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Modulation Bandwidth Reduction in Broadband High Speed Semiconductor Lasers due to Optical Confinement Factor Variation

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광통신용 광대역 고속 반도체 레이저의 광구속계수 변화에 의한 변조 대역폭 감소에 관한 연구

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ABSTRACT

A reduced effective differential gain is shown for the first time to arise in diode lasers by including the variation of the confinement factor with carrier density. This effective differential gain, and not the material gain, is the parameter which actually determines the resonance frequency and therefore the modulation bandwidth of diode lasers. For bulk lasers with short cavity length and thin active layer, the effective differential gain can be significantly reduced. The modulation of the confinement factor with carrier density can also adversely effect the differential gain of quantum-well lasers if the confining layer thickness is too thin and the threshold carrier density is too high.

要 約

반도체 레이저의 캐리어 밀도 변화에 의한 광구속계수 변화에 의해 레이저의 유효 미분 이득이 줄어듦을 처음으로 밝혔다. 반도체 레이저의 공진 주파수와 그에 따른 변조 대역폭은 이 새로운 유효 미분 이득에 의해 결정된다. FP 벌크 레이저의 경우, 공진기의 길이가 짧고 활성층의 두께가 얇을 때 유효 미분 이득치가 크게 줄어 들었으며, 양자-우물 반도체 레이저의 경우에는 제한층의 두께가 매우 얇고 임계 캐리어 밀도가 매우 높은 단일 양자-우물 반도체 레이저의 유효 미분 이득치가 캐리어 밀도 변화로 인한 광구속계수 변화에 의해 줄어듦을 밝혔다.

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I. INTRODUCTION

The change in optical confinement factor with carrier density has been proposed to explain the gain overshoot [1]-[2], and near field displacement [3] in ultrafast pump-probe experiments on semiconductor quantum-well optical amplifiers. It has always been assumed that the optical confinement factor (Γ) dose not change with the injected carrier density N, although it is well known that the refractive index changes with N. In this paper, we describe this overlooked factor, the variation of Γ with carrier density. A reduced effective differential gain is shown to arise in semiconductor lasers when the variation of Γ with carrier density is taken into account. This effective differential gain, and not the material gain, is the parameter actually determined from conventional measurements of the differential gain. This new term is in addition to the static reduction in Γ with carrier density, and can significantly reduce the resonance frequency and modulation bandwidth of diode lasers. For bulk lasers with short cavity length and thin active layers, the modulation bandwidth can be significantly reduced. It is shown that the variation of Γ with carrier density can also adversely effect the modulation bandwidth of quantum-well lasers if the confining layer thickness is too thin and the threshold carrier density is too high.

II. REDUCED EFFECTIVE DIFFERENTIAL GAIN IN DIODE LASERS DUE TO CONFINEMENT FACTOR VARIATION

The resonance frequency and modulation bandwidth of a semiconductor laser are well known to be proportional to the square root of the differential gain dg/dN. Therefore, we describe the modulation bandwidth reduction of diode lasers by showing the reduced effective differential gain when the variation of confinement factor with carrier density is considered.

The rate equations describe the interplay between the photon density and the carrier density in the active region of a semiconductor laser. We first derive the expression for the resonance frequency using the rate equations, assuming that the dependence of Γ with carrier density $d\Gamma/dN$ is nonzero. The rate equation for the carrier density N of a single mode laser is given by :

$$\frac{dN}{dt} = \frac{I_b}{qV} - R_T(N) - v_g\Gamma(N)g(N)(1-\varepsilon S)S \tag{1}$$

where I_b is the bias current, q is the electronic charge, and V is the active layer volume. R_T is the total recombination, vg is the group velocity, S is the cavity photon density of the laser, and g is the material gain. ε is the gain suppression parameter (assuming that the gain is suppressed by an amount εS). The rate equation for photon density S is given by :

$$\frac{dS}{dt} = v_g\Gamma(N)g(N)(1-\varepsilon S)S - v_g\alpha_T S + R_{sp} \tag{2}$$

where α_T is the total optical loss, and R_{sp} is the spontaneous emission into the guided modes. By ignoring the contribution of R_{sp}, a small-signal analysis of (2) under the conditions of $I = I_b + \delta_I$ and $S = S_b + \delta S$ yields :

$$\frac{d\delta S}{dt} = v_g\left\{\frac{d\Gamma(N)g(N)}{dN}\delta N - \Gamma(N_b)g(N_b)\varepsilon\delta S\right\}S_b \tag{3}$$

and

$$\frac{d^2\delta S}{dt} = v_g\left\{\frac{d\Gamma(N)g(N)}{dN}\frac{d\delta N}{dt} - \Gamma(N_b)g(N_b)\varepsilon\frac{d\delta S}{dt}\right\}S_b \tag{4}$$

Nb and Sb are the steady state carrier density and photon density, respectively. The carrier rate equation (1) gives :

$$\frac{d\delta N}{dt} = -\left\{\frac{1}{\tau_c} + v_g \frac{d\Gamma(N)g(N)}{dN} S_b\right\}\delta N - v_g\Gamma(N_b)g(N_b)\delta S \quad (5)$$

where differential carrier lifetime τ_c is defined as $1/\tau_c = \partial R/\partial N$. Using $d\delta N/dt$ from (5) and δN from (3), (4) can be written as :

$$\frac{d^2\delta S}{dt^2} + \gamma\frac{d\delta S}{dt} + \omega_0^2\delta S = driving\ term \quad (6)$$

The resonance frequency ω_0 is given by :

$$\omega_0^2 = v_g^2\ \Gamma(N_b)^2 g(N_b)\left(\frac{dg}{dN}\right)_{eff} S_b \quad (7)$$

and the damping rate γ by :

$$\gamma = \frac{1}{\tau_c} + v_g\Gamma(N_b)\left(\frac{dg}{dN}\right)_{eff} S_b + v_g\Gamma(N_b)g(N_b)\varepsilon S_b \quad (8)$$

where the effective differential gain is defined as :

$$\left(\frac{dg}{dN}\right)_{eff} = \frac{dg(N)}{dN} + \frac{g(N_b)}{\Gamma(N_b)}\frac{d\Gamma(N)}{dN} \quad (9)$$

Above lasing threshold, the carrier density is clamped. Therefore, $N_b \approx N_{th}$ and $g(N_{th})$ can be written as $g(N_{th}) = (dg/dN)_{mat}(N_{th}-N_0)$ where $(dg/dN)_{mat}$ is the material differential gain, N_{th} is the threshold carrier density, and N_0 is the carrier density at transparency. Thus,

$$\left(\frac{dg}{dN}\right)_{eff} = \left(\frac{dg}{dN}\right)_{mat}\left[\ 1 + (N_{th} - N_0)\frac{d\ln\Gamma(N)}{dN}\right] \quad (10)$$

This is the central result of this work. Since $d\Gamma/dN$ is negative, the second term in (9) and (10) always reduces the effective differential gain with respect to the true material differential gain. It is important to note that simply including the effect of carrier density on the static value of Γ(that is $\Gamma(N_0)$) is not sufficient; the new factor $d\Gamma/dN$ must also be included. The damping factor γ (8) is predominantly determined by the nonlinear gain term [4].
Therefore,

$$\gamma \approx v_g\Gamma(N_b)g(N_b)\varepsilon\ S_b \quad (11)$$

The K-factor, defined as K = γ/f02 [4] is then given by :

$$K = \frac{4\pi^2}{v_g\alpha_T} + \frac{4\pi^2\varepsilon}{v_g\Gamma(N_b)\left(\frac{dg}{dN}\right)_{eff}} \quad (12)$$

The K-factor is important in that it sets the upper limit to the maximum 3-dB bandwidth: $(f_{3\text{-}dB})_{max} = 8.8/K$ [5], [6]. Note that the resonance frequency f_0 (and therefore, the modulation bandwidth of the laser) and the K-factor are all governed by this effective differential gain $(dg/dN)_{eff}$. In the following (sections Ⅲ and Ⅳ) we show that $(dg/dN)_{eff}$ is significantly reduced for bulk and quantum-well lasers for certain cases. The details of the above analysis are given in Appendix Ⅰ.

Ⅲ. MODULATION BANDWIDTH REDUCTION IN BULK SEMICONDUCTOR LASERS

An analytical formula for $d\Gamma/dN$ can be derived for the case of bulk semiconductor lasers; the confinement factor Γ can be approximated for bulk lasers by [7] :

$$\Gamma = \frac{D^2}{2 + D^2} \quad (13)$$

where

$$D = \frac{2\pi}{\lambda}\ d(n_2^2 - n_1^2)^{1/2} \quad (14)$$

Here, n_2 and n_1 are the indices of the active and cladding layers, respectively, and d is the active layer thickness. Using (13) and (14), we calculate a closed form expression for the reduction in the effective differential gain :

$$\frac{g(N_b)}{\Gamma(N_b)}\frac{d\Gamma}{dN}=\frac{\alpha_T\lambda^2}{4\pi^2 d^2}\frac{1}{(n_2-n_1)^2}\frac{1}{n_2}\frac{dn_2}{dN} \tag{15}$$

The derivation of (15) from (13) and (14) is given in Appendix Ⅱ. The analysis of (15) requires accurate value of carrier-induced index change dn_2/dN. The values of the carrier-induced index change necessary for this calculation are measured very accurately by an injection-reflection technique in combination with carrier lifetime data [8].

In (15), the total optical loss α_T is given by $\alpha_T = i + (1/L)ln(1/R)$, where L is the cavity length and R is the facet reflectivity. Note that $(g/\Gamma)(d\Gamma/dN)$ increases as the cavity and active layer thickness decrease. Fig. 1 shows the magnitude of $(g/\Gamma)(d\Gamma/dN)$ (15) versus cavity length for lasers with various active region thickness. The parameters used in this calculation are listed in Table I. The value of dn_2/dN is obtained from the result of the carrier-induced index change measurement which is summarized in Table Ⅱ [8]. The internal loss is calculated from the relation $\alpha_i = \alpha_{cl}\times(1-\Gamma)+\alpha_{act}\times\Gamma$, where α_{cl} and α_{act} are the losses in the cladding and active layer, respectively. It is clear that the reduction in differential gain can be substantial for the laser with short cavity length and thin active layer thickness. The effect of $(g/\Gamma)(d\Gamma/dN)$ on the resonance frequency is shown in Fig. 2. The difference between the resonance frequency with $(g/\Gamma)(d\Gamma/dN)=0$ and the resonance frequency with reduced effective differential gain is more than 2GHz for the laser with short cavity length and thin active layer thickness. Therefore, when the effect of confinement factor variation is considered, the maximum allowable 3-dB modulation bandwidth of the laser decreases

Table Ⅰ. The parameters used in the calculation of $(g/\Gamma)(d\Gamma/dN)$ (15)

ParameterI	Value	Unit
Active layer thickness	0.1, 0.15, 0.2	μm
Active layer width	2.0	μm
dg/dN	3×10^{-16}	cm^2
α_{int}	$20\times(1-\Gamma)+130\times\Gamma$	cm^{-1}
Reflectivity	0.32	—
Index step(N=0)	0.26	—
Wavelength	1.3	μm

Table Ⅱ. The device parameters and results of the carrier-induced index changes

Pump laser wavelength (μm)	Probed laser wavelength (μm)structure	Active layer volume of probed laser($μm^3$)	δn_{act}=
1.533	1.545/bulk	275×1.8×0.21	$-6.1\times10^{-14}(N)^{0.66}$
1.533	1.307/bulk	275×1.5×0.15	$-9.2\times10^{-16}(N)^{0.72}$
1.313	1.307/bulk	275×1.5×0.15	$-1.3\times10^{-14}(N)^{0.68}$

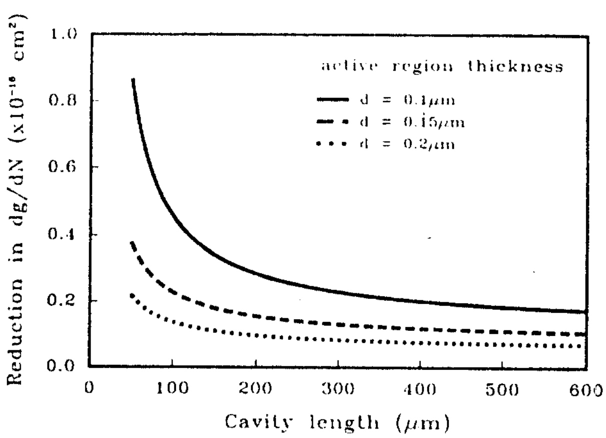

Fig. 1. The reduction in effective differential gain versus cavity length using (15). The parameters used are listed in Table Ⅰ.

by more than 3GHz (from the well-known relation $f_{3dB} = 1.55 f_0$) in the regimes where the bandwidth is not limited by electrical parasitics of damping.

Ⅳ. MODULATION BANDWIDTH REDUCTION IN QUANTUM-WELL (QW) SEMICONDUCTOR LASERS

Fig. 3 shows the energy diagram of the conduction and valence bands of a single QW laser structure. A well and confining layers are shown. The function of each layer is well known and will not be discussed here. It is well established that the dynamics of carriers related to carrier transport in the confining layers and carrier capture and emission in the quantum-well can substantially degrade the modulation response in QW lasers with wide confining layers [9], [10]. Here, we propose an independent effect which may also reduce the modulation response of QW lasers. We assume that the variation of the carrier density in the well δN_w produces a variation in the index in the well and confining layers, and a variation in the confinement factor $\delta\Gamma$. Γ is calculated by the usual manner by considering the index profile shown by the solid line in Fig. 4. The change in the confinement factor is determined by recalculating $\Delta\Gamma$ for the new index profile shown by the dashed lines in Fig. 4. The change in index in the well δn_w and confining layers δn_{conf} (differences between solid lines and dashed lines) due to a change in the carrier density in the well δN_w and in the confining layer δN_{conf} need to be determined in order to calculate $d\ln\Gamma/dN_w$ [see (10)]. This requires the knowledge of the carrier-induced index change, dn_{conf}/dN_{conf} and dn_w/dN_w at the lasing wavelength (1.53μm). An accurate calculation of the magnitude of $d\ln\Gamma/dN_w$ requires detailed knowledge of the carrier-induced index change of the confining layer at wavelength (well wavelength) displaced from the confining layer bandgap and the index change of the well at lasing wavelength. Moreover, the index change for a much wider range of car-

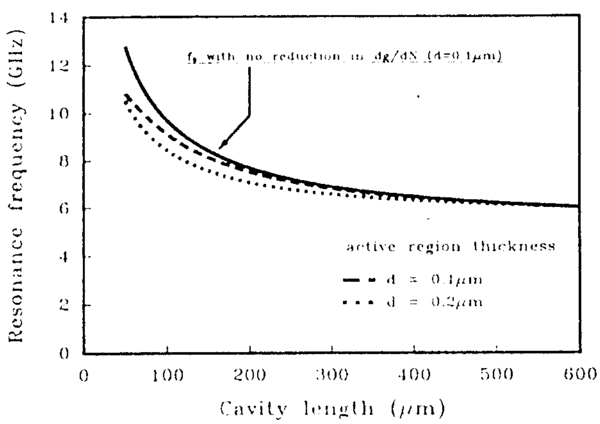

Fig. 2. The reduction in resonance frequency versus cavity length due to the reduced effective differential gain. The solid line shows the resonance frequency with no reduction in differential gain for the laser with active layer thickness of 0.1μm

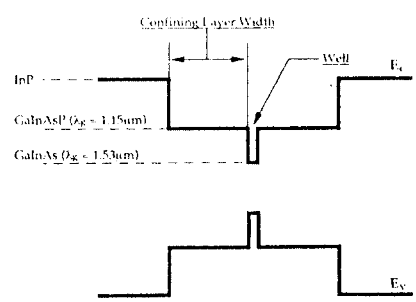

Fig. 3 The energy diagram of the conduction and valence bands of a single quantum-well laser structure.

rier density than has been previously measured [11] is required. In [8], a new technique for the accurate measurement of these previously unavailable quantities is given. We find for the first time that the index change is sublinear in carrier density [8] in accordance with a theory [12], and that the index change is significantly smaller for wavelengths displaced from the bandgap than for the bandgap wavelength as expected [see Table Ⅱ]. Our results for dn/dN (at bandgap wavelength) agree quite well with [11] and [13] at low carrier densities of 1×10^{18} cm^{-3} and 3×10^{18} cm^{-3}, respectively. We used Table II for calculating the factor $d\ln\Gamma/dN_w$.

For the sake of simplicity, we neglect the complicated nonsteady state behavior and detail spatial distribution of carriers in the confining and quantum-well layers [3], [9], [10]. We assume that a single Fermi-level describes the carrier distribution in the quantum-well and confining layers. The dependence of the change in the carrier density in the confining layer δN_{conf} on the change in δN_w is needed. The carrier densities in the well and confining layer are calculated by the following procedure: It is found that only one subband exists in the quantum-well from the analysis of time-independent Schr dinger equation with appropriate quantum-well boundary conditions. If the carrier density in the well is specified, the Fermi level E_{fc} is then given by :

$$E_{fc} = kT\ \ln(e^{N_w/N_{cqw}}-1) \qquad (16)$$

Here, N_{cqw} is given by $4\pi 2kTm_c^*/(\pi L_z h^2)$ where m_c^* is the electron effective mass, L_z is the well width, and h is the Plank's constant. The carrier density in the bulk confining layer is obtained from E_{fc} by [14] :

$$N_{conf}=N_c F_{1/2}((E_{fc}-\Delta E_c)/kT \qquad (17)$$

where N_c is the conduction band density of state for bulk material, and $F_{1/2}$ is the Fermi-Dirac integral which is given by :

$$F_{1/2}(\zeta)=\frac{2}{\pi^{1/2}}\int_0^\infty \frac{\xi^{1/2}d\xi}{1+\exp(\xi-\zeta)} \qquad (18)$$

where $\zeta=(E_{fc}-\Delta E_c)/kT$ and $\xi=(E-\Delta E_c)/kT$, and ΔE_c is the conduction band offset between the quantum well and the confining

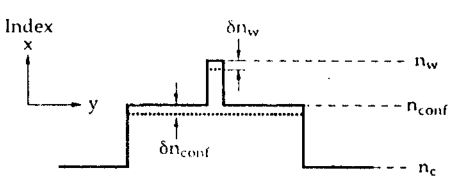

Fig. 4. The index profile of a single quantum-well laser. Carrier injection into the confining and quantum-well layers causes a reduction in the index (dashed lines).

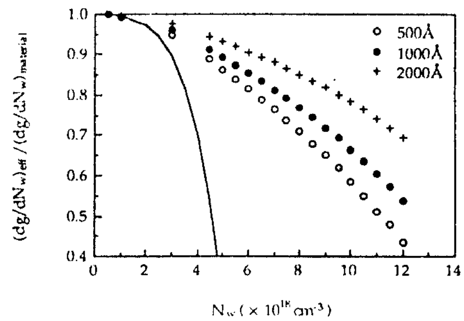

Fig. 5. The reduction of effective differential gain due to the dependence of confinement factor on carrier density. The confining layer widths (one side) are 500, 1000, and 2000Å. The well width is 100Å. The number of well is one. The solid line is the predicted ratio if dn/dN =-2.8×10^{-20} cm^3 is used.

layer. Therefore, the carrier-induced index changes δn_w and δn_{conf} can be obtained from Table Ⅱ with N_w and N_{conf} calculated from (16) and (17).

Fig. 5 shows the ratio of the effective differential gain versus the material differential gain as a function of threshold carrier density for different confining layer thickness in a 1.53μm wavelength single quantum-well laser with a 1.15μm wavelength confining layer. Here, N_{cqw} = 7.11$\times 10^{17}$ cm^{3}, ΔE_c = 0.5 Eg, and L_z = 100 . Note that the ratio decreases with well carrier density N_w and increases with confining layer thickness. This effect can significantly reduce $(dg/dN)_{eff}$ and the modulation bandwidth if the threshold density is high and the confining layer width is thin. The solid line is the predicted ratio for the laser with 500 confining layer width if the carrier-induced index change dn/dN =-2.8 $\times 10^{-20}$ cm^{3} (given in reference [11]) is used for describing the index change in the confining and quantum-well layers. As expected, the effect is overestimated at high carrier density. Note that our result and carrier transport mechanism [9] produce opposite effect with regard to the dependence of modulation bandwidth on confining layer thickness. However, both mechanisms degrade the modulation bandwidth of QW lasers if N_{th} is high. Fig. 6 shows the result for a QW laser with three wells. In this case, the effect isexpected to be less important because N_w is expected to be appreciably smaller than the case of single well.

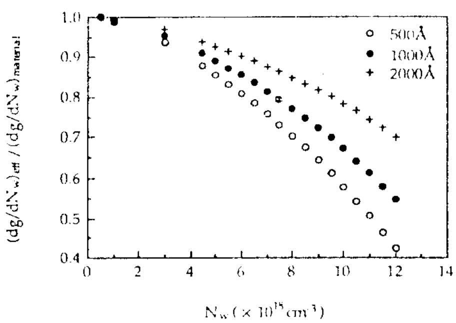

Fig. 6. The reduction of effective differential gain due to the dependence of confinement factor on carrier density. The confining layer widths (one side) are 500, 1000, and 2000Å. The well width is 100Å. The number of well is three.

Ⅴ. CONCLUSION

We have included for the first time the variation of the confinement factor with carrier density, and shown that it cannot simply be reduced to the static dependence of Γ on carrier density. From the small signal analysis of the rate equations taking into account the variation of Γ with carrier density, a reduced effective differential gain for bulk semiconductor lasers, particularly those with short cavity lengths or thin active layers is obtained. The variation of Γ with carrier density can also adversely effect the effective differential gain of single quantum-well lasers with high threshold carrier densities and narrow confining layer. It is less important for multiple-quantum-well lasers unless the threshold carrier density is very high. The square root of the reduced effective differential gain leads to a proportional decrease in the resonance frequency, and in the regimes where the bandwidth is not limited by electrical parasitics of damping, to a proportional decrease in the modulation bandwidth.

APPENDIX Ⅰ

SMALL SIGNAL ANALYSIS OF RATE EQUATIONS INCLUDING THE VARIATION OF CONFINEMENT FACTOR WITH CARRIER DENSITY

This Appendix presents the small signal analysis of the carrier and photon density rate equations when the confinement factor is considered as a function of carrier density. The rate equations are given by :

$$\frac{dN}{dt}=\frac{I_b}{qV}-R_T(N)-v_g\Gamma(N)g(N)(1-\varepsilon S)S \quad \text{(A1)}$$

$$\frac{dS}{dt}=v_g\Gamma(N)g(N)(1-\varepsilon S)S-v_g\alpha_T S+R_{sp} \quad \text{(A2)}$$

where N is the carrier density, I_b is the bias current, q is the electronic charge, and V is the active layer volume. R_T is the total recombination rate, v_g is the group velocity, S is the cavity photon density of the laser, and g is the carrier density dependent gain. is the gain suppression parameter (assuming that the gain is suppressed by an amount εS). α_T is the total optical loss and R_{sp} is the spontaneous emission into the guided modes. The rate equations are solved assuming the small signal solutions :

$$\begin{aligned} N(t) &= N_b + \delta N(t) \\ S(t) &= S_b + \delta S(t) \\ I(t) &= I_b + \delta I(t) \end{aligned} \quad \text{(A3)}$$

where N_b, S_b, and I_b are the dc bias conditions, and $\delta N(t)$, $\delta S(t)$, and $\delta I(t)$ are the small signals. In a steady state (when N is pinned at N_b (N_{th})), (A1) and (A2) becomes :

$$\frac{I_b}{qV}-R_T(N_b)-v_g\Gamma(N_b)g(N_b)(1-\varepsilon S_b)S_b=0 \quad \text{(A4)}$$

$$v_g\Gamma(N_b)g(N_b)(1-\varepsilon S_b)S_b-v_g\alpha_T S_b+R_{sp}=0 \quad \text{(A5)}$$

Now, assume $R_{sp}=0$ and $g(N_b)\gg g(N_b)\varepsilon S_b$. Then

$$\{v_g\Gamma(N_b)g(N_b)-v_g\alpha_T\}S_b=0 \quad \text{(A6)}$$

The small signal analysis of (A2) gives :

$$\begin{aligned}\frac{d\delta S}{dt} &= v_g\{\Gamma(N_b)g(N_b)(1-\varepsilon S_b)-\alpha_T\}\delta S \\ &\quad + v_g\left\{\frac{d\Gamma(N)g(N)}{dN}\delta N \right.\\ &\quad \left. -\frac{d\Gamma(N)g(N)}{dN}\varepsilon S_b\delta N\right\}S_b\end{aligned} \quad \text{(A7)}$$

From (A6) and $g(N_b)\gg g(N_b)\varepsilon S_b$, (A7) becomes :

$$\frac{d\delta S}{dt}=v_g\left\{\frac{d\Gamma(N)g(N)}{dN}\delta N-\Gamma(N_b)g(N_b)\varepsilon\delta S\right\}S_b \quad \text{(A8)}$$

and the second derivative of δS with respect to time is :

$$\frac{d^2\delta S}{dt^2}=v_g\left\{\frac{d\Gamma(N)g(N)}{dN}\frac{d\delta N}{dt} -\Gamma(N_b)g(N_b)\varepsilon\frac{d\delta S}{dt}\right\}S_b \quad \text{(A9)}$$

The carrier rate equation (A1) gives :

$$\begin{aligned}\frac{d\delta N}{dt} &= 0-\frac{dR_T(N)}{dN}\delta N-v_g\Gamma(N_b)g(N_b)(1 \\ &\quad -\varepsilon S_b)\delta S + v_g\left\{\frac{d\Gamma(N)g(N)}{dN}\delta N\right. \\ &\quad \left.-\frac{d\Gamma(N)g(N)}{dN}\varepsilon S_b\delta N\right\}S_b\end{aligned} \quad \text{(A10)}$$

Since I_b is constant, $d(I_b/qV)dt$ is zero. Using the definition of carrier lifetime (i.e. $1/\tau_c=\partial R/\partial N$) and $g(N_b)\gg g(N_b)\varepsilon S_b$, (A10) becomes :

$$\begin{aligned}\frac{d\delta N}{dt} &= -\left\{\frac{1}{\tau_c}+v_g\frac{d\Gamma(N)g(N)}{dN}S_b\right\}\delta N \\ &\quad -v_g\Gamma(N_b)g(N_b)\delta S\end{aligned} \quad \text{(A11)}$$

From (A8),

$$\delta N=\left[v_g\frac{d\Gamma(N)g(N)}{dN}S_b\right]^{-1}\left[\frac{d\delta S}{dt} + v_g\Gamma(N_b)g(N_b)\varepsilon\delta S S_b\right] \quad \text{(A12)}$$

Thus, (A11) becomes :

$$\frac{d\delta N}{dt}= -\beta\left[v_g\frac{d\Gamma(N)g(N)}{dN}S_b\right]^{-1}\times$$

$$\left[\frac{d\delta S}{dt} + v_g\Gamma(N_b)g(N_b)\varepsilon\delta SS_b \right]$$

$$- v_g\Gamma(N_b)g(N_b)\delta S \quad (A13)$$

where

$$\beta = \left\{ \frac{1}{\tau_c} + v_g \frac{d(\Gamma(N)g(N))}{dN} S_b \right\}$$

Using (A13), (A9) becomes :

$$\frac{d^2\delta S}{dt^2} = -\beta \left[\frac{d\delta S}{dt} + v_g\Gamma(N_b)g(N_b)\varepsilon\delta SS_b \right] - v_g\Gamma(N_b)g(N_b)\varepsilon\frac{d\delta S}{dt} S_b - v_g^2\Gamma(N_b)g(N_b)\frac{d(\Gamma(N)g(N))}{dN}\delta SS_b \quad (A14)$$

Here, $v_g\Gamma(N_b)g(N_b)\varepsilon\delta SS_b$ term is negligible, therefore, (A9) becomes :

$$\frac{d^2\delta S}{dt^2} + \left\{ \frac{1}{\tau_c} + v_g \frac{d\Gamma(N_b)g(N_b)}{dN} S_b + v_g\Gamma(N_b)g(N_b)\varepsilon S_b \right\} \frac{d\delta S}{dt} - v_g^2\Gamma(N_b)g(N_b)\frac{d(\Gamma(N)g(N))}{dN}\delta SS_b = 0 \quad (A15)$$

In simple expression,

$$\frac{d^2\delta S}{dt^2} + \gamma\frac{d\delta S}{dt} + \omega_0^2\delta S = 0 \quad (A16)$$

The damping rate γ is given by :

$$\gamma = \frac{1}{\tau_c} + v_g\Gamma(N_b)\left(\frac{dg}{dN}\right)_{eff} S_b + v_g\Gamma(N_b)g(N)\varepsilon S_b \quad (A17)$$

Note that the first two terms in (A17) are small comparing with the nonlinear gain term (the third term). The resonance frequency ω_0 is given by :

$$\omega_0^2 = v_g^2\Gamma(N_b)^2 g(N_b)\left(\frac{dg}{dN}\right)_{eff} S_b \quad (A18)$$

where the effective differential gain is defined as :

$$\left(\frac{dg}{dN}\right)_{eff} = \frac{dg(N)}{dN} + \frac{g(N_b)}{\Gamma(N_b)}\frac{d\Gamma(N)}{dN} \quad (A19)$$

The K-factor can be obtained by dividing (A17) by (A18), and is given by :

$$K = \frac{4\pi^2}{v_g\alpha_T} + \frac{4\pi^2\varepsilon}{v_g\Gamma(N_b)\left(\frac{dg}{dN}\right)_{eff}} \quad (A20)$$

(A18) to (A20) are the central results of this derivation.

APPENDIX II

DERIVATION OF EXPRESSION (15) FROM (13) AND (14)

The optical confinement factor of bulk semiconductor lasers can be approximated by :

$$\Gamma = \frac{D^2}{2+D^2} \quad (A21)$$

where

$$D = \frac{2\pi}{\lambda} d(n_2^2 - n_1^2)^{1/2} \quad (A22)$$

Differentiating (A22) with respect to carrier density N gives :

$$\frac{d\Gamma}{dN} = \frac{d\Gamma}{dD}\frac{dD}{dn_2}\frac{dn_2}{dD} = \frac{4D^2}{(2+D^2)^2}\frac{n_2}{n_2^2 - n_1^2}\frac{dn_2}{dN} \quad (A23)$$

Using $\alpha_T = \Gamma(N_0)g(N_0)$,

$$\frac{g(N_0)}{\Gamma(N_0)}\frac{d\Gamma}{dN} = \frac{\alpha_T}{\Gamma(N_0)^2}\frac{d\Gamma}{dN} \quad (A24)$$

Substituting (A21), (A22), and (A23) into (A24) gives :

$$\frac{g(N_0)}{\Gamma(N_0)}\frac{d\Gamma}{dN} = \frac{4\alpha_T}{k_0^2 d^2}\frac{1}{(n_2-n_1)^2}\frac{n_2}{(n_2+n_1)^2}\frac{dn_2}{dN} \quad (A25)$$

By applying the approximation, $n_2/(n_2+n_1)^2 = 1/4n_2$, (A25) becomes :

$$\frac{g(N_0)}{\Gamma(N_0)}\frac{d\Gamma}{dN}=\frac{\alpha_T\lambda^2}{4\pi^2 d^2}\frac{1}{(n_2-n_1)^2}\frac{1}{n_2}\frac{dn_2}{dN} \quad \text{(A26)}$$

Here, $k_0 = 2\pi/\lambda$ is used.

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