The structure of equalizers based on quantized sample space with non-linear MMSE

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ABSTRACT

In this paper, we introduce two types of equalizers, called equalizer-a and equalizer-b, applying to wireless communications having unknown channel characteristics. The equalizer-a, which has the single sample detector with equalizer system, is developed while the equalizer-b has the partition detectors with the same system used in equalizer-a. The methodology we adopt for designing the equalizers is that the sample space is partitioned into finite number of regions by using quantiles, which are estimated by robbins-monro stochastic approximation (RMSA) algorithm, and the coefficients of equalizers are calculated based on nonlinear minimum mean square error (MMSE) algorithm.. Through the computer simulation, the equalizers show much better performance in equiprobably partitioned sample subspaces of observations than the single sample detector and the detector, which has the conventional equalizer, in unquantized observation space under various noise environments.

I. 서 론

The use of statistical methods of detection and estimation theory has provided powerful signal processing techniques for communication system design, image processing and many other systems where noise and other forms of interference occur. In general, system engineers have employed a gaussian noise model as statistical model. No less important in the choice of the gaussian model is the ease of analysis and implementation. But, in most practical situations, the channel characteristics are not known beforehand, so the assumption of a gaussian environment will lead to poor detector and equalizer designs[1]-[2]. Due to unknown characteristics of multipath channel in wireless communication, the objective of the receiver is to identify the channel characteristics in order to reduce the intersymbol interference (ISI) and additive noise components. In order to improve bit error rate (BER), two methods (direct and indirect) have been studied in the past. In the direct method, the optimal detector is designed by estimating the noise probability density function (pdf) and conditional probability density function (cpdf). The functional form of the noise distribution is often assumed to be known, with only a finite number of unknown parameters but in many practical digital communication systems, the functional form of the noise distribution is unknown. In the indirect method, the optimal equalizer followed by a detector is designed to minimize the mean square error (MSE). In this paper, we consider two types of equalizer, called equalizer-a and equalizer-b, the design of equalizers having the ability to suppress problems caused by a broad class of noise and ISI. The structure of the equalizer-a is chosen to minimize MSE with respect to the transmitted information while the equalizer-b is chosen to minimize MSE with respect to the estimated mean under equiprobably partitioned subspaces of r-dimensional signal and past noise observations. The equalizers suggested here are based on piecewise approximations to a regression function involving only quantiles and partition moments which are estimated by a RMSA algorithm. For each partition, the coefficients of the equalizers are evaluated by using the conditional MMSE criterion. In our system, we employ a quarternary

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differential phase shift keying (QDPSK) as a modulation and detection scheme and use postdetection diversity system to combat fading caused by multipath waves with different time delays in radio channels. In postdetection diversity, diversity outputs are weighted according to each branch's channel conditions before combining so that the contribution of the weaker signal branch is minimized. We also set up a rician fading channel, which is consisted of three components, for the computer simulation as a channel model.

Ⅱ. Description of the System Model

We use a simple two branch diversity (Fig.1), called phase combining (PC)[3], reception providing a viable solution.

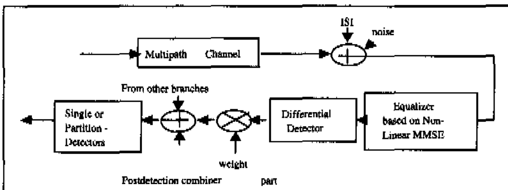

Fig 1. Postdetection combining diversity system

Basically there are two types of diversity combiners, predetection and postdetection combiners. Predetection diversity, which coherently combines the received faded signals before (differential) detection, may be difficult to implement because of the phase fluctuations of faded signals while postdetection diversity combines the (differential) detector outputs which are all in phase; thus co-phasing function is not required. In postdetection diversity, detector outputs are weighted according to each branch's channel condition before combining so that the contribution of the weaker signal branch is minimized. We express three components as y_{dl}, y_{fl}, y_{nl} which also represent the differential phase detection (DPD) detector input components in this paper. $M-ary$ DPSK transmitted signal can be expressed as

$$S(t)=\sum_{n=-\infty}^{\infty} a_n \cdot p(t-nT)\cos(w_c t+\theta_n) \qquad (1)$$

The DPD detector input $y_l(t)$ of the l^{th} branch can be expressed as

$$y_l = y_{dl}(t)+y_{fl}(t)+y_{nl}(t)$$

$$y_{dl}(t)=\sum_{n=-\infty}^{\infty} a_n\ v(t-nT)\cos(w_c t+\theta_n(t)),$$

$$y_{fl}(t)=\frac{1}{\beta}\sum_{n=-\infty}^{\infty} a_n\ c(t)\ v(t-nT)$$

$$\cos[w_c(t-\tau(t))+\theta_n(t-\tau(t))] \qquad (2)$$

where $h(t)$: channel filter, $v(t)=h(t)*p(t)$

Since the channel fading is considered to be sufficiently slow, which means that the time delay can be considered as constant over the time interval in question, we can assume without loss of generality that $\tau(t) \equiv constant\ (=0)$, so fading component can be expressed as following form ;

$$y_{fl}(t)=\frac{1}{\beta}\sum_{n=-\infty}^{\infty} a_n\ c(t)\ v(t-nT)\cos(w_c(t)+\theta_n(t)) \qquad (3)$$

When the effect of multipath delay spread can not be ignored, fading is termed frequency-selective for which complex stationary, zero-mean gaussian random process which has, therefore, a rayleigh amplitude distribution and an uniformly distributed phase. It is assumed that $c(t)$ has the autocorrelation function (Fourier transform : $C_c(F)$ is called delay spectrum) given by $R_c(\tau)=W C_c(\tau)$. $C_c(\tau)$ is the normalized correlation function of $c(t)$, W is the average power in the fading component $y_f(t)$. Various forms for $R_c(\tau)$ have been assumed in the literature[4]. The constant , β, is a normalized constant defined for pulse a given shape. An important parameter in comparing system performance in a rician fading environment is the ratio of direct component power s to the multipath power f. This ratio is often referred to as the *k-factor*[5]. $y(t)$ can be expressed by in-phase and quadrature components representing the input to the detector of the signal.

$$y_l(t) = y_{li}(t)\cos w_c t - y_{lq}(t)\sin w_c t$$
$$= \sqrt{y_{li}^2(t) + y_{lq}^2(t)}\cos[w_c t + \tan^{-1}(\frac{y_{lq}(t)}{y_{li}(t)})]$$
$$= A_l(t)\cos[w_c t + \tan^{-1}(\frac{y_{lq}(t)}{y_{li}(t)})] \quad (4)$$

In DPD, the phase of the detector input $y_l(t)$ relative to the local oscillator output with constant frequency is detected and sampled at time $t=nT$ which is ideally locked to the specular component; $t=nT$, this assumption holds for large k-factor values. The difference $\triangle\Psi_l$ of the received signal over an symbol duration can be represented as

$$\triangle\Psi_l = \triangle\varphi_s + \triangle\varphi_{fl} + \triangle\varphi_{nl}, \ \text{mod } 2\pi \quad (5)$$

where $\triangle\varphi_s$ is differential phase of transmitted signal, $\triangle\varphi_{nl}$, differential phase noise and $\triangle\varphi_{fl}$, random differential phase of fading. The resultant combiner output of the dual diversity system can be expressed by

$$\triangle\Psi = \xi_1\triangle\Psi_1 + \xi_2\triangle\Psi_2, \ \text{mod } 2\pi, \ \xi_1+\xi_2=1$$
$$\xi_l = \frac{A_l^m(nT)}{A_1^m(nT)+A_2^m(nT)}, \ l=1,2,$$
$$m^{th}\text{: power factor} \quad (6)$$

The weight ($\xi_l,\ l=1,2$) must be chosen to minimize the contribution of the weaker signal because the phase of noise becomes large when the received signal fades. The final $\triangle\Psi$ will be consisted of two components which are transmitted phase difference and weighted phase noise.

Ⅲ. The Structure of Equalizers

1. N-Equalizer

Consider MMSE estimation of the transmitted signal , a_n , from *(r+1)* present and past received signals. The optimum estimate is the well-known regression function;

$$E[a_n|y_n,\ y_{n-1},\ldots,y_{n-r}] \quad (7)$$

Since this function is nonlinear, and suboptimal linear estimator is often used, i.e.,

$$a' = \sum_{k=0}^{r} c_k\, y_{n-k} \quad (8)$$

Minimizing MSE, $E[|a_n - a'|^2]$, generates the coefficients of (8), $c=\Lambda^{-1}\cdot Y$ *where* $\Lambda=E[y_k y_l]$, $Y=E[a_k y_l]$, $c'=[c_0, c_1, \ldots, c_r]$. Now, we present a method designing the n-equalizer which has similar complexity and produce better performance as compared to suboptimal linear estimator. It is defined for *SNR*(dB) >0, so we can observe pure noise samples during the transmitting mode. Assuming the detected symbols are correct, noise samples can be obtained from $n_{n-k} = y_{n-k} - a^*_{n-k}$ where $k=1,2,3,\ldots,r$ and a^*_{n-k} detected symbols. So the MSE can be expressed by

$$MSE = E[[a_n - (\beta_{cf} + \gamma_{cf}y_n + \sum_{k=1}^{r} a_{cf,j}n_{n-k})]^2] \quad (9)$$

Using the orthogonality principle, we can get the following of the estimator;

$$E[a_n|y_n, n_{n-1},\ldots n_{n-r}] = \beta_{cf} + \gamma_{cf}y_n + \sum_{k=1}^{r} a_{cf,k}n_{n-k} \quad (10)$$

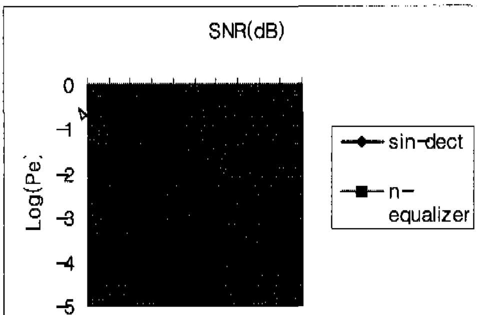

Fig.2 Pe of n-equalizer, k=22, m=2 for dep gaussian noise

Fig.2 shows that the performance of the n-equalizer is significantly better as compared to that of the single sample detector. In Fig.3, we plotted the performance of the n-equalizer according to changing values of m weight factor to find the optimum value of m. We can conclude that $m=2$ is the optimum value of the weight factor through the computer simulation.

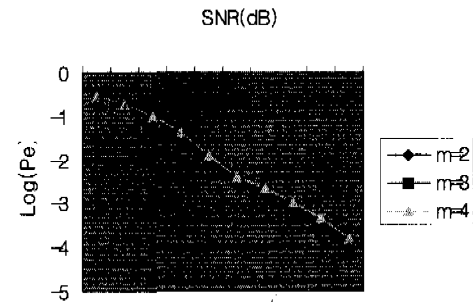

Fig.3 Pe of n-equalizer according to m's value, k=20 for dep gaussian noise

2. Equalizer-a, N-Equalizer based on Quantized Sample Space

Using the training mode, we can estimate quantiles which partition the observation space into m independent regions. The proposed recursive estimation is for , $n=0,1,2,..$

$$e_{n+1}=e_n-q_n[u(e_n-y_{n+1})-p] \tag{11}$$

where $u(y)$ is Heaviside step function and q_n is the sequence of positive numbers[6]-[7]. For each partitioned sets, the partition moments, which are required to calculate the coefficients of non-linear MMSE equalizer, are estimated by RMSA algorithm[6]. There is a recursive estimation scheme for estimating the partition moments defined by

$$m_i\ (a,b]=E\ [\chi_{(a,b]}(y)y^i],\ i=1,2,\ldots \tag{12}$$

where $\chi_{(a,b]}$ is the indicator function of the interval $(a,b]$, and $\chi_{(a,b]}y^i$ will also be denoted by H. From the available sequence $[y_n]$, $n=0,1,2,\ldots,$ the proposed partition moments are

$$\mu_{n+1}=\mu_n+q_n[H_{n+1}-\mu_n] \tag{13}$$

where $H_{n+1}=\chi_{(a,b]}(y)y^i$ and q_n satisfies the previous conditions described. Using Taylor series expansion, we approximate the function $E[a_n\mid y_n,y_{n-1},\ldots y_{n-r}]$ in the neighborhood of a point $(y_n^*,y_{n-1}^*,\ldots,y_{n-m}^*)$ as

$$E[a_n|y_n,y_{n-1},\ldots,y_{n-r}]=E[a_n|y_n^*,y_{n-1}^*,\ldots,y_{n-r}^*]+\nabla E[a_n|y_n,y_{n-1},\ldots,y_{n-r}]|_{(y_n^*,\ldots,y_{n-r}^*)}\cdot[y_n-y_n^*,y_{n-1}-y_{n-1}^*,\ldots,y_{n-r}-y_{n-r}^*] \tag{14}$$

Based on estimated quantiles and vector quantization(VQ), the conditioning vector $(y_n,y_{n-1},\ldots,y_{n-m})$ is represented by a finite set with elements called pseudo states of the conditional vector. We can express the MMSE estimation of a_n (9) as ;

$$MSE=E[\,[a_n-(\beta_{qk}+\gamma_{qk}y_n+\sum_{j=1}^{r}a_{qk,j}n_{n-j})]^2\mid S_k] \tag{15}$$

Using the orthogonality principle(see **appendix**) and the RMSA algorithm for calculating partition moments, we can get the coefficient $(a_{qk},\ \beta_{qk},\ \gamma_{qk})$ of equalizer for each partitioned sets, and the non-linear estimation based on quantized sample space can be represented by the following form;

$$\begin{aligned} r\ E[a_n|y_n,n_{n-1},n_{n-2},\ldots,n_{n-r}] &= \sum_{k=1}^{L}\chi_{Sk}[y_n,n_{n-1},n_{n-2},\ldots,n_{n-r}] \\ &\quad [\beta_{qk}+\gamma_{qk}y_n+\sum_{x=1}^{r}a_{qk,j}\ n_{n-x}] \end{aligned} \tag{16}$$

where k : number of partition set.

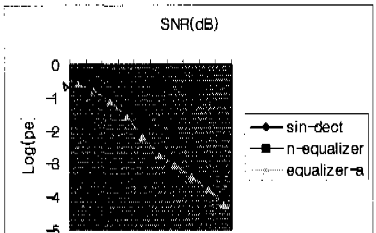

Fig4. Pe of equalizer-a, k=22,m=2 for dep gaussian

Fig.4 shows that the system(equalizer-a) in quantized subspaces of observations yields better performance when comparing to system(called n-equalizer) in unquantized observation space. For computer simulation, we use 1,125, total number of partitions, and 1,000,000 training samples.

3. Equalizer-b, N-Equalizer with Partition Detectors

We can observe the pure noise samples n_{n-k}, $k=1,2,3,\ldots,r$ assuming that the detected symbols are correct. An r-dimensional noise sample space is equiprobably partitioned into a finite number of regions. Having the past noise samples partitioned for each hypothesis, the received signal is subsequently partitioned similarly as was done in previous section. Using the RMSA procedure, we can get the estimated value of mean $\eta_{i,k}$ under the each hypothesis and partitioned cells of past noise samples

$$\eta_{i,k} = E[y_n | VQ(w_{n-1}, w_{n-2}, \ldots, w_{n-r}) \in \rho, H_i] \quad (17)$$

where $i=1,2$ (for hypothesis) and k : level of partition. The estimated threshold under each partition ρ and each hypothesis can be expressed as

$$\lambda_{j,k} = \frac{\eta_{1,k} + \eta_{0,k}}{2} \quad (18)$$

where $j=1,2$ represent the number of channel path. Equalizer-b use the non-linear MMSE equalizer with respect to the estimated mean not the transmitted information, and the outputs of equalizer-b pass through partition detectors having average value of the thresholds. The conditional MSE can be expressed by

$$MSE = E[[\eta_{a,k} - (\beta_{qk} + \gamma_{qk} y_n + \sum_{j=1}^{r} a_{qk,j} n_{n-j})]^2 | S_k], \quad a=0,1 \quad (19)$$

Through the same procedure done in previous section, we will generate the coefficients a_{qk}, β_{qk}, γ_{qk}. The output of the equalizer-b is

$$\eta_k^* = \sum_{k=1}^{L} \chi_{S_k}[y_n, n_{n-1}, \ldots, n_{n-r}][\beta_{qk} + \gamma_{qk} + \sum_{j=1}^{r} a_{qk,j} w_{n-j}] \quad (20)$$

Fig5. shows that the equalizer-b generates significantly improved result as compared to n-equalizer. For this case we use 625 as total partition number.

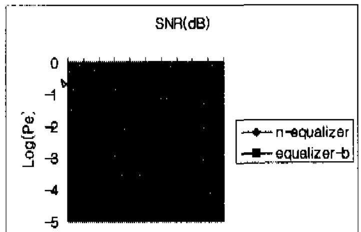

Fig5. Pe of equalizer-b, k=20, m=2 for dep gaussian noise

IV. Discussion of Results

We use the dependent noise model generated by the auto-regressive (*AR*) model as a noise component in a rician fading channel, i.e.,

$$n(n) = \rho_1 n_{n-1} + \rho_2 n_{n-2} + \ldots + \rho_m n_{n-k} + v_n \quad (21)$$

and take a component having a rayleigh amplitude distribution as a fading component for computer simulation. Through the computer simulation, we demonstrate that the performance of these two systems are much better than that of other systems(i.e., n-equalizer and single sample detector) in dependent gaussian noise case in terms of $Log_{10}(Pe)$ as shown in Fig.3-Fig.5. The amount of improvement, which is defined by

$$\text{improvement of } P_e(dB) = 10\log_{10}\frac{p_e(\text{single sample detector})}{p_e(\text{equalizer}-a \text{ or } \text{equalizer}-b)} \quad (22)$$

, compared to a single sample detector is from 0.8 to 9.7 dB for equalizer-a and from 0.4 to 10.4 for equalizer-b under dependent gaussian noise environment. From the definition of *k*, we can also expect that as it increases the system performance will improve until a saturation is reached. It is clear that below a certain level of the average power in the fading component or above certain level of *k*, the performance remains constant because the power of the direct component is constant (see Fig.6). We use 1,125, total number of partitions, for equalizer-a and 625 for equalizer-b and take 1,000,000 training samples.

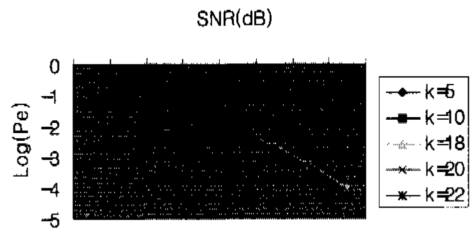

Fig6. Pe of equalizer-a according to k-factor, m=2 for dep gaussian noise

Ⅵ. Concluding Remarks

The main purpose of this study is to propose new equalizer systems which have the ability to apply any kind of unknown functional form of noise environments because the channel characteristic are not known beforehand in most cases. In this paper, we examine two types of equalizers(called equalizer-a & equalizer-b) and show how an adaptive system can achieve near optimal performance by estimating certain key statistical quantities from the available data. When comparing two systems, there are two different aspects in approach; a conditional MMSE is used with respect to the estimated mean under equiprobably partitioned subspaces of *(r+1)*-dimensional signal & past noise observations and the outputs of the system pass through partition detectors in case of equalizer-b while the equalizer-a uses a conditional MMSE with respect to transmitted information and the outputs of that system pass through a single sample detector. Using computer simulations, we show that the BER of equalizer-a & equalizer-b is improved significantly when comparing to the single sample detector and to the n-equalizer and we also demonstrate the changing of the value of *BER* according to the *k*-factor value. We can also expect that as total partition number increases the system performance will improve until a saturation is reached.

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Appendix

- $\frac{de}{d\beta_{qk}}$:

$$E\,[(a_n-(\beta_{qk}+\gamma_{qk}y_n+\sum_{j=1}^{m}a_{qk,j}\,n_{n-j}))\,(-1)|S_k]=0$$

$$E\,[a_n|S_k]=\beta_{qk}+\gamma_k E[y_n|S_k]+\sum_{j=1}^{m}a_{qk,j}\,E[n_{n-j}|S_k]$$

- $\frac{de}{d\gamma_{qk}}$:

$$E\left[\left(a_n-\left(\beta_{qk}+\gamma_{qk}y_n+\sum_{j=1}^{m}\alpha_{qk,j}\, n_{n-j}\right)\right)(-y_n)|S_k\right]=0$$

$$E\left[a_n y_n|S_k\right]=$$

$$\beta_{qk}E[y_n|S_k]+\gamma_{qk}E[y_n^2|S_k]+\sum_{i=1}^{m}\alpha_{qk,i}\,E[y_n n_{n-i}|S_k]$$

- $\frac{de}{d\alpha_{qk}}$:

$$E\left[\left(a_n-\left(\beta_{qk}+\gamma_{qk}y_n+\sum_{j=1}^{m}\alpha_{qk,j}\, n_{n-j}\right)\right)(-n_{n-j})|S_k\right]=0$$

$$E\left[a_n n_{n-j}|S_k\right]=\beta_{qk}\sum_{j=1}^{m}E[n_{n-j}|S_k]$$

$$+\gamma_{qk}\sum_{j=1}^{m}E[y_n n_{n-j}|S_k]+\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_{qk,i}\,E[n_{n-i}n_{n-j}|S_k]$$

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