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Approximations to blocking probability in Two-Stage Queueing Model

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ABSTRACT

We investigate a two-stage queueing system which frequently arises in the study of overflow problems. A primary service facility consists of multiple primary queues where blocked calls are overflowed to a secondary queue. By approximating the input to the secondary queue with a two-state Markov Modulated Poisson Process (MMPP), we derive the blocking probability of the secondary queue. For the approximation, we employ the well-known Heffes' method and the SAM procedure.

요 약

본 논문에서는 오버플로우에 관한 연구에서 흔히 나타나는 이단계 대기모형을 연구하였다. 첫번째 단계는 여러 개의 큐(queue)로 이루어져 있고, 이 큐들의 수용능력을 넘어서는 순간에 두번째 단계로 오버플로우가 발생한다. 두번째 단계로의 입력과정을 두 개의 상태를 가지는 MMPP로 근사함으로써 손실확률에 대한 계산을 이끌어 내었다. 근사방법으로써, Heffes의 방법과 SAM procedure를 이용하였다.

Ⅰ. Introduction

In this paper we study a two-stage queueing system with multiple primary queues and a single secondary queue, where blocked calls in primary queues are overflowed to a secondary queue. This model has potential applications in telecommunication networks where several routing strategies are used for congestion control and improvement of system's availability. According to [6], the computational complexity of the exact analysis often makes it necessary to develop accurate approximation techniques.

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Within this framework, many researchers have replaced each overflow process from the primary queues by Interrupted Poisson Process(IPP) through the moment matching method. See [1]. Then input to the secondary queue is the superposition of an exogenous Poisson and multiple IPP's. Matsumoto [4] exactly analyzed a queueing model where one Poisson and two IPP's are offered simultaneously. The model with a Poisson and more than three IPP's was studied by Hellstern [5]. She showed that the overflow process of the model is a Markov Modulated Poisson Process(MMPP).

We focus on the reduction of computational burden to obtain the blocking probability in the secondary queue. To this end, we approximate the result of Hellstern to a two-state MMPP by employing two techinques. One is Heffes' method [3] and the other is SAM procedure [7].

The remainder of this paper is structured as follows. In section 2, we describe a two-stage queueing model and its exact analysis. Section 3 discusses two approximation techniques and the application to our model. We present numerical studies in section 4 and the conclusion is given in section 5. Finally, simulation results are summarized in section 6.

Ⅱ. Model and Analysis

The two-stage queueing model considered in this paper is given as follows. The primary service facility is composed of $M/M/c_i/K_i$, $(i=1, 2, \cdots, m)$ queues. Let λ_i and μ_i^{-1} be the arrival rate and the mean service time of the queue indexed by i. Overflow calls from the primary queues are allowed to be offered to the secondary queue modeled by $M/M/c/K$ with parameters λ and μ^{-1}. From now on, we assume μ_i's are normalized to 1.

From the arguments in [1] and [2], overflow process from each primary queue can be approximated accurately by a suitable IPP(γ_i, ω_i, λ_i), where $\gamma_i(\omega_i)$ is the transition rate out of the ON(OFF) state and λ_i is the Poisson rate in the ON state. It is well known that IPP(γ_i, ω_i, λ_i) is a two-state MMPP(Q_i, Λ_i), where Q_i and Λ_i have the form of

$$Q_i=\begin{pmatrix} -\gamma_i & \gamma_i \\ \omega_i & -\omega_i \end{pmatrix} \text{ and } \Lambda_i=\begin{pmatrix} \lambda_i & 0 \\ 0 & 0 \end{pmatrix}$$

Since the superposition of independent MMPP's is also an MMPP, the input to the secondary queue, which is the superposition of an exogenous Poisson and several IPP's, can be represented by an MMPP with

$$Q=Q_1 \oplus Q_2 \oplus \cdots \oplus Q_m$$
$$\Lambda=\lambda \cdot I_{2^m}+\Lambda_1 \oplus \Lambda_2 \oplus \cdots \oplus \Lambda_m$$

where I_k denotes the identity matrix of order k and $\oplus$ denotes the Kronecker sum. Thus, the secondary queue becomes an MMPP(Q, Λ)/$M/c/K$ queue.

Now, we consider the overflow process from the secondary queue. MMPP(Q, Λ)/$M/c/K$ queue which models the secondary queue may be studied as a Markov process on the state space $\{(j, j');0 \le j \le K, 1 \le j' \le 2^m\}$. The number j corresponds to the number of calls at the queue, and j' corresponds to the state of the Markov process with infinitesimal generator Q. It will be convenient to let $\boldsymbol{j}$($(\boldsymbol{j}=0, 1, 2, \cdots, \boldsymbol{K})$ denote the set of states $\{(j, j');1 \le j' \le 2^m\}$. The infinitesimal generator and the arrival rate matrix of this process are given by the followings. (see details in [5])

$$\bar{Q}=\left[\begin{array}{ccccccccc} Q-\Lambda & \Lambda & & & & & & & \\ \mu I & A_1 & \Lambda & & & & & & \\ & 2\mu I & A_2 & \Lambda & & & & & \\ & & & \cdots & & & & & \\ & & & c\mu I & A_c & \Lambda & & & \\ & & & & c\mu I & A_c & \Lambda & & \\ & & & & & & \cdots & & \\ & & & & & & c\mu I & A_c & \Lambda \\ & & & & & & & c\mu I & Q-c\mu I \end{array}\right] \quad (1)$$

and

$$\bar{\Lambda} = diag(0_{2^m}, \cdots, 0_{2^m}, \Lambda) \tag{2}$$

where 0_k denotes the $k \times k$ matrix in which all elements are zero and $A_n = Q - \Lambda - n\mu I$ for $n = 1, 2, \cdots, c$. Then, we can obtain the blocking probability in the secondary queue

$$b = \frac{\bar{\pi}_K \Lambda e}{\sum_{n=0}^{K} \bar{\pi}_K \Lambda e} \tag{3}$$

where $\bar{\pi} = (\bar{\pi}_0, \bar{\pi}_1, \cdots, \bar{\pi}_K)$ is the stationary vector of $\bar{Q}$ and $e = (1, \cdots, 1)^t$. However, the amount of computation for $\bar{\pi}$ explodes as the number m of primary queues or the size of K gets large.

Ⅲ. Approximations

3.1 Heffes' Method

Heffes' technique [3] is to approximate a multistate MMPP(Q, Λ) to a two-state MMPP with

$$Q_h = \begin{pmatrix} -\gamma_h & \gamma_h \\ \omega_h & -\omega_h \end{pmatrix} \text{ and } \Lambda_i = \begin{pmatrix} \lambda_{h1} & 0 \\ 0 & \lambda_{h2} \end{pmatrix}$$

For the derivation of γ_h, ω_h, λ_{h1} and λ_{h2}, Heffes gives a following matching algorithm.

· Step 1 - We can get the first three noncentral moments and the time constant of the instantaneous arrival rate of the MMPP(Q, Λ) as

$$\alpha_n = \pi \Lambda^n e, \; n = 1, 2, 3$$
$$\beta = (\alpha_2 - (\alpha_1)^2)^{-1} [\pi \Lambda (e\pi - Q)^{-1} \Lambda e - (\alpha_1)^2]$$

where π is the stationary vector of Q.

· Step 2 - The corresponding moments and constant of the MMPP(Q_h, Λ_h) are explicitly given by

$$\alpha_n^h = \lambda_{h1}^n \pi_1^h + \lambda_{h2}^n \pi_2^h, \; n = 1, 2, 3$$
$$\beta_h = (\gamma_h + \omega_h)^{-1}$$

where the $\pi_h = (\pi_1^h, \pi_2^h)$ is the stationary vector of Q_h.

· Step 3 - Matching the three moments and the time constant of the instantaneous arrival rates of both MMPP's, we can derive γ_h, ω_h, λ_{h1} and λ_{h2}.

Through the above matching algorithm, MMPP$(Q, \Lambda)/M/c/K$ queue can be approximated by MMPP$(Q_h, \Lambda_h)/M/c/K$ queue.

In the same way in section 2, we can get the overflow process from the secondary queue, say, MMPP $(\bar{Q}_h, \bar{\Lambda}_h)$. Then, the blocking probability b can be approximated by

$$b_h = \frac{\bar{\pi}_K^h \Lambda_h e}{\sum_{n=0}^{K} \bar{\pi}_K^h \Lambda_h e} \tag{4}$$

where $\bar{\pi}_h = (\bar{\pi}_0^h, \bar{\pi}_1^h, \cdots, \bar{\pi}_K^h)$ is the stationary vector of $\bar{Q}_h$.

3.2 SAM Procedure

The SAM procedure is originally designed to approximate the superposition of N homogeneous ON-OFF sources by a two-state MMPP. See [7]. The main idea of the SAM procedure is to divide the state space of the superposed process into two regions, say, underload(UL) and overload(OL) region. If we consider an IPP to be an ON-OFF source, we can replace MMPP(Q, Λ) by a two-state MMPP with suitable parameters

$$Q_s = \begin{pmatrix} -\gamma_s & \gamma_s \\ \omega_s & -\omega_s \end{pmatrix} \text{ and } \Lambda_s = \begin{pmatrix} \lambda_{s1} & 0 \\ 0 & \lambda_{s2} \end{pmatrix}$$

where $\omega_s(\gamma_s)$ is the transition rate out of the OL(UL) state and $\lambda_{s2}(\lambda_{s1})$ is the Poisson arrival rate in the OL(UL) state. We can derive ω_s, γ_s, λ_{s1} and λ_{s2} in the following way.

· Step 1 - We rearrange Q and Λ according to the size of arrival rates given in Λ.

· Step 2 - Compare the arrival rates with the capacity of the secondary queue to divide the state space $S = \{1, 2, \cdots, 2^m\}$ of the above Markov process Q into two regions: UL region $R_{UL} = \{1, 2, \cdots, L\}$ and OL region $R_{OL} = \{L+1, \cdots, 2^m\}$.

· Step 3 - OL region duration can be identified with the absorption time in a transient Markov process with the state space $\{L\} \cup R_{OL}$ and the infinitesimal generator Q^*

$$q^*_{i,j} = \begin{cases} q_{i,j} & , i \in R_{OL}, j \in R_{OL} - \{L+1\} \\ q_{i,j} + \sum_{j=1}^{L} q_{i,j}, & i \in R_{OL}, j = L+1 \end{cases}$$

where all $q_{i,j}$ are elements of the above matrix Q.

· Step 4 – Using the similar arguments to [7], the parameters of MMPP(Q_s, Λ_s) must be

$$\omega_s = \eta$$

$$\lambda_{s1} = \frac{\sum_{j \in R_{UL}} \Lambda_j \pi_j}{\sum_{j \in R_{UL}} \pi_j}$$

$$\lambda_{s2} = \frac{\sum_{j \in R_{OL}} \Lambda_j \pi_j}{\sum_{j \in R_{OL}} \pi_j}$$

$$\gamma_s = \frac{\omega_s (\lambda_{s1} - \sum_{j \in S} \Lambda_j \pi_j)}{\sum_{j \in S} \Lambda_j \pi_j - \lambda_{s2}}$$

where $-\eta$ is the unique maximal real part eigenvalue of Q^* and Λ_j is the j^{th} diagonal element of Λ. The vector $\pi = (\pi_1, \cdots, \pi_{2^*})$ is the stationary vector of Q.

Through the above matching algorithm, MMPP(Q, Λ) $/M/c/K$ queue can be approximated by MMPP(Q_s, Λ_s) $/M/c/K$ queue.

In the same way in section 2, we can get the overflow process from the secondary queue, say, MMPP $(\overline{Q}_s, \overline{\Lambda}_s)$. Then, the blocking probability b can be approximated by

$$b_s = \frac{\overline{\pi}^s_K \Lambda_s e}{\sum_{n=0}^{K} \overline{\pi}^s_n \Lambda_s e} \quad (5)$$

where $\overline{\Lambda}_s = (\overline{\pi}^s_0, \overline{\pi}^s_1, \cdots, \overline{\pi}^s_K)$ is the stationary vector of $\overline{Q}_s$.

Ⅳ. Simulation

For the numerical study, we consider the various case where the primary service facility is characterized by Table 1 and the secondary queue is characterized in all cases by $\lambda = 5$, $c = 3$, and $\mu = 3$. Computations are made varying the size of K and the results are given as tables in section 6.

Table 1. Simulation values

parameter	$\lambda^*_1\ c_1\ K_1$	$\lambda^*_2\ c_2\ K_2$	$\lambda^*_3\ c_3\ K_3$	$\lambda^*_4\ c_4\ K_4$
Model 1	1 2 5	1 3 7	2 4 7	3 6 11
Model 2	1 2 5	2 3 7	3 4 7	5 6 11
Model 3	2 2 5	3 3 7	4 4 7	6 6 11
Model 4	1 2 5	3 3 7	4 4 7	8 6 11
Model 5	1 2 5	3 3 7	5 4 7	8 6 11
Model 6	3 2 5	4 3 7	5 4 7	7 6 11

Over the whole range of our simulation, we can find some features.

1) The SAM procedure always underestimates the blocking probability. Since SAM procedure assumes the ON-OFF sources which generates calls deterministically, it cannot explain the randomness of arriavals from the actual input which generates calls according to a Poisson process.

2) Absolute errors($|b - b_h|$ and $|b - b_s|$) show us that Heffes(b_h) is more accurate than SAM(b_s). Roughly speaking, the accuracy of two approximation techniques differs by one or two decimal points.

Watching the behavior of result in detail, another features are found.

3) The sign of error due to Heffes' method oscillates according to the size of K. If we have much interest in getting upper(or lower) bound of the blocking probability, the behavior like this is unstable and somewhat undesirable in spite of the absolute errors being within a tolerable level. In this aspect, SAM(b_s) shows more stable behavior than Heffes(b_h). SAM(b_s) always gives lower bound over the whole range of simulation and moreover, errors decrease monotonically to 0.

4) The resulting values of Model 1 give us hope that Heffes(b_h) and SAM(b_s) give an upper bound and a lower bound, respectively. If this is true in all cases, we can get excellent information about the exact blocking probability. Unfortunately, this is not the

case with other Models. In Model 2-6, the same behavior as Model 1 comes true until $K-c=6,12,30,$ 66, and 100(except 1), respectively. But from the practical point of view, we have informative results for the cases in which the load and the size of waiting rooms are common in real situations, for example, Model 3-6 with moderate size of K.

V. Conclusion

We have analyzed a two-stage queueing system which frequently arises in the study of overflow problems. By modeling the overflow processes from the primary queues by IPP's, it is possible to represent the input to the secondary service facility as an MMPP and to analyze the queueing system exactly. Our study is focused on the approximation of the input MMPP to the secondary service facility by means of a suitably chosen two-state MMPP. We employ two approximation techniques. One is the general technique of Heffes and the other is SAM procedure. Note that we have extended SAM procedure to the case of heterogeneous sources.

Though the resulting model obtained by approximation is a quite rough representation of a complex real situation, it provides good accuracy as compared to simulation results. We compared two approximate blocking probabilities with exact one varying the size of waiting rooms. Heffes' method gives more accurate estimate of the blocking probability than the SAM procedure. The SAM procedure shows the more stable behavior than Heffes' method and, in some common situations, the SAM procedure and Heffes' method give us a lower bound and an upper bound for the exact blocking probability respectively, which is very informative.

VI. Tables

Table 2. Model 1-extreme low offered load

$K-c$	b	b_h	b_s	$b-b_h$	$b-b_s$
1	9.93384E-02	9.94164E-02	9.63133E-02	-7.800E-05	3.0251E-03
2	5.70755E-02	5.71628E-02	5.43337E-02	-8.730E-05	2.7418E-03
3	3.36819E-02	3.37651E-02	3.13949E-02	8.320E-05	2.2870E-03
4	2.01869E-02	2.02585E-02	1.83840E-02	-7.160E-05	1.8029E-03
5	1.22121E-02	1.22695E-02	1.08478E-02	-5.740E-05	1.3643E-03
6	7.43040E-03	7.47420E-03	6.42950E-03	-4.380E-05	1.0009E-03
7	4.53740E-03	4.56970E-03	3.82080E-03	-3.230E-05	7.1660E-04
8	2.77730E-03	2.80030E-03	2.27400E-03	-2.300E-05	5.0330E-04
9	1.70250E-03	1.71870E-03	1.35470E-03	-1.620E-05	3.4780E-04
10	1.04470E-03	1.05580E-03	8.07500E-04	-1.110E-05	2.3720E-04
11	6.41500E-04	6.49100E-04	4.81500E-04	-7.600E-06	1.6000E-04
12	3.94100E-04	3.99200E-04	2.87100E-04	-5.100E-06	1.0700E-04
13	2.42200E-04	2.45600E-04	1.71200E-04	-3.400E-06	7.1000E-05
14	1.48900E-04	1.51100E-04	1.02100E-04	2.200E-06	4.6800E-05
15	9.15000E-05	9.30000E-05	6.09000E-05	1.500E-06	3.0600E-05
16	5.63000E-05	5.72000E-05	3.63000E-05	9.000E-07	2.0000E-05
17	3.46000E-05	3.52000E-05	2.17000E-05	-6.000E-07	1.2900E-05
18	2.13000E-05	2.17000E-05	1.29000E-05	-4.000E-07	8.4000E-06
19	1.31000E-05	1.33000E-05	7.71500E-06	-2.000E-07	5.3850E-06
20	8.04670E-06	8.21140E-06	4.60220E-06	-1.647E-07	3.4445E-06
21	4.94880E-06	5.05440E-06	2.74540E-06	-1.056E-07	2.2034E-06
22	3.04360E-06	3.11110E-06	1.63770E-06	6.750E-08	1.4059E-06
25	7.08100E-07	7.25590E-07	3.47650E-07	1.749E-08	3.6045E-07
30	6.23260E-08	6.41220E-08	2.62610E-08	-1.796E-09	3.6065E-08
40	4.83000E-10	5.00800E-10	1.44900E-10	1.780E-11	3.3810E-10
50	3.74100E-12	3.99100E-12	8.55400E-13	-2.500E-13	2.8856E-12
55	3.28500E-13	3.44600E-13	6.55100E-14	-1.610E-14	2.6299E-13
60	2.71600E-14	2.93600E-14	5.23800E-15	2.200E-15	2.1922E-14

Table 3. Model 2-overall low offered load

$K-c$	b	b_h	b_s	$b-b_h$	$b-b_s$
1	1.071863E-01	1.072384E-01	1.016558E-01	-5.2100E-05	5.53050E-03
2	6.392500E-02	6.399430E-02	5.850630E-02	-6.9300E-05	5.41870E-03
3	3.941910E-02	3.948720E-02	3.453260E-02	-6.8100E-05	4.88650E-03
4	2.483400E-02	2.488760E-02	2.067590E-02	-5.3600E-05	4.15810E-03
5	1.587440E-02	1.590780E-02	1.248330E-02	-3.3400E-05	3.39110E-03
6	1.025270E-02	1.026600E-02	7.574500E-03	-1.3300E-05	2.67820E-03
7	6.672700E-03	6.669700E-03	4.609800E-03	3.0000E-06	2.06290E-03
8	4.368400E-03	4.353900E-03	2.810600E-03	1.4500E-05	1.55780E-03
9	2.873300E-03	2.852000E-03	1.715500E-03	2.1300E-05	1.15780E-03
10	1.897100E-03	1.872800E-03	1.047800E-03	2.4300E-05	8.49300E-04
11	1.256700E-03	1.232100E-03	6.402000E-04	2.4600E-05	6.16500E-04
12	8.347000E-04	8.117000E-04	3.913000E-04	2.3000E-05	4.43400E-04
13	5.558000E-04	5.353000E-04	2.392000E-04	2.0500E-05	3.16600E-04
14	3.708000E-04	3.532000E-04	1.462000E-04	1.7600E-05	2.24600E-04
15	2.479000E-04	2.333000E-04	8.940000E-05	1.4600E-05	1.58500E-04
16	1.660000E-04	1.541000E-04	5.470000E-05	1.1900E-05	1.11300E-04
17	1.114000E-04	1.018000E-04	3.340000E-05	9.6000E-06	7.80000E-05
18	7.480000E-05	6.730000E-05	2.040000E-05	7.5000E-06	5.44000E-05
19	5.030000E-05	4.450000E-05	1.250000E-05	5.8000E-06	3.78000E-05
20	3.390000E-05	2.940000E-05	7.638500E-06	4.5000E-06	2.62615E-05
25	4.774400E-06	3.721000E-06	6.527200E-07	1.0534E-06	4.12168E-06
30	6.864600E-07	4.710700E-07	5.577700E-08	2.1539E-07	6.30683E-07
40	1.475700E-08	7.553100E-09	4.073000E-10	7.2039E-09	1.43497E-08
50	3.265000E-10	1.211000E-10	2.975000E-12	2.0540E-10	3.23525E-10
70	1.669000E-13	3.105000E-14	1.589000E-15	1.3585E-13	1.65311E-13

Table 4. Model 3-overall middle offered load

$K-c$	b	b_h	b_s	$b-b_h$	$b-b_s$
1	1.600056E-01	1.601902E-01	1.464871E-01	-1.8460E-04	1.351850E-02
2	1.114911E-01	1.118835E-01	9.633830E-02	-3.9240E-04	1.515280E-02
3	8.118660E-02	8.172040E-02	6.548660E-02	-5.3380E-04	1.570000E-02
4	6.085770E-02	6.144950E-02	4.544280E-02	-5.9180E-04	1.541490E-02
5	4.653330E-02	4.711120E-02	3.196290E-02	-5.7790E-04	1.457040E-02
6	3.608370E-02	3.659840E-02	2.268780E-02	-5.1470E-04	1.339590E-02
7	2.826830E-02	2.869290E-02	1.620590E-02	-4.2460E-04	1.206240E-02
8	2.231550E-02	2.264120E-02	1.162710E-02	-3.2570E-04	1.068840E-02
9	1.771920E-02	1.794910E-02	8.368100E-03	-2.2990E-04	9.351100E-03
10	1.413350E-02	1.427790E-02	6.035900E-03	-1.4440E-04	8.097600E-03
11	1.131370E-02	1.138610E-02	4.360700E-03	-7.2400E-05	6.953000E-03
12	9.082400E-03	9.097300E-03	3.154000E-03	-1.4900E-05	5.928400E-03
13	7.308000E-03	7.278900E-03	2.283100E-03	2.9100E-05	5.024900E-03
14	5.891300E-03	5.830400E-03	1.653700E-03	6.0900E-05	4.237600E-03
15	4.756600E-03	4.674100E-03	1.198300E-03	8.2500E-05	3.558300E-03
16	3.845300E-03	3.749600E-03	8.686000E-04	9.5700E-05	2.976700E-03
17	3.111900E-03	3.009400E-03	6.297000E-04	1.0250E-04	2.482200E-03
18	2.520600E-03	2.416400E-03	4.566000E-04	1.0420E-04	2.064000E-03
19	2.043200E-03	1.940800E-03	3.311000E-04	1.0240E-04	1.712100E-03
20	1.657200E-03	1.559200E-03	2.402000E-04	9.8000E-05	1.417000E-03
21	1.344900E-03	1.252900E-03	1.742000E-04	9.2000E-05	1.170700E-03
22	1.091900E-03	1.006900E-03	1.264000E-04	8.5000E-05	9.655000E-04
23	8.868000E-04	8.094000E-04	9.170000E-05	7.7400E-05	7.951000E-04
25	5.855000E-04	5.230000E-04	4.820000E-05	6.2500E-05	5.373000E-04
30	2.082000E-04	1.758000E-04	9.688600E-06	3.2400E-05	1.985114E-04
40	2.650000E-05	1.990000E-05	3.910600E-07	6.6000E-06	2.610894E-05
50	3.388100E-06	2.248000E-06	1.578400E-08	1.1401E-06	3.372316E-06
70	5.544300E-08	2.876500E-08	2.572000E-11	2.6678E-08	5.541728E-08
100	1.161000E-10	4.163000E-11	2.197000E-15	7.4470E-11	1.160978E-10

Table 5. Model 4-1 low, 2 middle, and 1 high offered load

$K-c$	b	b_h	b_s	$b-b_h$	$b-b_s$
1	.2300049	.2301012	.20815410000	-.0000963	.02185080000
2	.1804567	.1808503	.15422500000	-.0003936	.02623170000
3	.1476507	.1482820	.11865990000	-.0006313	.02899080000
4	.1240163	.1247968	.09363040000	-.0007805	.03038590000
5	.1059875	.1068341	.07522310000	-.0008466	.03076440000
6	.0916842	.0925339	.06125030000	-.0008497	.03043390000
7	.0800202	.0808313	.05038840000	-.0008111	.02963180000
8	.0703175	.0710659	.04178810000	-.0007484	.02852940000
9	.0621254	.0627999	.03487910000	-.0006745	.02724630000
10	.0551291	.0557269	.02926400000	-.0005978	.02586510000
15	.0317293	.0320132	.01279950000	-.0002839	.01892980000
20	.0191091	.0192252	.00585900000	-.0001161	.01325010000
25	.0117955	.0118325	.00273570000	-.0000370	.00905980000
30	.0073866	.0073886	.00128900000	-.0000020	.00609760000
31	.0067344	.0067324	.00110960000	.0000020	.00562480000
32	.0061418	.0061365	.00095520000	.0000053	.00518660000
33	.0056029	.0055949	.00082250000	.0000080	.00478040000
40	.0029637	.0029480	.00028920000	.0000157	.00267450000
41	.0027078	.0026919	.00024910000	.0000159	.00245870000
42	.0024742	.0024584	.00021460000	.0000158	.00225960000
43	.0022611	.0022454	.00018490000	.0000157	.00207620000
45	.0018888	.0018736	.00013720000	.0000152	.00175160000
50	.0012064	.0011934	.00006510000	.0000130	.00114130000
70	.0002027	.0001983	.00000331420	.0000044	.00019938580
100	.0000141	.0000135	.00000003805	.0000006	.00001406196

Table 6. Model 5-1 low, 1 middle, and 2 high offered load

$K-c$	b	b_h	b_s	$b-b_h$	$b-b_s$
1	.2560758	.2561078	.2313954	-.0000320	.0246804
2	.2069146	.2073603	.1772946	-.0004457	.0296200
3	.1741071	.1748996	.1413091	.0007925	.0327980
4	.1502537	.1512807	.1157002	-.0010270	.0345535
5	.1318745	.1330264	.0966023	-.0011519	.0352722
6	.1171357	.1183263	.0818605	-.0011906	.0352752
7	.1049774	.1061471	.0701765	-.0011697	.0348009
8	.0947382	.0958506	.0607208	-.0011124	.0340174
9	.0859791	.0870147	.0529383	-.0010356	.0330408
10	.0783936	.0793442	.0464438	-.0009506	.0319498
15	.0519060	.0524752	.0257631	-.0005692	.0261429
20	.0362485	.0365838	.0152587	-.0003353	.0209898
25	.0261413	.0263434	.0093592	-.0002021	.0167821
30	.0192557	.0193799	.0058577	-.0001242	.0133980
35	.0143930	.0144699	.0037112	-.0000769	.0106818
40	.0108715	.0109189	.0023691	-.0000474	.0085024
50	.0063339	.0063504	.0009775	-.0000165	.0053564
55	.0048690	.0048778	.0006301	-.0000088	.0042389
60	.0037550	.0037588	.0004066	-.0000038	.0033484
65	.0029031	.0029038	.0002626	-.0000007	.0026405
66	.0027581	.0027584	.0002407	-.0000003	.0025174
67	.0026206	.0026204	.0002205	.0000002	.0024001
70	.0022487	.0022475	.0001697	.0000012	.0020790
75	.0017443	.0017421	.0001097	.0000022	.0016346
80	.0013546	.0013518	.0000709	.0000028	.0012837
81	.0012879	.0012851	.0000650	.0000028	.0012229
82	.0012246	.0012218	.0000596	.0000028	.0011650
83	.0011644	.0011615	.0000546	.0000029	.0011098

84	.0011072	.0011043	.0000501	.0000029	.0010571
85	.0010528	.0010499	.0000459	.0000029	.0010069
88	.0009054	.0009025	.0000353	.0000029	.0008701
89	.0008610	.0008582	.0000324	.0000028	.0008286
90	.0008188	.0008160	.0000297	.0000028	.0007891
100	.0004960	.0004936	.0000124	.0000024	.0004836

Table 7. Model 6-overall high offered load

$K-c$	b	b_h	b_s	$b-b_h$	$b-b_s$
1	.2790447	.2789483	.2580881	.0000964	.0209566
2	.2303774	.2307301	.2050544	-.0003527	.0253230
3	.1980130	.1987717	.1696950	-.0007587	.0283180
4	.1745803	.1756425	.1444082	-.0010622	.0301721
5	.1565887	.1578421	.1254206	-.0012534	.0311681
6	.1421904	.1435393	.1106388	-.0013489	.0315516
7	.1303177	.1316907	.0988054	-.0013730	.0315123
8	.1203073	.1216561	.0891194	-.0013488	.0311879
9	.1117220	.1130167	.0810457	-.0012947	.0306763
10	.1042594	.1054833	.0742135	-.0012239	.0300459
15	.0777907	.0786350	.0514503	-.0008443	.0263404
20	.0615188	.0620950	.0386153	-.0005762	.0229035
25	.0504711	.0508801	.0303978	-.0004090	.0200733
30	.0424796	.0427823	.0247015	-.0003027	.0177781
40	.0317059	.0318887	.0173573	-.0001828	.0143486
50	.0247983	.0249193	.0128655	-.0001210	.0119328
70	.0165090	.0165718	.0077468	-.0000628	.0087622
100	.0100968	.0101265	.0040846	-.0000297	.0060122

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