$p_{v_{m,l}} = \frac{2v_{m,l}}{b_m} \exp\left\{-\frac{v_{m,l}^2}{b_m}\right\}, v_{m,l} \geq 0, \ \forall l, \ \forall m,$$

where bm is a constant which satisfies

$$\int p_{v_{m,l}} dv_{m,l} = 1.$$

3. Receiver Model

The FFH/SS/BFSK receiver is depicted in Figure 1. It consists of a frequency dehopper followed by an envelope detector. The dehopping signal $U(t)$ introduces random phase φ_l during the time intervals between hops:

$$U(t) = \sum_{l=-\infty}^{\infty} P(t - lT) \cos(2\pi f_l t + \varphi_l) \quad (3)$$

l modulo K.

The only assumption made is that the receiver is time synchronous with the main(first arriving) path(i.e., τ_0=0). The normalizing condition assumed that the mean received energy per bit at the input of the envelope detector is equal to E_b, i.e.

$$E\left\{\frac{2E_b}{T}\int_{-\infty}^{\infty}\left[\sum_{m=0}^{M-1} v_{m,l} P(t - lT - \tau_m) \cos(2\pi(f_c + a_l\beta/2T)t - \theta_{m,l} - \varphi_l)\right]^2 dt\right\} = E_b \quad (4)$$

where it can be shown that $\theta_{m,l} + \varphi_l$ is uniformly distributed between 0 and 2π and therefore, left hand side of Equation (4) is reduced to

$$\frac{2E_b}{T}\sum_{m=0}^{M-1} E(v_{m,l}^2)\frac{T}{2}, \quad (5)$$

$b_m = E(v^2_{m,l})$ and Equation(5) = Eb, it can be deduced that

$$\sum_{m=0}^{M-1} b_m = 1.$$

The square-law envelope detector has two branches each with in-phase and quadrature subbranchs. The output of the branch corresponding to "+1" data bits is denoted by $X(n)$ and the output corresponding to "-1"data bits by $Y(n)$ where

$$X(n) = \sqrt{[X_1(n)]^2 + [X_2(n)]^2}$$
$$Y(n) = \sqrt{[Y_1(n)]^2 + [Y_2(n)]^2}.$$

$X_1(n)$ and $Y_1(n)$ are the outputs of the in-phase subbranchs, and $X_2(n)$ and $Y_2(n)$ are the outputs of the quadrature subbranchs:

$$X_1(n) = \frac{4}{T}\int_{nT}^{(n+1)T} R(t)U(t)\cos\left[2\pi(f_c + f_d + \frac{1}{2T})t\right] dt \quad (6)$$

$$X_2(n) = \frac{4}{T}\int_{nT}^{(n+1)T} R(t)U(t)\sin\left[2\pi(f_c + f_d + \frac{1}{2T})t\right] dt \quad (7)$$

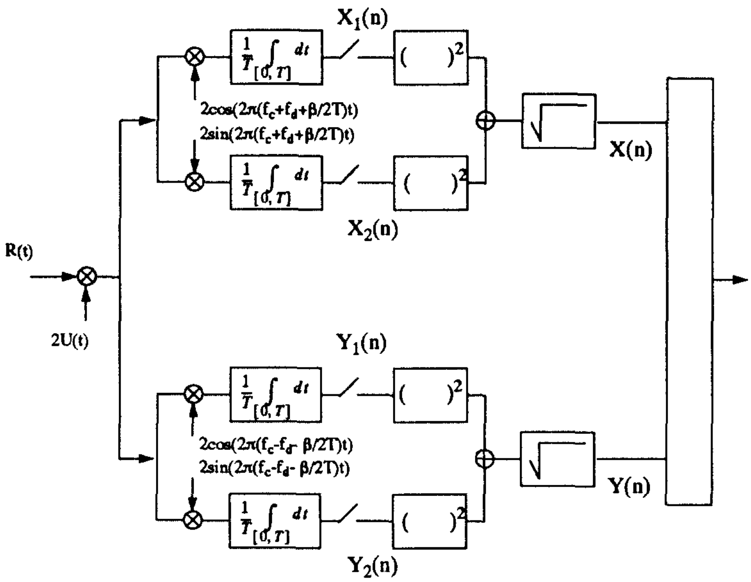

Figure 1. Envelope detector structure

where f_d is frequency offset caused by frequency tracking loop. $Y_1(n)$ and $Y_2(n)$ have the same expression as $X_1(n)$ and $X_2(n)$ with replacing $1/2T$ by $-1/2T$.

3.1 Statistics of In-phase and Quadrature Subbranchs Output

The marginal probability density functions of in-phase and quadrature subbranchs output, *i.e.*, X_1, X_2, Y_1 and Y_2 are computed in this section. The output of in-phase subbranchs of Xn, X_1, is represented by substitute Equation (3) to (6),

$$X_1(n)=\frac{1}{T}\sqrt{\frac{2E_b}{T}}\int_{nT}^{(n+1)T}\sum_{m=0}^{M-1}\sum_{l=-\infty}^{\infty} v_{m,l}P(t-lT - \tau_m)\cos[2\pi(f_c+f_l+a_l\beta/2T)t-\theta_{m,l}]$$

$$\times 2\sum_{l'=-\infty}^{\infty} P(t-l'T)\cos(2\pi f l' t+\phi_{l'})2\cos(2\pi(f_c+f_d + \frac{\beta}{2T})t)dt + \frac{1}{T}\int_{nT}^{(n+1)T} N(t)\times 2\sum_{l'=-\infty}^{\infty} P(t-l'T) \cos(2\pi f l' t+\phi_{l'})2\cos(2\pi(f_c+f_d+\frac{\beta}{2T})t)dt. \quad (8)$$

Assuming $F_0 \gg 1/T$ and $F_0 \gg f_d$, we separate Equation(8) to signal part(term of m=0), intersymbol interference(terms of nonzero m), and noise after simple integration:

$$X_1(n)=\frac{1}{T}\sqrt{\frac{2E_b}{T}}\frac{\sqrt{2-2\cos(2\pi(f_d-(a_n-1)\beta/2T)T)}}{2\pi(f_d-(a_n-1)\beta/2T)} v_{o,n}\cos(\theta_{0,n}+\varphi_n+\psi_{x,a_n}) + I_{x_1(n)}+N_{x_1(n)}, \quad (9)$$

where

$$\psi_{x,a_n} = \tan^{-1}\frac{1-\cos(2\pi(f_d-(a_n-1)\beta/2T)T)}{\sin(2\pi(f_d-(a_n-1)\beta/2T)T)},$$

and

$$I_{x_1(n)} = \frac{4}{T}\sqrt{\frac{2E_b}{T}}\int_{nT}^{(n+1)T}\sum_{m=1}^{M-1}\sum_{l=-\infty}^{\infty} v_{m,l}P(t-lT - \tau_m)\cos[2\pi(f_c+f_l+a_l\beta/2T)t-\theta_{m,l}] \times 2\sum_{l'=-\infty}^{\infty}P(t-l'T)\cos(2\pi f l' t+\varphi_{l'})2 \cos(2\pi(f_c+f_d+\frac{\beta}{2T})t)dt, \quad (10)$$

$N_{x1(n)}$ is a Gaussian r.v. with zero mean and variance $2N_0/T$. To evaluate Equation (10), the time delay τ_m of the mth path can be expressed as

$$\tau_m = JT + t_{m,J}$$

where $t_{m,J}$ is uniformly distributed between 0 and T_0 and $J \in \{0, 1, \cdots, (K-2)\}$. Ruling out the high frequency term and taking into account the definition of the function $P(t)$, $I_{x1(n)}$ becomes

$$I_{x_1(n)} = \frac{4}{T}\sqrt{\frac{2E_b}{T}}\sum_{J=0}^{K-2}\sum_{m\in N_J}\Bigg[\int_{nT+t_{m,J}}^{(n+1)T} v_{m,n-J} \cos(2\pi(f_c+f_{n-J}+a_{n-J}\beta/2T)t-\theta_{m,n-J}) \times\cos(2\pi f_n t+\varphi_n)\cos(2\pi(f_c+f_d-\frac{\beta}{2T})t) dt + v_{m,n-J-1}\times\int_{nT}^{nT+t_{m,J}}\cos\left(2\pi(f_c + f_{n-J-1}+a_{n-J-1}\frac{\beta}{2T}\right)t-\theta_{m,n-J-1}) \cos(2\pi f_n+\varphi_n)\times\cos\left(2\pi(f_c+f_d-\frac{\beta}{2T})t\right)dt\Bigg]$$

After finishing integrals of above equation, it can be easily shown that

$$I_{x_1(n)} = \sqrt{\frac{2E_b}{T}}\sum_{J=0}^{K-2}\sum_{m\in N_J}\frac{v_{m,n-J}}{A(n,J,f_d)}\{\sin(A(n,J, f_d)(n+1)-\theta_{m,n-J}-\varphi_n) - \sin(A(n, J,f_d)(n+t_{m,J}/T)-\theta_{m,n-J}-\varphi_n)\} + v_{m,n-J-1}\{\sin(A(n,J+1,f_d)(n+t_{m,J} /T)-\theta_{m,n-J-1}-\varphi_n) - \sin(A(n,J +1,f_d)n-\theta_{m,n-J-1}-\varphi_n)\} \quad (11)$$

where $A(n,J,f_d) \triangleq 2\pi((f_{n-J}-f_n - f_d) + (a_{n-J}-1)\beta/2T)T$. In Equation(11), $v_{m,n-J}$ is Rayleigh distributed and $\theta_{m,n-J} - \varphi_n$ is uniformly distributed, therefore $v_{m,n-J}\sin(\theta_{m,n-J} - \varphi_n)$ is mean zero Gaussian random variable and linear combination of Gaussian random variables is Gaussian. Given a_n and $\{a_{n-i}\}$, $v_{0,n}\cos(\varphi_{0,n}+\varphi_n+\varphi_{x,an})$ is Gaussian with zero mean and $I_{x1(n)}$ is approximated by a Gaussian r.v. with zero mean variance α^2. To determine α^2, assumption is made that Nj denotes all paths arriving with time delay between $[jT, (j+1)T]$ and variance of the strength coefficients $v_{m,J}$, b_m, is the same for all the N_J paths and $S_J \triangleq \Sigma_{m\in NJ} b_m$, and the variable $t_{m,J}$ are uniformly distributed over $[0, T]$.

$$\alpha^2 = E(|I_{x_1(n)}|^2) = \frac{2E_b}{T}\sum_{J=0}^{K-2}\sum_{m\in N_J}\frac{v_{m,n-J}^2}{A^2(n,J,f_d)}\{\sin(A(n,J,f_d)(n+1)-\theta_{m,n-J}-\varphi_n) - \sin(A(n,J,f_d)(n+t_{m,J}/T)-\theta_{m,n-J}-\varphi_n)\}^2 + v_{m,n-J-1}^2\{\sin(A(n,J+1,f_d)(n+t_{m,J}/T)-\theta_{m,n-J-1}-\varphi_n) - \sin(A(n,J+1,f_d)n-\theta_{m,n-J-1}-\varphi_n)\}^2$$

Note that $E\{v^2_{m,n-J}\} = b_m$, and $S_J \triangleq \Sigma_{m\in NJ} b_m$. Then α^2 can be represented as follows,

$$\alpha^2 = \frac{2E_b}{T}\sum_{J=0}^{K-2}S_J\left\{\frac{1}{A^2(n,J,f_d)}\left(1 - \frac{\sin(A(n,J,f_d))-\sin(A(n,J,f_d)(1-T_0/T))}{A(n,J,f_d)T_0/T}\right)\right.$$

$$+\frac{1}{A^2(n,J+1,f_d)}\left(1 - \frac{\sin(A(n,J+1,f_d)T_0/T)}{A(n,J+1,f_d)T_0/T}\right)\Bigg\}$$

Likewise, the output of quadrature subbranchs of $X(n)$, $X_2(n)$, can be evaluated as follows,

$$X_2(n) = \frac{1}{T}\sqrt{\frac{2E_b}{T}}\frac{\sqrt{2-2\cos(2\pi(f_d-(a_n-1)\beta/2T)T)}}{2\pi(f_d-(a_n-1)\beta/2T)} v_{0,n}\sin(\theta_{0,n}+\varphi_n+\psi_{x,a_n}) + I_{x_2(n)} + N_{x_2(n)},$$

where $N_{x2(n)}$ is a Gaussian r.v. with zero mean and variance $2N_0/T$. $I_{x2(n)}$ is approximated by a Gaussian r.v. with zero mean and variance α^2 and v0,n $\cos(\varphi_{0,n}+\varphi_n+\varphi_{x,an})$ given an is Gaussian with zero mean and variance $b_0/2$. $I_{x2(n)}$ can be expressed in the similar way that $I_{x1(n)}$ is expressed:

$$I_{x_2(n)} = \sqrt{\frac{2E_b}{T}}\sum_{J=0}^{K-2}\sum_{m\in N_J}\frac{v_{m,n-J}}{A(n,J,f_d)}\{\cos(A(n,J,f_d)(n+1)-\theta_{m,n-J}-\varphi_n) - \cos(A(n,J,f_d)(n+t_{m,J}/T)-\theta_{m,n-J}-\varphi_n)\} + v_{m,n-J-1}\{\cos(A(n,J+1,f_d)(n+t_{m,J}/T)-\theta_{m,n-J-1}-\varphi_n) - \cos(A(n,J+1,f_d)n-\theta_{m,n-J-1}-\varphi_n)\}. \quad (12)$$

Likewise, we can compute pdf of inphase and quadrature components of $Y(n)$. The output of in-phase subbranch of $Y(n)$, $Y_1(n)$ can be expressed as

$$Y_1(n) = \frac{1}{T}\sqrt{\frac{2E_b}{T}}\frac{\sqrt{2-2\cos(2\pi(f_d-(a_n+1)\beta/2T)T)}}{2\pi(f_d-(a_n+1)\beta/2T)} v_{o,n}\cos(\theta_{0,n}+\varphi_n+\psi_{y,a_n}) + I_{y_1(n)} + N_{y_1(n)} \quad (13)$$

where

$$\psi_{y,a_n} = \tan^{-1}\frac{1-\cos(2\pi(f_d-(a_n+1)\beta/2T)T)}{\sin(2\pi(f_d-(a_n+1)\beta/2T)T)}.$$

Given a_n and $\{a_{n-i}\}$, $v_{0,n}\cos(\theta_{0,n}+\varphi_n+\phi_{y,an})$ is a Gaussian r.v. with zero mean and variance $b_0/2$. $N_{y1(n)}$ is a Gaussian r.v. with zero mean and variance $2N_0/T$. $I_{y1(n)}$ is expressed as follows,

$$I_{y_1(n)} = \sqrt{\frac{2E_b}{T}}\sum_{J=0}^{K-2}\sum_{m\in N_J}\frac{v_{m,n-J}}{B(n,J,f_d)}\{\sin(B(n,J,f_d)(n+1)-\theta_{m,n-J}-\varphi_n) - \sin(B(n,J,f_d)(n+t_{m,J}/T)-\theta_{m,n-J}-\varphi_n)\} + v_{m,n-J-1}\{\sin(B(n,J+1,f_d)(n+t_{m,J}/T)-\theta_{m,n-J-1}-\varphi_n) - \sin(B(n,J+1,f_d)n-\theta_{m,n-J-1}-\varphi_n)\}. \quad (14)$$

and approximated by a Gaussian r.v. with zero mean and variance ζ^2:

$$\zeta^2 = \frac{2E_b}{T}\sum_{J=0}^{K-2}S_J\left\{\frac{1}{B^2(n,J,f_d)}\left(1 - \frac{\sin(B(n,J,f_d)) - \sin(B(n,J,f_d)(1-T_0/T))}{B(n,J,f_d)T_0/T}\right) + \frac{1}{B^2(n,J+1,f_d)}\left(1 - \frac{\sin(B(n,J+1,f_d)T_0/T)}{B(n,J+1,f_d)T_0/T}\right)\right\},$$

where $B(n,J,f_d) = 2\pi(f_{n-J}-f_n-f_d+(a_{n-J}+1)/2T)T$. The output of quadrature subbranch of $Y(n)$, $Y_2(n)$ is

$$Y_2(n) = \frac{1}{T}\sqrt{\frac{2E_b}{T}}\frac{\sqrt{2-2\cos(2\pi(f_d-(a_n+1)\beta/2T)T)}}{2\pi(f_d-(a_n+1)\beta/2T)} v_{o,n}\sin(\theta_{0,n}+\varphi_n+\psi_{y,a_n}) + I_{y_2(n)} + N_{y_2(n)}$$

where $v_{0,n}\sin(\theta_{0,n}+\varphi_n+\psi_{y,an})$ is zero mean Gaussian with variance $b_0/2$ and $Iy_{2(n)}$ is approximated as zero mean Gaussian with variance ζ^2 and $N_{y2(n)}$ is zero mean Gaussian with variance $2N_0/T$.

3.2 Decomposition of Joint Pdf of $X_1(n)$ and $Y_1(n)$ using Laguerre Polynomials

We show joint pdf of $X_1(n)$ and $Y_1(n)$ which is decomposed by Laguerre polynomials. This form is useful to compute bit error probability. $X_I(n)$ is correlated with $Y_I(n)$ and $X_2(n)$ is correlated with $Y_2(n)$ by frequency offset. Correlation coefficient of two in-phase components can be computed by the definition which is,

$$\rho_{x_1(n),y_1(n)} = \frac{E(X_1(n)Y_1(n))}{\sigma_{x_1}\sigma_{y_1}}.$$

Standard variances can be evaluated from variances of $Y_I(n)$ and $X_I(n)$ by taking square root. Variance of $X_I(n)$ can be easily computed using Equation(9). Given an and {an-i}, signal term, ISI term, and noise term are statistically independent so that variance of $X_I(n)$ is just sum of variances of each terms and expressed as

$$\sigma_{x_1}^2 = \frac{2E_b}{T^3}\frac{2-2\cos(2\pi(f_d-(a_n-1)\beta/2T)T)}{(2\pi(f_d-(a_n-1)\beta/2T))^2}\frac{b_0}{2} + \alpha^2 + \frac{2N_0}{T}.$$

Likewise, variance of $Y_I(n)$ can be computed and expressed as,

$$\sigma_{y_1}^2 = \frac{2E_b}{T^3}\frac{2-2\cos(2\pi(f_d-(a_n+1)\beta/2T)T)}{(2\pi(f_d-(a_n+1)\beta/2T))^2}\frac{b_0}{2} + \xi^2 + \frac{2N_0}{T},$$

where α^2 and ζ^2 are defined at the last section. The cross correlation of $X_I(n)$ and $Y_I(n)$ can be computed by multiplying and taking expectation of Equation(9) and Equation(13) which is as follows,

$$\begin{aligned} E(X_1(n)Y_1(n)) \quad = \\ \frac{E_b}{T^3}b_0\frac{1}{2\pi(f_d-(a_n+1)\beta/2T)2\pi(f_d-(a_n-1)\beta/2T)} \\ \times[\sin(2\pi(f_d-(a_n-1)\beta/2T)T) \\ \sin(2\pi(f_d-(a_n+1)\beta/2T)T) \\ +(1-\cos(2\pi(f_d-(a_n-1)\beta/2T) \\ ))(1-\cos(2\pi(f_d-(a_n+1)\beta/2T) \\ ))] \\ +E(I_{x_1(n)}I_{y_1(n)}), \end{aligned}$$

in which cross correlation of $I_{x1(n)}$ and $I_{y1(n)}$ can be computed from Equation (11) and Equation (14) which is,

$$\begin{aligned} E(I_{x_1}I_{y_1}) \quad = \quad & \frac{2E_b}{T}\sum_{J=0}^{K-2} S_J\frac{1}{A(n,J,f_d)B(n,J,f_d)}\Bigg[1 \\ & -\frac{1}{2}\frac{1}{B(n,J,f_d)T_0/T}\{-\sin(2\pi f_d T) \\ & +\sin(2\pi f_d T + B(n,J,f_d)T_0/T)\} \\ & -\frac{1}{2A(n,J,f_d)T_0/T}+\frac{1}{2\times 2\pi T_0/T} \\ & \times(-\sin(2\pi T_0/T))] \\ & +S_J\frac{1}{A(n,J+1,f_d)B(n,J+1,f_d)} \\ & \Bigg[\frac{1}{2\times 2\pi T_0/T}\times(-\sin(2\pi T_0/T)) \\ & -\frac{1}{2A(n,J+1,f_d)T_0/T}(-\sin(A(n, \\ & J+1,f_d)T_0/T) \\ & +\frac{1}{2B(n,J+1,f_d)T_0/T}\sin(B(n,J \\ & +1,f_d)T_0/T)+1\Bigg]. \end{aligned}$$

It is easy to show that $\rho_{x1(n),y1(n)} = \rho_{x2(n),y2(n)}$. Since $X_I(n)$ is independent of $X_2(n)$ and $X_2(n)$ is independent of $Y_2(n)$, $X(n)$ is Rayleigh distributed, so is $Y(n)$. Therefore we can obtain decomposed form of joint pdf of $X(n)$ and $Y(n)$ by using results of[2],

$$p_{x,y}(x,y) = \sum_{k=0}^{\infty}\frac{xy}{\sigma_{x_1}^2\sigma_{y_1}^2}\rho^{2k}L_k\left(\frac{x^2}{2\sigma_{x_1}^2}\right)\left(\frac{y^2}{2\sigma_{y_1}^2}\right)\exp\left\{-\frac{x^2}{2\sigma_{x_1}^2}-\frac{y^2}{2\sigma_{y_1}^2}\right\},$$

where

$$\rho \quad = \quad \frac{E(X_1Y_1)}{\sigma_{x_1}\sigma_{y_1}}$$

and ρ, σ_x and σ_y will be evaluated. Let

$$\sigma_x^2 = \sigma_{x-}^2, \quad \sigma_y^2 = \sigma_{y-}^2, \quad \rho = \rho_- \quad \text{if } a_n = -1$$
$$\sigma_x^2 = \sigma_{x+}^2, \quad \sigma_y^2 = \sigma_{y+}^2, \quad \rho = \rho_+ \quad \text{if } a_n = 1.$$

where time index n is omitted.

4. Bit Error Probability in Rayleigh Selective Fading Channel

The bit error rate performance of the system under consideration is evaluated when a maximum likelihood symbol by symbol detector is used. In the bit interval$[nT, (n+1)T]$ (synchronized to the main path), the received waveform $R(t)$ depends on the symbols a_n, a_{n-1}, ⋯, $a_{n-(K-1)}$. This, in fact, is due to the intersymbol interference(ISI) introduced by the multipath propagation.

The bit error probability given $\{a_{n-i}\}$ is

$$P_e|_{\{a_{n-i}\}} = \frac{1}{2}Pr[X(n) > Y(n)|a_n = -1, \{a_{n-i}\}] + \frac{1}{2}Pr[X(n) < Y(n)|a_n = 1, \{a_{n-i}\}] \quad (15)$$

where $\{a_{n-i}\}$ represents the sequence a_{n-1}, ⋯, $a_{n-(K-1)}$. It will be shown in Section 4.1 that conditional error probability can be expressed as infinite summation form which is numerically computable:

$$Pr[X(n) > Y(n)|a_n = -1, \{a_{n-i}\}] = \frac{1}{2\sigma_{y-}^2}\sum_{k=0}^{\infty}\frac{\rho_-^{2k}}{k!} \left(\frac{\lambda_-\mu_-}{b^2_-}\right)^k \left[\frac{\Gamma(2k+1)}{k!b_-} - \frac{\Gamma(2k)}{(k-1)!\mu_-}\right] \quad (16)$$

whereλ-$=1/(2\sigma^2_{x-})$, $-=1/(2\sigma^2_{y-})$, and $b_-=\lambda_-+\mu_-$ and

$$Pr[X(n) < Y(n)|a_n = +1, \{a_{n-i}\}] = 1 - \frac{1}{2\sigma_{y+}^2}\sum_{k=0}^{\infty}\frac{\rho_+^{2k}}{k!} \left(\frac{\lambda_+\mu_+}{b^2_+}\right)^k \left[\frac{\Gamma(2k+1)}{k!b_+} - \frac{\Gamma(2k)}{(k-1)!\mu_+}\right] \quad (17)$$

where $\lambda_+=1/(2\sigma^2_{x+})$, $\mu_-=1/(2\sigma^2_{y+})$, and $b_+=\lambda_++\mu_+$. By inserting Equation (16) and (17) to Equation (15), we can get

$$\begin{aligned} P_e|_{\{a_{n-i}\}} = & \frac{1}{2}\left[\frac{1}{2}\frac{1}{\sigma_{y-}^2}\sum_{k=0}^{\infty}\rho_-^{2k}\frac{1}{k!}\left(\frac{\lambda_-\mu_-}{b_-^2}\right)^k \left\{\frac{\Gamma(2k+1)}{k!b_-} - \frac{\Gamma(2k)}{(k-1)!\mu_-}\right\}\right] + \frac{1}{2} \\ & -\frac{1}{2}\left[\frac{1}{2}\frac{1}{\sigma_{y+}^2}\sum_{k=0}^{\infty}\rho_+^{2k}\frac{1}{k!}\left(\frac{\lambda_+\mu_+}{b_+^2}\right)^k \left\{\frac{\Gamma(2k+1)}{k!b_+} - \frac{\Gamma(2k)}{(k-1)!\mu_+}\right\}\right]. \end{aligned} \quad (18)$$

Then average bit error probability is computed by averaging Equation (18) over all possible $\{a_{n-i}\}$, *i.e.*,

$$P_e = E_{\{a_{n-i}\}}\{P_e|_{\{a_{n-i}\}}\}.$$

When there is no frequency offset, *i.e.*, $f_d=0$, the bit error rate given $\{a_{n-i}\}$ is obtained by letting ρ be 0, *i.e.*,

$$P_e|_{\{a_{n-i}\}} = \frac{1}{4}\frac{1}{\sigma_{y-}^2}\left(\frac{1}{b_-}\right) + \frac{1}{2} - \frac{1}{4}\frac{1}{\sigma_{y+}^2}\left(\frac{1}{b_+}\right) \quad (19)$$

$$= \frac{\sigma_{x-}^2 + \sigma_{y+}^2}{2(\sigma_{x+}^2 + \sigma_{y+}^2)}. \quad (20)$$

Equation (20) corresponds to result in[1].

4.1 Conditional Error Probability

The conditional error probability given a_n, ⋯, a_n-$(K-1)$ is evaluated by using decomposed form of joint pdf of correlated Rayleigh pdf. When a_n=-1 and $\{a_{n-i}\}$ is given, conditional error probability is

$$Pr(X(n) > Y(n)|a_n = -1, \{a_{n-i}\}) = \int_0^{\infty}\int_y^{\infty}\frac{4xy}{\sigma_{x-}^2\sigma_{y-}^2}exp\left(-\frac{x^2+y^2}{\sigma_{x-}\sigma_{y-}(1-\rho_-^2)}\right) I_0\left(\frac{2xy\rho_-}{\sigma_{x-}\sigma_{y-}(1-\rho_-^2)}\right)dxdy$$

$$= \sum_{k=0}^{\infty} \frac{1}{\sigma_{x-}^2 \sigma_{y-}^2} \rho_-^{2k} \int_0^\infty \int_y^\infty x L_k \left(\frac{x^2}{2\sigma_{x-}^2}\right) \exp\left(\frac{x^2}{2\sigma_{x-}^2}\right) dx \; y L_k \left(\frac{y^2}{2\sigma_{y-}^2}\right) \exp\left(\frac{y^2}{2\sigma_{y-}^2}\right) dy. \quad (21)$$

Putting

$$\frac{x^2}{2\sigma_{x-}^2} = t,$$

then

$$\int_y^\infty x L_k \left(\frac{x^2}{2\sigma_{x-}^2}\right) \exp\left(\frac{x^2}{2\sigma_{x-}^2}\right) dx = \int_{y^2/2\sigma_{x-}^2}^\infty L_k(t) \exp(-t) dt \sigma_{x-}^2. \quad (22)$$

By the identity [3, p884]:

$$\int_y^\infty \exp(-x) L_n^\alpha dx = \exp(-y)[L_n^\alpha(y) - L_{n-1}^\alpha(y)],$$

Equation(22) becomes

$$\sigma_{x-}^2 \exp\left(-\frac{y^2}{2\sigma_{x-}^2}\right) \left[L_k\left(\frac{y^2}{2\sigma_{x-}^2}\right) - L_{k-1}\left(\frac{y^2}{2\sigma_{x-}^2}\right)\right], \quad (23)$$

for k≥1,

$$\int_y^\infty x \exp\left(-\frac{x^2}{\sigma_{x-}^2}\right) dx = \frac{\sigma_{x-}^2}{2} \exp\left(-\frac{y^2}{\sigma_{x-}^2}\right) \quad (24)$$

for k=0. By substituting Equation(23) to Equation(21),

$$Pr(X(n) > Y(n)|a_n = -1, \{a_{n-i}\}) = \sum_{k=0}^{\infty} \frac{\rho_-^{2k}}{\sigma_{x-}^2 \sigma_{y-}^2} \int_0^\infty y \sigma_{x-}^2 \exp\left(-\frac{y^2}{2\sigma_{x-}^2}\right) \left[L_k\left(\frac{y^2}{2\sigma_{x-}^2}\right)\right] L_k\left(\frac{y^2}{2\sigma_{x-}^2}\right) \exp\left(-\frac{y^2}{2\sigma_{y-}^2}\right) dy$$

$$= \sum_{k=0}^{\infty} \frac{\rho_-^{2k}}{\sigma_{x-}^2 \sigma_{y-}^2} \int_0^\infty y \sigma_x^2 - \exp\left(-\frac{y^2}{2\sigma_{x-}^2}\right) \left[L_{k-1}\left(\frac{y^2}{2\sigma_{x-}^2}\right)\right] L_k\left(\frac{y^2}{2\sigma_{x-}^2}\right) \exp\left(-\frac{y^2}{2\sigma_{y-}^2}\right) dy. \quad (25)$$

Consider the first term of Equation(25) and putting

$$y^2 = t,$$

and by the identity [3, p.884]

$$\int_0^\infty \exp(-bx) x^\alpha L_n^\alpha(\lambda x) L_m^\alpha(\mu x) dx = \frac{\Gamma(n+m+\alpha+1)}{m!n!} \frac{(b-\lambda)^n (b-\mu)^m}{b^{m+n+\alpha+1}} \times F\left[-m, -n; -m-n-\alpha; \frac{b(b-\lambda-\mu)}{(b-\lambda)(b-\mu)}\right]$$

where $Re(\alpha)$ > -1 and $Re(b)$ > 0, the first term of Equation(25) is reduced to

$$\sum_{k=0}^{\infty} \frac{\rho_-^{2k}}{\sigma_{y-}^2} \frac{1}{2} \frac{\Gamma(2k+1)\lambda_-^k \mu_-^k}{k!k!b^{2k+1}} F[-k; -k; -2k; 0] \quad (26)$$

where hypogeometric function reduces to 1. Similarly, the second term of Equation(25) becomes

$$\sum_{k=0}^{\infty} \frac{\rho_-^{2k}}{\sigma_{y-}^2} \int_0^\infty \exp\left(-\frac{t}{2\sigma_{x-}^2}\right) \exp\left(-\frac{t}{2\sigma_{x-}^2}\right) L_{k-1}\left(\frac{t}{2\sigma_{x-}^2}\right) L_k\left(\frac{t}{2\sigma_{y-}^2}\right) \frac{1}{2} dy$$

$$= \sum_{k=0}^{\infty} \frac{\rho_-^{2k} \Gamma(2k) \mu_-^{k-1} \lambda_-^k}{\sigma_{y-}^2 (k-1)! k! b_-^{2k}} \frac{1}{2}. \quad (27)$$

Therefore conditional error probability is obtained by subtracting Equation(27) form Equation(26):

$$Pr[X(n) > Y(n)|a_n = -1, \{a_{n-i}\}] = \frac{1}{2\sigma_{y-}^2} \sum_{k=0}^{\infty} \rho_-^{2k} \frac{1}{k!} \left(\frac{\lambda_- \mu_-}{b_-^2}\right)^k \left[\frac{\Gamma(2k+1)}{k! b_-} - \frac{\Gamma(2k)}{(k-1)! \mu_-}\right].$$

This proves Equation(16). Similarly we can compute another conditional probability.

$$Pr[X(n) < Y(n)|a_n = +1, \{a_{n-i}\}] = 1 - \frac{1}{2\sigma_{y+}^2} \sum_{k=0}^{\infty} \rho_+^{2k} \frac{1}{k!} \left(\frac{\lambda_+ \mu_+}{b_+^2}\right)^k \left[\frac{\Gamma(2k+1)}{k! b_+} - \frac{\Gamma(2k)}{(k-1)! \mu_+}\right].$$

5. Numerical Computation

The path gain b_m is assumed to be the same for all the N_J paths and K is assumed to be 2, *i.e.*, the number of frequency for hopping is two. We also assume that all path delays are smaller than the symbol duration T. This is the case of Indoor Wireless Communications(IWC)[1]. This model is one of the simplest fading models to analyze. The performance of the system for this channel model serves as a reference with which the performance for more realistic but more complex models can be compared.

Figure 2 and 3 show bit error probabilities for some range of frequency offset time products for SIR=0dB, C=54, β=70, T_0/T=0.1, and T_0/T=1. When f_dT=0.4, about 3dB degradation is shown. This implies that when bit rate is 100 bps, the 40Hz frequency offset causes 3dB performance degradation. When f_dT is smaller than 0.8, bit error probability for T_0/T=0.1 is large than that of T_0/T=1. However when f_dT is greater than 0.8, bit error probability for T_0/T=1 is less than that of T_0/T=0.1. This implies that when carrier frequency is shifted by $1/T$, dispersive channel is superior to nondispersive channel. In

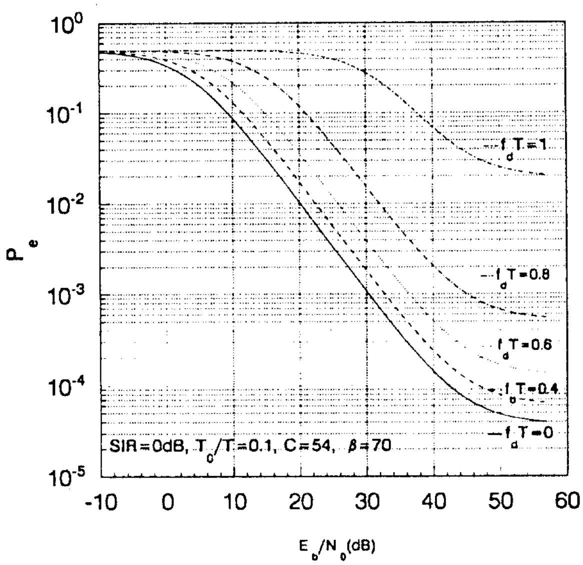

Figure 2. Average bit error probability in Rayleigh selective fading channel, $f_dT \in [0,1]$and T_0/T=0.1, SIR-0dB, C=54, β=70

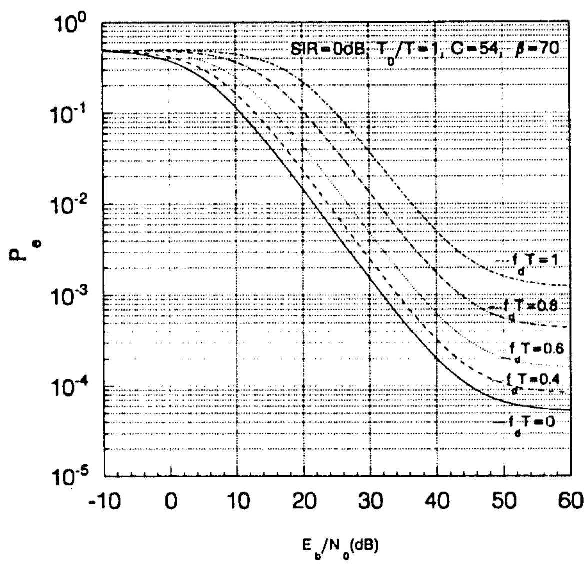

Figure 3. Average bit error probability in Rayleigh selective fading channel, $f_dT \in [0,1]$ and T_0/T=1, SIR-0dB, C=54, β=70

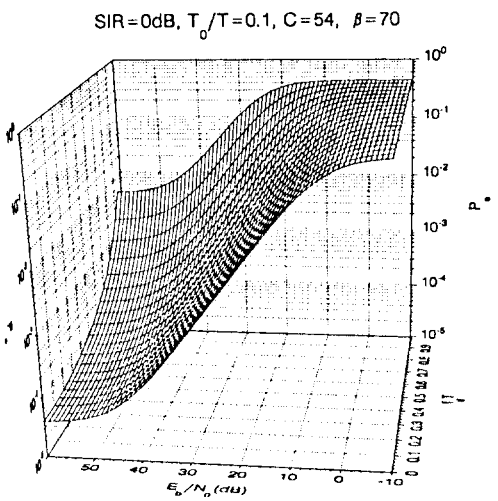

Figure 4. 3-D description of average bit error probability in Raylleigh selective fading channel T_0/T=0.1, SIR=0dB, C=54, β=70

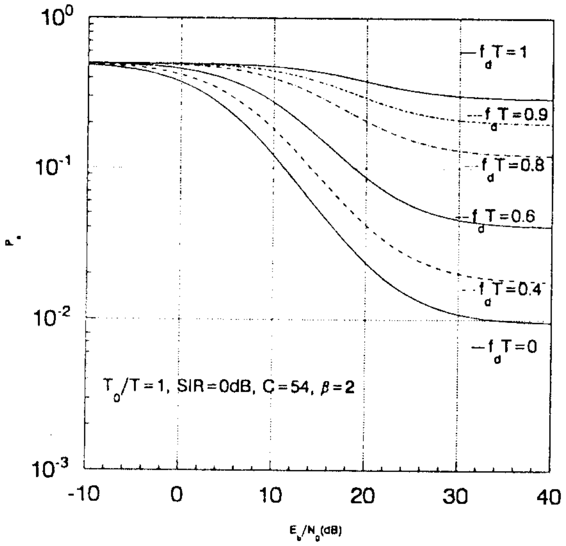

Figure 5. Average bit error probability in Rayleigh selective fading channel, $f_dT \in [0,1]$ and T_0/T=1, SIR-0dB, C=54, β=2

other words, $X(n)$ and $Y(n)$ have only noise when f_dT=1 and T_0/T=0. When T_0/T=1, however, signal component of main path is orthogonal to references of correlators and time delayed replicas of transmitted signal are not orthogonal to reference signal of correlators. Therefore these dispersed transmitted energy gives better performance than nondispersive channel.

Figure 5 shows bit error probability for the case that β is 2, *i.e.*, the minimum value of β. In this case, the distance of information carrying frequencies is $1/T$. The $X(n)$ having $1/T$ frequency offset corresponds to $Y(n)$. Therefore we can expect 0.5 error probability for equally likelihood data. Figure 6 shows bit error probability in the case of T_0/T=1. Error probability on highly dispersive channel is almost same as low dispersive channel when f_dT is 1 and E_b/N_o is low. Figure 4 shows 3-D graphs for average SNR and frequency offset time product for 10dB SIR, T_0/T=0.1, C=54 and β=70.

Reference

1. Bassel Solaiman, Alain Glavieux and Alain Hillion, "Error Probability of Fast Frequency Hopping Spread Spectrum with BFSK Modulation in Selective Rayleigh and Selective Rician Fading Channels" IEEE Transaction on Communications, Vol 38, No. 2, 233-240, Feb. 1990.
2. Dongku Kim, "Use of Discrete Models for Evaluating Codes for Fading Channels and Error/Erasure Decoder Co-operating with an

Interleaver and Hidden Markov Chain Modeler". Ph. D. thesis University of Southern California, 1992.

3. I.S. Gradshteyn, I.M. Ryzhik. *Table of Integrals, Series, and Products, corrected and enlarged edition*. Academic Press, 1980.

金 東 九(Dong Ku Kim) 정회원

1960년 4월 20일생
1983년 2월 : 한국항공대학교 통신공학과 졸업(공학사)
1985년 5월 : Univeristy of Southern California 전기공학과 졸업(공학석사)
1992년 5월 : Univeristy of Southern California 전기공학과 졸업(공학박사)
1992년 5월~1992년 9월 : 미국 Univeristy of Southern California Post Doc.(연구원)
1992년 9월~1994년 2월 : 미국 Motorola cellular infra-structure group 연구소 연구원
1994년 3월~현재 : 연세대학교 공과대학 전파공학과 조교수