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Intersymbol Interference Due to Sampling-Time Jitter and Its Approximations in a Raised Cosine Filtered System

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ABSTRACT

This paper studies the effect of intersymbol interference due to sampling-time jitter on the worst-case bit error probability in a digital modulation over an additive white Gaussian noise channel, with the squared-root raised-cosine filters in the transmitter and the receiver. It derives approximation formulas using the Taylor series approximations. The principal results of this paper is the relationship between the worst-case bit error probability, the degree of jitter, the roll-off factor of the raised cosine filter, and other quantities. Numerical results show, as expected, that the intersymbol interference decreases as the roll-off factor increases and the jitter decreases. They also show that the approximation formulas are accurate for small intersymbol interference, i.e., for large roll-off factors and for small jitter. Surprisingly these formulas are more accurate for small or moderate signal-to-noise ratio $\frac{E_b}{N_0} \lessapprox 7$ dB and begin to lose accuracy for larger signal-to-noise ratio.

Index terms: sampling-time jitter, square-root raised-cosine filters, bit error probability, intersymbol interference, additive Gaussian noise channel, the Taylor series approximation

I. Introduction

Consider a baseband digital communication system in Fig. 1. The source emits either 1 or -1 in every T second and its output denoted by $\{X_k\}_{k=-\infty}^{\infty}$ is assumed to be a binary independent identically distributed (iid) random sequence. The transmitter is considered a pulse-shaping filter with the impulse response $h_s(t)$. The output $s(t)$ of the transmitter is sent over the channel, which adds a continuous-time random noise $n(t)$. The receiver consists of a receiver filter, a sampler, and a detector: the receiver filter with the impulse response $h_r(t)$ takes in the signal $r(t)$; the sampler samples the output $z(t)$ of the receiver filter at nominal time $t=mT$ to produce $z(mT)$, which is compared with threshold $\gamma=0$ to estimate the transmitted source sequence.

The Problem Statement In a practical digital communication system, imperfect timing is bound to occur in the sampling of the receiver filter output This sampling-time jitter causes the sampling-time to be out of integer multiples of the source symbol duration T and the system to suffer intersymbol interference. A reasonable approach to capturing jitter is to model the sampling-time as a random sequence, e.g., model the sampling time $\{t_m\}_{k=-\infty}^{\infty}$ as a random

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sequence

$$t_m = mT + T_m, \qquad (1)$$

where $\{T_m\}_{m=-\infty}^{\infty}$ is an iid Gaussian random sequence with mean 0 and variance σ_T^2. We are concerned with the evaluation of the bit error probability $P(E) \triangleq \Pr(\hat{X}_k \neq X_k)$. It is pointed out that the pulse-shaping filter $h_s(t)$ and the receiver filter $h_r(t)$ are individually of a squared-root raised cosine characteristic and the overall response $h_o(t) \triangleq h_s(t) * h_r(t)$ constitutes a raised cosine filter.

Previous work for sampling-time jitter has been reported in the literature, e.g., [1, 2, 3]. For the case of binary signaling over an additive white Gaussian noise channel, Lindsey and Simon in [1] discuss the jitter effect on the bit error probability as a function of signal-to-noise ratio for various degrees of jitter. There the correlator synchronization error effect is discussed for baseband signals: nonreturn-to-zero pulses, the Manchester code, return-to-zero signals, and the Miller code. Since they assumed implicitly infinite channel bandwidth, jitter affects at most three source bits. Huang and et al. in [2] deals with pulses in connection with intersymbol interference and jitter. Raised cosine and/or square-root raised cosine filters are discussed as a method of relieving the adverse effect of sampling-time jitter and a general introduction can be found in [3] among others.

This paper resembles [1] in that the purpose and results are in the same vein, but differs from the previous work including [1] in that herein is studied jitter with square-root raised-cosine pulse shaping filters. And to the best of authors' knowledge, there has been no report on the study of the effect of the sampling-time jitter combined with (square-root) raised-cosine filters on the bit error probability from the Taylor series approximation viewpoint.

The principal results of this paper are (a) the relationship between the worst-case bit error probability and the affecting factors such as the roll-off factors and the degree of sampling-time jitter, and (b) two approximation formulas for the worst-case bit error probability derived from the Taylor series approximation. Numerical results show that the error probability decreases as the roll-off factor increases and the jitter decreases, as expected. They also show that the derived formulas become more accurate for a large roll-off factor and for small jitter. Surprisingly these formulas are more accurate for small or moderate signal-to-noise ratio $\frac{E_b}{N_0} \lesssim 7$ dB and begin to lose accuracy for larger signal-to-noise ratio.

The rest of the paper is organized as follows: Section 2 formulates the problem in mathematical terms and presents the analysis; Section 3 presents numerical results and discusses them; and the conclusions follow in Section 4.

II. Formulation and Analysis

The goal of this paper is to compute the worst-case bit error probability $P(E) = \Pr(\hat{X}_k \neq X_k)$ and to derive an approximation formula for it. Consider the output $s(t)$ of the pulse-shaping filter $h_s(t)$ in Fig. 1 due to the source sequence: $s(t) = \sum_{k=-\infty}^{\infty} X_k h_s(t-kT)$. Then the receiver filter output $z(t)$ is given as follows:

$$z(t) = (s(t) + n(t)) * h_r(t) = \sum_{k=-\infty}^{\infty} X_k h_s(t-kT) * h_r(t) + n(t) * h_r(t),$$

where $h_s(t)$ and $h_r(t)$ are square-root raised-cosine filters given by

$$h_{\sqrt{}}(t) = \frac{\sqrt{T}}{1-(\frac{4\alpha t}{T})^2} \Big(\frac{1-\alpha}{T} \operatorname{sinc}(\frac{t}{T}(1-\alpha)) + \frac{4\alpha}{\pi T} \cos(\frac{\pi t}{T}(1+\alpha)) \Big). \qquad (2)$$

and where $n(t)$ is an additive white Gaussian noise with its power spectral density $S_n(f) = \frac{N_0}{2}$ and is statistically independent of the source sequence $\{T_k\}_{k=-\infty}^{\infty}$. For notational convenience, $h_o(t) \triangleq h_s(t) * h_r(t)$ and $n_r(t) \triangleq n(t) * h_r(t)$. (This paper assumes that the filter $h_o(t)$ is appropriately delayed and truncated for causality, though the notation does not so indicate. And the truncation length will be specified explicitly when needed.) Then, since $h_s(t-kT) * h_r(t) = h_o(t-kT)$, we can rewrite $z(t)$ as

$$z(t) = \sum_{k=-\infty}^{\infty} X_k h_o(t-kT) + n_r(t). \tag{3}$$

It is assumed that the jitter random sequence $\{T_m\}_{m=-\infty}^{\infty}$ is independent of the source sequence $\{X_k\}_{k=-\infty}^{\infty}$ and the channel noise $n(t)$. Without the loss of generality due to the strict sense stationarity of $\{X_k\}_{k=-\infty}^{\infty}$, $n(t)$, and $\{T_m\}_{m=-\infty}^{\infty}$, we can carry out the analysis for X_0 and the resulting bit error probability will be valid for an arbitrary k. Therefore, we will assume that the source bit X_0 is transmitted and that the decision for X_0 is based on $z(t)$ sampled at $t = t_0 \triangleq 0 \cdot T + T_0$, where T_0 is the random variable that accounts for sampling-time jitter at the supposed sampling time $t=0$. We also note that the bit error probability $P(E)$ can be simplified to be

$$P(E) = \Pr(z(T_0) < 0 \mid X_0 = 1). \tag{4}$$

The decision for X_0 will be

$$\hat{X}_0 = \begin{cases} 1, & \text{if } z(T_0) > \gamma, \\ -1, & \text{if } z(T_0) < \gamma. \end{cases}$$

The decision variable $z(T_0)$ is broken into several terms: from (3)

$$z(T_0) = z(t)|_{t=t_0} = \sum_{k=-\infty}^{\infty} X_k h_o(t_0 - kT) + n_r(t_0)$$

$$= X_0 h_o(T_0) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} X_k h_o(T_0 - kT) + n_r(T_0), \tag{5}$$

where the first term in (5) is due to the corresponding source bit X_0, the second the source bits other than X_0, and the last the noise. The second term is the intersymbol interference. The last term $n_r(t)$ is the noise due to the channel noise and is given by $n_r(t) = h_r(t) * n(t) = \int_{-\infty}^{\infty} h_r(u) n(t-u)\, du$. Since the linear operation on a Gaussian random process yields a Gaussian random process, $n_r(t)$ is Gaussian and $n_r(T_0)$ is a Gaussian random variable with mean 0 and variance

$$\frac{N_0}{2} \int_{-\infty}^{\infty} |h_r(u)|^2\, du. \tag{6}$$

We note that $n_r(T_0)$ is independent of T_0 and hence we use n_r instead when notational convenience is preferred. In case that it is not truncated, from the Parseval theorem we can get

$$E\{n_r^2(T_0)\} = \frac{N_0}{2} \int_{-\infty}^{\infty} |h_r(u)|^2\, du = \frac{N_0}{2} \int_{-\infty}^{\infty} |H_r(f)|^2\, df.$$

Let σ_n^2 denote $E\{n_r^2(T_0)\}$, the variance of $n_r(T_0)$. Then from the above argument we have $n_r(T_0)$ Gaussian with mean 0 and variance σ_n^2 given in (6).

The truncated $h_{\sqrt{},2L+1}(t)$ with the truncation length $2L+1$ is defined by

$$h_{\sqrt{},2L+1}(t) \triangleq h_{\sqrt{}}(t)\, \Pi\left(\frac{t}{(2L+1)T}\right) = \begin{cases} h_{\sqrt{}}(t), & |t| \le (L+\frac{1}{2})T, \\ 0, & \text{elsewhere.} \end{cases} \tag{7}$$

It is noted that $h_{\sqrt{},2L+1}(t)$ approaches $h_{\sqrt{}}(t)$ as L gets arbitrarily large. In practice $L=10$ suffices for almost all α including $\alpha=0$, since $L=10$ accounts for 98% or more of $L=\infty$. For mathematical tractability, this paper will use the untruncated $\sqrt{E_b}\, h_{\sqrt{}}(t)$ in (2) as the pulse-shaping ilter $h_s(t)$ at the transmitter, i.e., $h_s(t) = \sqrt{E_b}\, h_{\sqrt{}}(t)$. With this untruncated pulse-shaping filter at the transmitter, the overall filter response $h_0(t)$ will be given by $h_0(t) = \sqrt{E_b}\, h_{\sqrt{}}(t) * h_{\sqrt{},2L+1}(t)$. For a large L the response $h_0(t)$ can be approximated by the untruncated raised-cosine filter $h(t)$ for $|t| \le \frac{1}{2}(2L+1)$. For this reason we will use in the rest of the paper $h_o(t)=\sqrt{E_b}\,h_{2L+1}(t)$.

With the truncation length $2L+1$, the decision variable in (5) is now simplified to yield

$$z(T_0)=X_0h_o(T_0)+\sum_{\substack{k=-L\\k\neq 0}}^{L}X_kh_o(T_0-kT)+n_r(T_0), \tag{8}$$

Now we apply the Taylor series approximation to each $h_o(T_0-kT)$ for $k=-L,\ldots,L$. The first three terms of the Taylor series approximation is used in the paper: at around $t=kT$

$$h_o(t)\approx h_o(kT)+h_o'(kT)(t-kT)+\frac{1}{2}h_o''(kT)(t-kT)^2.$$

By the first-order and the second-order approximations we mean the ones obtained taking up to the linear term, i.e., the first derivative term, and the quadratic term, i.e., the second derivative term, respectively.

For the truncated raised-cosine filter $h_o(t)=h_{2L+1}(t)$, we obtain Table 1. Then $X_kh_o(T_0-kT)$ can be approximated as

$$X_kh_o(T_0-kT)\approx\sum_{l=0}^{2}c_{kl}X_kT_o^l, \tag{9}$$

where c_{kl} is the first l coefficients of the Taylor series of $h_o(t)$ evaluated at $t=-kT$, i.e.,

$$c_{kl}\triangleq\frac{1}{l!}h_o^{(l)}(-kT). \tag{10}$$

Summing (9) over k from $-L$ to L, we get

$$\sum_{k=-L}^{L}\sum_{l=0}^{2}c_{kl}X_kT_o^l=\sum_{l=0}^{2}\sum_{k=-L}^{L}c_{kl}X_kT_o^l.$$

Let W_l denote $\sum_{k=-L}^{L}c_{kl}X_k$. Then from (8) the decision variable can be written as

$$z(T_0)\approx W_0+W_1T_0+W_2T_0^2+n_r(T_0), \tag{11}$$

where

(a) the random variables W_0, W_1 and W_2 are determined by iid binary random sequence $X_{-L}X_{-L+1}\cdots X_L$ and $h_o(t)$,

(b) the random variable T_0 is a Gaussian random variable that is independent of X_k's and hence of W_i's and that the mean 0 and variance σ_T^2,

(c) and the random variable $n_r(T_0)$ is a Gaussian random variable, independent of T_0 and X_k's, with the mean 0 and the variance σ_n^2 given in (6).

2.1 The Worst-Case Analysis: The First-Order Effect of Jitter

The worst-case of the intersymbol interference due to the sampling-time jitter can be studied from a close look at (4). From (8) the worst-case interference occurs when each individual term in

$$\sum_{\substack{k=-L\\k\neq 0}}^{L}X_kh_o(T_0-kT)$$

affects adversely, i.e., $X_kh_o(T_0-kT)=|h_o(T_0-kT)|$ for each k. Then

$$P(E)=\Pr(z(T_0)<0\mid X_0=1)\le\Pr\Big(h_o(T_0)-\sum_{\substack{k=-L\\k\neq 0}}^{L}|h_o(T_0-kT)|+n_r(T_0)<0\mid X_0=1\Big).$$

We denote the bounding probability in the right hand side of the above inequality by $P_b(E)$, which will be called the worst-case bit error probability,

$$P_b(E)\triangleq\Pr\Big(h_o(T_0)-\sum_{\substack{k=-L\\k\neq 0}}^{L}|h_o(T_0-kT)|+n_r(T_0)<0\mid X_0=1\Big).$$

Approximation formulas for $P_b(E)$ can be derived from (11). The first-order approximation of $P_b(E)$ is obtained by treated W_2 as 0 for the following reason. It can be conceivable that one can approximate the overall filter $h_o(t)$ to the degree of precision that one wants by taking the desired number of terms in the Taylor series approximation. In this regard, by the first-order effect, we will mean the effect of jitter when the first-order approximation is used and hence the terms due to the second-order derivative of $h_o(t)$ or higher are ignored. This then dictates setting W_2 in

Table 1. The derivatives of $h_o(t)$ evaluated at $t = kT$

	$k=0$	$k = \pm 1, \pm 2, \dots, \pm L$
$h_o(kT)/\sqrt{E_b}$:	1	0
$h_o'(kT)/\sqrt{E_b}$:	0	$\frac{(-1)^k \cos(\alpha\pi k)}{kT\left(1-4\alpha^2k^2\right)}$, if $2\alpha k \neq 1$ $\frac{(-1)^k \pi}{4kT}$, if $2\alpha k = 1$
$h_o''(kT)/\sqrt{E_b}$:	$\frac{\left(\alpha^2(24-3\pi^2)-\pi^2\right)}{3T^2}$	$\frac{2(-1)^k\left((12\alpha^2k^2-1)\cos(\alpha\pi k)+\alpha\pi k(4\alpha^2k^2-1)\sin(\alpha\pi k)\right)}{k^2T^2\left(1-4\alpha^2k^2\right)^2}$, if $2\alpha k \neq 1$ $-\frac{(-1)^k}{T^2}\frac{3\pi}{4k^2}$, if $2\alpha k = 1$

(11) to 0.

Then, using n_r for $n_r(T_0)$ due to the independence of $n_r(T_0)$ and T_0 as noted previously, the first-order approximation $P_1(E)$ of $P_b(E)$ can be derived from the following expression:

$$P_1(E) \stackrel{\triangle}{=} \max_{X_{-L},\dots,X_L} \Pr(W_0 + W_1 T_0 + n_r < 0 \mid X_0 = 1)$$

$$= \max_{X_{-L},\dots,X_L} \iint_{W_0+W_1u+v<0} f_{T_0 n_r|X_0}(u, v \mid X_0 = 1)\,du\,dv, \qquad (12)$$

where $f_{T_0 n_r|X_0}(u, v \mid X_0=1)$ is the joint probability density function of T_0 and n_r conditioned on $X_0 = 1$. The first-order approximation $P_1(E)$ of $P_b(E)$ is obtained when W_1 takes on a value that maximizes (12). The decision region will be determined by

$$W_0 + W_1 T_0 + n_r < 0 \text{ given } X_0 = 1.$$

For the computation of W_0 and W_1, one can use $X_0 = 1$, the Taylor series coefficients c_{kl} in (10) and Table 1 to obtain

$$W_0 = \sqrt{E_b} \text{ given } X_0 = 1,$$

$$W_1 = \frac{\sqrt{E_b}}{T} \sum_{k=1}^{L} \frac{(-1)^k \cos(\alpha\pi k)}{k(1-4\alpha^2 k^2)} (X_{-k} - X_k).$$

It is noted that W_1 is still a random variable while $W_0 = 1$ due to the conditioning on $X_0 = 1$. We will treat W_1 for the time being as a constant. (Or one can use enough condition so that it may be treated as a constant.)

The evaluation can be carried out without performing the double integral in the above expression, by noting that T_0 and n_r are independent Gaussian random variables. Let $Y \stackrel{\triangle}{=} W_1 T_0 + n_r$. Then Y is Gaussian with the mean 0 and the variance $\sigma_Y^2 = W_1^2 \sigma_T^2 + \sigma_n^2$.

$$\iint_{W_1u+v+\sqrt{E_b}<0} f_{T_0 n_r|X_0}(u, v \mid X_0 = 1)\,du\,dv$$

$$= \int_{-\infty}^{-\sqrt{E_b}} \frac{1}{\sqrt{2\pi}\,\sigma_Y} e^{-\frac{y^2}{2\sigma_Y^2}} dy = Q\left(\frac{\sqrt{E_b}}{\sigma_Y}\right).$$

The maximum probability is obtained, when W_1 achieves either its maximum or its minimum over the choice of the source bit sequence $X_{-L}X_{-L+1}\cdots X_{-1}X_1\cdots X_L$, where each X_k can take on either 1 or -1. This is due to the fact that the maximum or minimum value of W_1 will make σ_Y largest and the resulting probability maximum. (The maximum and minimum have the same value with the opposite signs.) The maximum of W_1, denoted by μ, is

$$\mu \triangleq \frac{2\sqrt{E_b}}{T}\sum_{k=1}^{L}\left|\frac{\cos(\alpha\pi k)}{k(1-4\alpha^2k^2)}\right|. \tag{13}$$

For $\alpha=0$, the source bit sequence that achieves this maximum is $X_1=-X_{-1}=1$, $X_2=-X_{-2}=-1$, and so on. Therefore the first-order approximation $P_1(E)$ of $P_b(E)$ is given by

$$P_1(E)=Q\left(\frac{\sqrt{E_b}}{\left[\left(\frac{2\sqrt{E_b}\,\sigma_T}{T}\sum_{k=1}^{L}\left|\frac{\cos(\alpha\pi k)}{k(1-4\alpha^2k^2)}\right|\right)^2+\sigma_n^2\right]^{\frac{1}{2}}}\right). \tag{14}$$

In case that L is large, the variance σ_n^2 approximately equals $\frac{N_0}{2}$. For this $\sigma_n^2=\frac{N_0}{2}$, we rewrite (14) as follows: for large L,

$$P_1(E)=Q\left(\frac{\sqrt{\frac{2E_b}{N_0}}}{\left[\frac{2E_b}{N_0}\left(\frac{2\sigma_T}{T}\sum_{k=1}^{L}\left|\frac{\cos(\alpha\pi k)}{k(1-4\alpha^2k^2)}\right|\right)^2+1\right]^{\frac{1}{2}}}\right). \tag{15}$$

We can make several observations from (14)-(15).

(a) When there is no jitter in sampling-time and hence no intersymbol interference, this case corresponds to $\sigma_T=0$. In such a case the worst-case bit error probability $P_b(E)$ reduces to the optimum bit error probability for antipodal signaling, i.e.,

$$P_b(E)=Q\left(\sqrt{\frac{2E_b}{N_0}}\right),$$

as is expected. This can be a partial support that the result agrees well with the known analysis.

(b) If there is no channel noise, this corresponds to $N_0=0$ and accordingly $\sigma_n^2=0$. This would be a situation one may use to study the intersymbol interference alone. In this case we have

$$P_1(E)|_{N_0=0}=Q\left(\frac{\frac{T}{2\sigma_T}}{\sum_{k=1}^{L}\left|\frac{\cos(\alpha\pi k)}{k(1-4\alpha^2k^2)}\right|}\right)$$

$$=\lim_{\frac{E_b}{N_0}\to\infty}P_1(E). \tag{16}$$

From (16) we can study the effect of the roll-off factor α on the bit error probability. A couple of examples $\alpha=0$ and 1 are discussed.

The case of $\alpha=0$: This choice of α yields

$$\sum_{k=1}^{L}\left|\frac{\cos(\alpha\pi k)}{k(1-4\alpha^2k^2)}\right|_{\alpha=0}=\sum_{k=1}^{L}\frac{1}{k},$$

which grows without bound as L becomes arbitrarily large. This implies that, although $\frac{\sigma_T}{T}$ is small, the worst-case bit error probability can be very large. In theory it becomes $\frac{1}{2}$ for $L=\infty$ and can nullify the communication system. Through a computer evaluation, $\sum_{k=1}^{10}\frac{1}{k}\approx 2.929$.

The case of $\alpha=1$: For this case

$$\sum_{k=1}^{L}\left|\frac{\cos(\alpha\pi k)}{k(1-4\alpha^2k^2)}\right|_{\alpha=1}$$

converges with L. To confirm the convergence one can use the alternating series theorem, see for example [5, p.78]. A computer evaluation yields

$$\sum_{k=1}^{10}\left|\frac{\cos(\pi k)}{k(1-4k^2)}\right|\approx 0.3852.$$

The comparison with $\alpha=0$ for the truncation length $2L+1=2\cdot 20+1=21$ gives the ratio of

$\frac{2.929}{0.3852} \approx 7.604$. Therefore, for $L=10$, the signal-to-interference ratio is better with $\alpha=1$ than with $\alpha=0$. Quantitatively, the roll-off factor $\alpha=1$ is $10\log_{10}7.604 \approx 8.81$ dB better than $\alpha=0$. However, it must be noted that this gain does not come free since the filter with $\alpha=1$ uses double the bandwidth of $\alpha=0$.

2.2 The Worst-Case Analysis : The Second-Order Effect of Jitter

The first-order approximation $P_1(E)$ of the worst-case bit error probability $P_b(E)$ loses accuracy as the degree of jitter increases. Numerical results for $\alpha=1$ show that, when $\frac{\sigma T}{T} \lessapprox 0.03$, the approximation $P_1(E)$ is approximately 70% of $P_b(E)$ on the average over the range of $\frac{E_b}{N_0}=0\sim10$ dB. (The subsequent section has a more detailed discussion on the accuracy.)

To have a better approximation formula than $P_1(E)$, we can take more terms in the Taylor series approximation. We define the second-order approximation $P_2(E)$ of the worst-case bit error probability $P_b(E)$ by the following formula: from (11)

$$P_2(E) \triangleq \max_{X_{-L},\cdots,X_L} \Pr(W_0+W_1T_0+W_2T_0^2+n_r<0\,|\,X_0=1), \qquad (17)$$

where

$$W_0=\sqrt{E_b} \text{ given } X_0=1,$$

$$W_1=\frac{\sqrt{E_b}}{T}\sum_{k=1}^{L}\frac{(-1)^k\cos(\alpha\pi k)}{k(1-4\alpha^2k^2)}(X_{-k}-X_k),$$

$$W_2=\frac{\sqrt{E_b}}{2T^2}\left(\frac{24\alpha^2-3\alpha^2\pi^2-\pi^2}{3}+2\sum_{k=1}^{L}\frac{(-1)^k((12\alpha^2k^2-1)\cos(\alpha\pi k)+\alpha\pi k(4\alpha^2k^2-1)\sin(\alpha\pi k))}{k^2(1-4\alpha^2k^2)^2}(X_{-k}-X_k)\right).$$

The source sequence $X_{-L}-X_L$ that achieves $P_2(E)$ in general depends on the roll-off factor α and is not simple to find. However, for the case of $\alpha=0$ and 1, simpler expressions for $P_2(E)$ can be found from a straightforward evaluation and simplification. Since deriving an useful information from (17) is not easy, we close the section noting that in the subsequent section the two approximations will be compared for their accuracy.

In the following section numerical results are presented based on the analysis developed here.

Ⅲ. Numerical Results

Through numerical evaluation of (15) and (17) we obtain several results, which are tabulated in Tables 2 and 3, and plotted in Figs. 2-3. The chosen roll-off factors are 0, 0.2, 0.35, 0.5, and 1. Since there is a convergence problem when $\alpha=0$, the truncation length is chosen to be $2L+1=21$. We believe that $L=10$ accounts for communication systems in practice and is considered big enough to justify the approximation that $\sigma_n^2 \approx \frac{N_0}{2}$. The chosen ratio $\frac{\sigma_T}{T}$ of the Gaussian jitter deviation to the source symbol duration are 0.01, 0.03, 0.05, and 0.1.

Fig. 2 shows $P_1(E)$, $P_2(E)$ and $P_b(E)$ versus $\frac{E_b}{N_0}$ for the roll-off factor $\alpha=1$ as the degree of jitter, $\frac{\sigma_T}{T}$, is varied, while Fig. 3 shows $P_1(E)$ and $P_b(E)$ versus $\frac{E_b}{N_0}$ for $\frac{\sigma_T}{T}=0.01$ for various roll-off factors. The value $\frac{\sigma_T}{T}=0.01$ is chosen, since for this value the approximation formulas are considered accurate enough to discuss the effect of various roll-off factors on the worst-case bit error probability and to compare with the simulated $P_b(E)$.

Fig. 2 shows that the general trend is that the intersymbol interference worsens and the disparity between the simulated error probability and the appro-

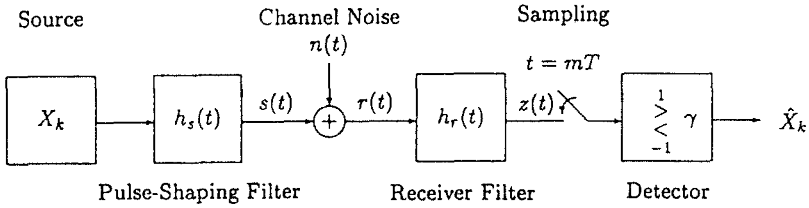

Fig. 1 The block diagram of the baseband digital communication system

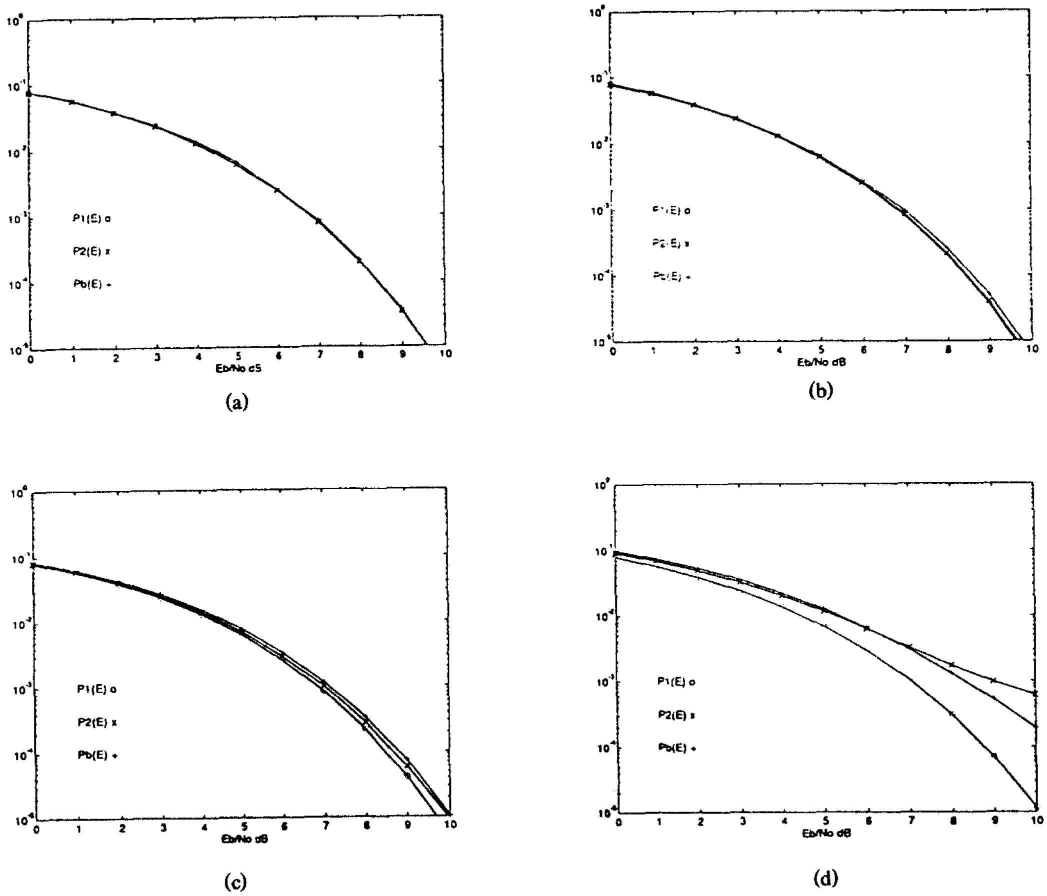

Fig. 2 $P_1(E)$, $P_2(E)$ and $P_b(E)$ vs. $\frac{E_b}{N_0}$ for $\alpha = 1$ (a) $\frac{\sigma_T}{T} = 0.01$

(b) $\frac{\sigma_T}{T} = 0.03$ (c) $\frac{\sigma_T}{T} = 0.05$ (d) $\frac{\sigma_T}{T} = 0.1$.

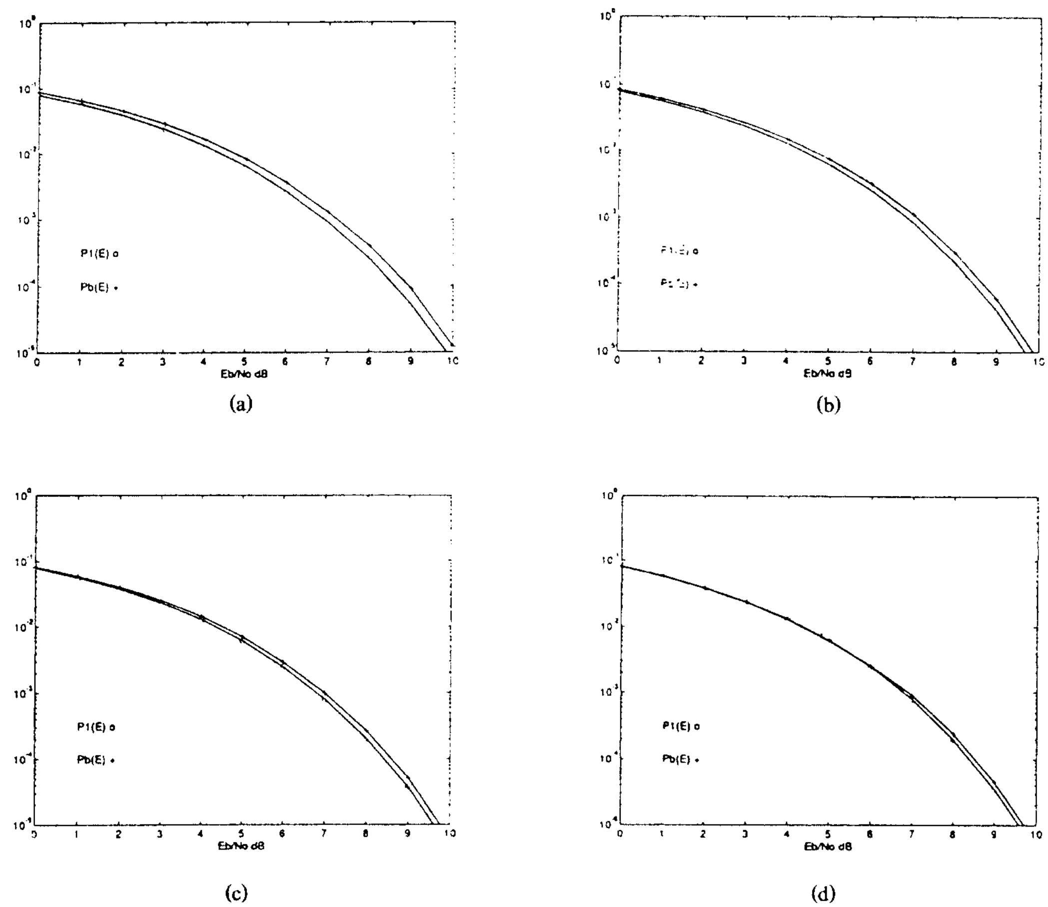

Fig. 3 $P_1(E)$ and $P_b(E)$ vs. $\frac{E_b}{N_0}$ for $\frac{\sigma_T}{T}=0.01$ (a)$\alpha=0$ (b)$\alpha=0.2$ (c)$\alpha=0.35$ (d)$\alpha=0.5$.

ximations increases with $\frac{\sigma_T}{T}$. The latter means that the approximations lose accuracy for large $\frac{\sigma_T}{T}$. Also we can make the following observations from these plots.

(a) We have, for the most cases, $P_1(E) < P_2(E) < P_b(E)$. One exceptional case is for $\frac{\sigma_T}{T}=0.1$. In that case $P_2(E)$ starts out smaller and becomes bigger than $P_b(E)$ at around $\frac{E_b}{N_0}\approx 7$ dB.

(b) The approximation formulas maintain reasonable accuracy up to $\frac{\sigma_T}{T}\approx 0.03$ for all the range of $\frac{E_b}{N_0}=0\sim10$ dB. For $\frac{\sigma_T}{T}\approx 0.01$, $P_1(E)$ is at least 90% and on the average approximately 95% of $P_b(E)$, while $P_2(E)$ is more accurate. For $\frac{\sigma_T}{T}\approx 0.03$, $P_1(E)$ is over 90% of $P_b(E)$ for up to $\frac{E_b}{N_0}=6$ dB, about 80% for $\frac{E_b}{N_0}=7\sim8$ dB, and about 70% for $\frac{E_b}{N_0}=9\sim10$ dB, while $P_2(E)$ is closer to $P_b(E)$ than $P_1(E)$ in that $P_2(E)$ stays within 78% of P_1

Table 2. $P_1(E)$, $P_2(E)$ and $P_b(E)$ for $\alpha=1$

$\alpha = 1$ $\frac{\sigma_T}{T} = 0.01$			
$\frac{E_b}{N_o}$ dB	$P_1(E)$	$P_2(E)$	$P_b(E)$
0	7.8662E-2	7.8769E-2	8.0410E-2
1	5.6295E-2	5.6388E-2	5.7820E-2
2	3.7520E-2	3.7595E-2	3.8480E-2
3	2.2891E-2	2.2947E-2	2.3870E-2
4	1.2512E-2	1.2549E-2	1.3590E-2
5	5.9619E-3	5.9840E-3	6.6500E-3
6	2.3933E-3	2.4042E-3	2.5233E-3
7	7.7519E-4	7.7960E-4	8.3760E-4
8	1.9188E-4	1.9324E-4	2.1010E-4
9	3.3895E-5	3.4196E-5	3.7500E-5
10	3.9206E-6	3.9645E-6	4.0000E-6

$\alpha = 1$ $\frac{\sigma_T}{T} = 0.03$			
$\frac{E_b}{N_o}$ dB	$P_1(E)$	$P_2(E)$	$P_b(E)$
0	7.8760E-2	7.9739E-2	8.3370E-2
1	5.6403E-2	5.7255E-2	6.0100E-2
2	3.7629E-2	3.8324E-2	3.9600E-2
3	2.2994E-2	2.3516E-2	2.4210E-2
4	1.2598E-2	1.2953E-2	1.3560E-2
5	6.0258E-3	6.2376E-3	6.6700E-3
6	2.4332E-3	2.5411E-3	2.7000E-3
7	7.9540E-4	8.4051E-4	9.6200E-4
8	1.9973E-4	2.1449E-4	2.5600E-4
9	3.6088E-5	3.9648E-5	5.0700E-5
10	4.3248E-6	4.9157E-6	6.3000E-6

$\alpha = 1$ $\frac{\sigma_T}{T} = 0.05$			
$\frac{E_b}{N_o}$ dB	$P_1(E)$	$P_2(E)$	$P_b(E)$
0	7.8957E-2	8.1751E-2	8.6190E-2
1	5.6618E-2	5.9071E-2	6.2900E-2
2	3.7848E-2	3.9875E-2	4.2680E-2
3	2.3200E-2	2.4751E-2	2.6610E-2
4	1.2772E-2	1.3852E-2	1.5120E-2
5	6.1546E-3	6.8242E-3	7.6400E-3
6	2.5142E-3	2.8746E-3	3.3400E-3
7	8.3685E-4	1.0004E-3	1.1500E-3
8	2.1614E-4	2.7666E-4	3.3500E-4
9	4.0802E-5	5.8442E-5	7.4400E-5
10	5.2350E-6	9.1514E-6	1.0400E-5

$\alpha = 1$ $\frac{\sigma_T}{T} = 0.1$			
$\frac{E_b}{N_o}$ dB	$P_1(E)$	$P_2(E)$	$P_b(E)$
0	7.9878E-2	9.2459E-2	9.9310E-2
1	5.7622E-2	6.9129E-2	7.5110E-2
2	3.8876E-2	4.8935E-2	5.3740E-2
3	2.4169E-2	3.2516E-2	3.6140E-2
4	1.3598E-2	2.0134E-2	2.2110E-2
5	6.7752E-3	1.1589E-2	1.2650E-2
6	2.9137E-3	6.2546E-3	6.4160E-3
7	1.0494E-3	3.2566E-3	3.0360E-3
8	3.0585E-4	1.7221E-3	1.3000E-3
9	6.9415E-5	9.7862E-4	5.2800E-4
10	1.1775E-5	6.1480E-4	1.9350E-4

(E) over the range of $\frac{E_b}{N_0} = 0 \sim 10$ dB. For $\frac{\sigma_T}{T} \lessapprox 0.05$ the gap between the first and second-order approximations is noticeable, while for smaller jitter this gap is insignificant. This noticeable gap is an indication that a higher-order approximation may yield better accuracy. On the other hand, we believe that higher-order approximations lose simplicity and may not be so useful.

(c) It can be seen with the aid of Table 2 that in general the accuracy of the approximation formulas decreases as $\frac{E_b}{N_0}$ increases. The loss of accuracy accelerates at around $\frac{E_b}{N_0} \approx 7$ dB and wor-

sens for large $\frac{\sigma_T}{T}$. We think that it is caused by the fact that, for large $\frac{\sigma_T}{T}$, the increase in $\frac{E_b}{N_0}$ affects severely the argument of the Q function in (15).

Fig. 3 shows plots of $P_1(E)$ and $P_b(E)$ for a fixed $\frac{\sigma_T}{T}=0.01$ and for various roll-off factors. The first impression from it is that the intersymbol interference due to sampling-time jitter decreases as the roll-off factor increases. This is a widely known result and in fact this is the reason that the raised-cosine pulse-shaping filters are used. They also show that the accuracy of the first-order approximation gets better as the roll-off factor increases, as can be seen that the size of the gap between the curves marked by 'o' for $P_1(E)$ and '+' for $P_b(E)$ becomes smaller.

Table 3. $P_1(E)$ and $P_b(E)$ for $\frac{\sigma_T}{T}=0.01$

	$\alpha=0$		$\alpha=0.2$	
$\frac{E_b}{N_o}$ dB	$P_1(E)$	$P_b(E)$	$P_1(E)$	$P_b(E)$
0	7.9361E-2	8.8888E-2	7.8942E-2	8.5199E-2
1	5.7058E-2	6.5137E-2	5.6600E-2	6.2067E-2
2	3.8298E-2	4.4991E-2	3.7831E-2	4.2244E-2
3	2.3623E-2	2.8623E-2	2.3183E-2	2.6468E-2
4	1.3131E-2	1.6466E-2	1.2758E-2	1.4980E-2
5	6.4229E-3	8.3400E-3	6.1443E-3	7.4461E-3
6	2.6851E-3	3.6540E-3	2.5077E-3	3.1615E-3
7	9.2615E-4	1.3170E-3	8.3348E-4	1.1018E-3
8	2.5273E-4	3.9600E-4	2.1479E-4	3.0290E-4
9	5.1928E-5	8.9200E-5	4.0408E-5	6.0300E-5
10	7.5871E-6	1.2200E-5	5.1569E-6	7.6000E-6

	$\alpha=0.35$		$\alpha=0.5$	
$\frac{E_b}{N_o}$ dB	$P_1(E)$	$P_b(E)$	$P_1(E)$	$P_b(E)$
0	7.8797E-2	8.3250E-2	7.8729E-2	8.2220E-2
1	5.6443E-2	6.0334E-2	5.6368E-2	5.9150E-2
2	3.7670E-2	4.0828E-2	3.7594E-2	3.8720E-2
3	2.3032E-2	2.5368E-2	2.2961E-2	2.3740E-2
4	1.2631E-2	1.4185E-2	1.2570E-2	1.3220E-2
5	6.0497E-3	6.9648E-3	6.0052E-3	6.3600E-3
6	2.4482E-3	2.9065E-3	2.4203E-3	2.5100E-3
7	8.0301E-4	9.9410E-4	7.8886E-4	9.1000E-4
8	2.0272E-4	2.6550E-4	1.9718E-4	2.4300E-4
9	3.6933E-5	5.1300E-5	3.5371E-5	4.5400E-5
10	4.4838E-6	6.1000E-6	4.1912E-6	5.4000E-6

Ⅳ. Conclusions

This paper addresses the effect of sampling-time jitter on the error probability in a baseband communication system that uses square-root raised-cosine filters. Approximation formulas of the worst-case bit error probability are derived from the Taylor series approximation of the overall filter response. For an additive white Gaussian noise channel, these formulas can be used to investigate the relationship of the worst-case bit error probability and the affecting factors: the degree of jitter $\frac{\sigma_T}{T}$; the signal-to-noise ratio $\frac{E_b}{N_0}$; the roll-off factor α; and the truncation length L. These formulas are reasonably accurate when the degree of jitter is approximately within 3% in the case of $\alpha=1.0$. Their accuracy suffers for large jitter and large signal-to-noise ratio.

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