

## 레일레이 페이딩 채널에서 수신신호의 신호대잡음비에 근거한 적응부호화 시스템의 성능 분석

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## Performance Analysis of the Adaptive Coding System Based on Received SNR over a Rayleigh Fading Channel

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요 약

본 논문에서는 수신신호의 신호대잡음비를 이용하여 작동되는 적용부호화 시스템을 레일레이 페이딩 채널 환경에서 분석한다. 분석에서는 레일레이 페이딩 채널을 수신신호의 신호대잡음비를 K개의 구간으로 나누어서 유한 상태 마르코프 채널로 모델링한다. 채널 상태를 예측하는 과정에서 발생하는 오류확률을 고려하여 적용부호화 시스템의 평균 BER과 throughput이 계산된다. 본 논문에서 제안한 분석 방법을 이용하여 천공 길쌈부호를 이용하는 적용부호화 시스템의 성능을 분석한 결과가 예제로 보여진다.

#### **ABSTRACT**

In this paper, an adaptive coding system is analyzed over a Rayleigh fading channel for digital communication without retransmission, which operates based on a received signal to noise ratio (SNR). For the analysis, a finite-state Markov channel model is used for a Rayleigh fading channel, which is constructed by dividing the value of a received SNR into K intervals. Taking the types of errors at the prediction process into account, the average bit error probability and the average throughput are computed for the adaptive coding system. Numerical results are shown, which uses the punctured convolutional codes as a coding scheme whose code rates are simple to be changed. It is shown that the adaptive coding system based on a received SNR improves the performance of the system.

## I. INTRODUCTION

In mobile communication environments, there are many electrical wave reflectors between transmitter and receiver, such as plain, terrain, buildings and vehicles. When a signal is transmitted in this wireless channel, many paths of electrical wave are formed between transmitter and receiver. Although a transmitted signal has a constant envelope, the

envelope of a received signal is not constant, which is called as multipath fading<sup>[1-2]</sup>. This phenomenon degrades reliability of communication in wireless channel.

To overcome signal fading, many methods were proposed. One of them is to use error control codes<sup>[3-9]</sup>. The design of an error correction coding system usually consists of selecting a code with the fixed code rate and the error correction capability is matched to the worst channel conditions to be

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expected. The code with fixed code rate provides bit error probability which is always below a specified value. However, the data throughput is lower than that achievable if we select optimum code in each state. So, a more efficient method is to use an adaptive error control coding scheme that operates according to the actual channel state by selecting the optimum code rate.

In the adaptive coding system, followings are necessary: criterion for demarcating channel state, channel state estimator, channel state predictor, return channel, variable rate encoder and decoder. Some of the adaptive coding schemes are introduced<sup>[5,8-15]</sup>. In [8], the considered system is hybrid ARO, and the criterion for type- I demarcating channel state is the number of erroneous blocks at the decoder. In [9], the considered system is FEC system, and the criterion for demarcating channel state is a path gain, which has Rayleigh distribution. In this paper, the analyzed system is FEC system, and the criterion for demarcating channel state is a received SNR. It is considered over a Rayleigh fading channel for digital communication.

In the analysis, a Rayleigh fading channel is modeled into a finite state Markov channel model. The value of a received SNR is divided into a finite number of intervals, and each interval represents a state of Markov channel model. In each state of Markov channel model, a new probability density function(PDF) is introduced and is used for calculating the crossover probability, mean and variance of a received SNR of that state. Taking types of errors at the prediction process into account, their probabilities are calculated, and then the bit error probability and throughput of each state are calculated. Then the average bit error probability and the average throughput of the adaptive coding system are calculated. Using these, numerical results are shown, which uses the punctured convolutional codes as a variable rate coding scheme whose code rates are simple to be changed without changing the basic codec structure, and zeroth order prediction. The results show that this adaptive coding system based on a received SNR using channel prediction has advantages relative to nonadaptive coding system. Furthermore, results show the method to find optimum threshold to satisfy the required BER and to achieve as high throughput as possible. Most of all, the focus of this paper is to provide a method to analyze an adaptive coding system which operates according to a received SNR.

This paper is organized as follows. Section II describes the channel model for a Rayleigh fading channel, and Markov chain is considered. In section III, system configuration and code description are given. The punctured convolutional code is described. In section IV, the system analysis is described, and in section V, numerical results are shown for the adaptive coding system using punctured convolutional codes. Finally, section VI concludes this paper.

## II. CHANNEL MODEL

A channel model is generally used to describe the error statistics of a channel. The simplest channel model is binary symmetric channel (BSC) model. It describes stationary, memoryless channels which have binary input and output, and it is completely determined by crossover probability<sup>[4,8]</sup>.

Rayleigh fading channel has time varying characteristics, and so BSC model is not used. But over a short time interval, the channel parameters are reported to remain constant [8,16]. Like this, the channel, which has constant channel parameters over a short time interval although channel is time varying over long time interval, is called as quasistationary channel. This characteristic enable time varying channel to be represented as finite-state channel model, each state of which is stationary [16]. Furthermore, if perfect interleaving is assumed and it is also assumed that each received symbols are independent, as the characteristic 'memoryless' is satisfied, BSC model can be used in each state of time varying channel [8, 16].

In this paper, the criterion for demarcating channel state is a received SNR which is expressed as (1), where  $E_c$  is a transmitted code symbol energy, and  $N_0/2$  is two sided power spectral

density of an additive white Gaussian noise, and  $\alpha$  is a multiplicative distortion which is Rayleigh distributed.

$$\gamma = \frac{E_c}{N} \alpha^2 \tag{1}$$

The probability density functions of  $\alpha$  and  $\gamma$  are (2) and (3) respectively. In (3),  $\rho = E[\gamma] = \frac{E_c}{N_c} E[\alpha^2] = \frac{E_c}{N_c} (2\sigma^2)$ .

$$p_{a}(a) = \begin{cases} \frac{\alpha}{\sigma^{2}} \exp\left(-\frac{\alpha^{2}}{2\sigma^{2}}\right), & \alpha \geq 0 \\ 0, & \alpha \leq 0 \end{cases}$$
 (2)

$$p_{\gamma}(\gamma) = \begin{cases} \frac{1}{\rho} \exp\left(-\frac{\gamma}{\rho}\right), & \gamma \ge 0 \\ 0, & \gamma < 0 \end{cases}$$
 (3)

To make a finite-state Markov channel model, the value of a received SNR is divided into K intervals such as  $[a_0, a_1)$ ,  $[a_1, a_2)$ , ...,  $[a_{K-1}, a_K)$ , where  $a_0 = 0$  and  $a_K = \infty$ . Here, each interval represents a state of the model. Let a state  $s_k$  denote an interval  $[a_k, a_{k+1})$ . To find statistics in some probability space, the probability density function (PDF) is needed. Suppose PDF of a received SNR in state  $s_k$  as

$$p_{\gamma,k}(\gamma) = \begin{cases} \frac{p_{\gamma}(\gamma)}{p_k}, & \gamma \in [a_k, a_{k+1}) \\ 0, & \text{elsewhere} \end{cases}$$
 (4)

where  $p_k$  is the probability of state  $s_k$  which is calculated as (5).

$$p_k = \int_{a_k}^{a_{k+1}} p_{\gamma}(\gamma) \, d\gamma \tag{5}$$

Let a new notation  $E[f(\gamma_k)]$  be the expectation of a function  $f(\gamma)$  in state  $s_k$  and  $E[g(\alpha_k)]$  be the expectation of a function  $g(\alpha)$  in state  $s_k$ . Then they are given by

$$E[f(\gamma_k)] = \int_{a_k}^{a_{k+1}} f(\gamma) \, p_{\gamma,k}(\gamma) \, d\gamma \tag{6a}$$

$$E[g(a_k)] = \int_{b_k}^{b_{k+1}} f(a) p_{a,k}(a) da$$
 (6b)

where  $b_k = \sqrt{\frac{N_a}{E_c}} a_k$  is the threshold value converted to ' $\alpha$ ' domain and  $p_{a,k}(a)$  is defined similarly to (4) replacing  $p_{\gamma}(\gamma)$  with  $p_{\alpha}(\alpha)$ . Then mean and variance of a received SNR in state  $s_k$  are computed as (7a) and (7b).

$$m_{r,k} = E[\gamma_k], \tag{7a}$$

$$\sigma_{r,k}^2 = E[(\gamma_k - m_{r,k})^2] = E[\gamma_k^2] - E^2[\gamma_k]$$
 (7b)

Now the important element in BSC model, crossover probability, can be calculated in state  $s_k$  using material given above. For example, when BPSK is used as a modulation scheme, using (6a) the crossover probability in state  $s_k$  is given by

$$e_{k} = E[Q(\sqrt{2\gamma_{k}})]$$

$$= \int_{a_{k}}^{a_{k+1}} Q(\sqrt{2\gamma}) p_{\gamma,k}(\gamma) d\gamma.$$
(8)

Then, when no error correction coding scheme is applied, the average bit error probability of BPSK is given by

$$e = \sum_{k=0}^{K-1} p_k e_k$$

$$= \sum_{k=0}^{K-1} p_k E[Q(\sqrt{2\gamma_k})]$$

$$= \int_0^\infty Q(\sqrt{2\gamma}) p_{\gamma}(\gamma) d\gamma.$$
(9)

To explain the time varying characteristics of Rayleigh fading channel using finite K states which are stationary, Markov process is used in which a first-order Markov chain is used for state transition. Let  $\gamma_n$  denote a received SNR at time n and  $S_n$  denote a channel state at time n. Then "" $S_n$ "" denotes Markov process representing Rayleigh fading channel, and if  $\gamma_n$  is included in an interval  $[a_k, a_{k+1})$ ,  $S_n$  has state value  $s_k$ .

In the Markov process, two elements are needed: state transition probability and initial state probability. First, the state transition probability, denoted as  $t_{i,k}$ , is the probability of state transition from  $s_i$  to  $s_k$ , which is written as (10).

$$t_{j,k} = \Pr^{n} S_{n+1} = s_k | S_n = s_j^n$$
 (10)

Under the assumption of slow variation of channel, transition is possible only to one of the adjacent channel states. All the state transition probabilities are as follows<sup>[16]</sup>.

$$t_{i,k} = 0, \quad \forall |j-k| > 1, \tag{11a}$$

$$t_{k,k+1} = \frac{N_{k+1}}{R_i \times p_k}$$
,  $k=0,1,\dots,K-2$ , (11b)

$$t_{k,k-1} = \frac{N_k}{R \times b_k}, k=1, 2, \dots, K-1,$$
 (11c)

$$t_{k,k} = 1 - t_{k,k-1} - t_{k,k+1}, k=1, 2, \dots, K-2, (11d)$$

$$t_{0,0} = 1 - t_{0,1}, (11e)$$

$$t_{K-1,K-1} = 1 - t_{K-1,K-2} \tag{11f}$$

In (11b) and (11c),  $R_t$  is the transmission rate and  $N_t$  is level crossing rate which is given by

$$N_k = \sqrt{\frac{2\pi a_k}{\rho}} f_m \exp\left(-\frac{a_k}{\rho}\right) \tag{12}$$

where  $f_m$  is the maximum Doppler frequency shift. Second,  $p_k$  in (5) is used for the initial state probability. To verify that it is reasonable to use  $t_{j,k}$  in (11) and  $p_k$  in (5) as the state transition probability and the initial state probability respectively, two equations should be examined. Let  $V_s$  be the initial probability vector and  $M_T$  be the transition probability matrix as follows.

$$V_s = [p_0, p_1, \cdots, p_{K-1}]$$
 (13a)

$$M_T = \begin{bmatrix} t_{0,0} & \cdots & t_{0,K-1} \\ \vdots & \ddots & \vdots \\ t_{K-1,0} & \cdots & t_{K-1,K-1} \end{bmatrix}$$
 (13b)

Then two equations such as (14a) and (14b) are satisfied.

$$V_s \cdot M_T = V_s \tag{14a}$$

$$V_{\bullet} \cdot [1, 1, \dots, 1]^T = 1$$
 (14b)

So  $t_{j,k}$  and  $p_k$  can be used for Markov process as the state transition probability and the initial state probability respectively [8,16].

# III. SYSTEM AND CODE DESCRIPTION

The model of an adaptive coding system is shown in Fig.1. Data from a data source is encoded, interleaved and modulated at the transmitter. Coherent BPSK is used as a modulation scheme. At the receiver, the channel state estimator (CSE) determines a current channel state by estimating a received SNR. The channel state predictor decides a next channel state by predicting a received SNR of the next packet and the code rate for next packet is determined based on this, which is sent to the encoder at the transmitter via a return channel. It is assumed that there is no delay in the return channel.

For the simplicity of the encoder and decoder, a punctured convolutional code is used as a coding scheme because it does not require to change their basic structure during code rate transition. When a 1/n convolutional code is punctured with period P, a code rate of punctured code is given by [10,11]

$$r = \frac{P}{P+I} \tag{15}$$

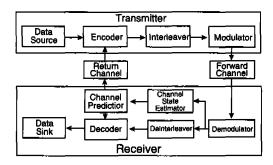


Fig. 1 Adaptive coding system.

where l is the number of redundant bits during puncturing period, which is in the range of [1, (n-1)P].

Fig.2 shows an encoder of a punctured convolutional code of code rate 2/3, n=3, P=4, l=2, whose parent code is a (3,1,6) convolutional code. Its puncturing pattern in a matrix form is given by

$$Q = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

When a code with fixed code rate is used, the first error event probability  $P_{E,k}$  in state  $s_k$  is given by [3-4,8]

$$P_{E,k} \le \sum_{d=d_{k-1}}^{\infty} a_d P_{d,k} \tag{17a}$$

where  $d_{free}$  is the free distance of a punctured convolutional code,  $a_d$  is the number of paths at distance d from a correct path, and  $P_{d,k}$  is the probability that a wrong path at distance d from a correct path is selected in state  $s_k$ , which is called as pairwise probability. For the case of fixed code rate, the bit error probability  $P_{b,k}$  in state  $s_k$  is given by

$$P_{b,k} \le \frac{1}{i} \sum_{d=d_{co}}^{\infty} b_d P_{d,k} \tag{17b}$$

where  $b_d$  is the total number of nonzero information bits on all paths at distance d from a correct path and i is the number of information bits in a code symbol.

The value of pairwise probability depends on channel type, modulation scheme and decision scheme. When a hard decision is applied at the decoder, pairwise probability in state  $s_k$  is given by [3-4,8]

$$P_{d,k} = \begin{cases} \sum_{r=\frac{d}{2}1}^{d} {d \choose r} e_k^r (1-e_k)^{d-r}, & d \text{ odd} \\ \frac{1}{2} {d \choose d/2} p^{d/2} (1-p)^{d/2} \\ + \sum_{r=\frac{d}{2}+1}^{d} {d \choose r} e_k^r (1-e_k)^{d-r}, & d \text{ even} \end{cases}$$
(18a)

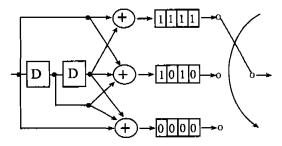


Fig. 2 Encoder of a punctured convolutional code.

where  $e_k$  is crossover probability at state  $s_k$ . When a soft decision is applied at the decoder, pairwise probability in state  $s_k$  is given by [8]

$$P_{d,k} \le \frac{1}{2} \left( 1 + 2 \frac{E_c}{N_o} \sigma_{d,k}^2 \right)^{-\frac{1}{2}} \exp \left( - \frac{d \frac{E_c}{N_c} m_{a,k}^2}{1 + 2 \frac{E_c}{N_c} \sigma_{d,k}^2} \right)$$
 (18b)

where

$$m_{a,k} = E[a_k]$$

and

$$\sigma_{a,k}^2 = E[(\alpha_k - m_{a,k})^2].$$

By transforming a random variable  $\alpha$  to  $\gamma$  using (1), (18b) becomes as following.

$$P_{d,k} \leq \frac{1}{2} \left( 1 + 2(E[\gamma_k] - E^2[\sqrt{\gamma_k}]) \right)^{-\frac{1}{2}} \exp\left( -\frac{dE^2[\sqrt{\gamma_k}]}{1 + 2(E[\gamma_k] - E^2[\sqrt{\gamma_k}])} \right)$$
(19)

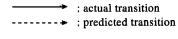
## IV. SYSTEM ANALYSIS

As stated at section III, a current channel state is determined by a received SNR of the current packet and next channel state is decided by predicting received SNR based on current value. According to the predicted state, the code rate for transmission of next data packet is determined.

Let  $\hat{S}_n$  be a predicted state for time n and  $\hat{\gamma}_n$ , be the predicted received SNR for time n. As it is assumed that channel variation is slow, when  $S_n$ , actual channel state at time n, is  $s_k$ ,  $\hat{\gamma}_{n+1}$  is in the range of  $[a_{k-1}, a_{k+2})$ . When  $\hat{\gamma}_{n+1}$  is in the range  $[a_{k-1}, a_k)$ ,  $\hat{S}_{n+1}$  is decided as  $s_{k-1}$ . When  $\hat{\gamma}_{n+1}$  is in the range  $[a_{k+1}, a_{k+2})$ , then  $\hat{S}_{n+1}$  is decided as  $s_{k+1}$ . When  $\hat{\gamma}_{n+1}$  is in the other range,  $\hat{S}_{n+1}$  is decided as  $s_k$ .

In the adaptive coding system based on a received SNR, there are four types of prediction errors: types I, II, III and IV. Assume that the current channel state is  $s_k$ . In the prediction error of type I, the receiver predicts the next state as  $s_{k+1}$  while the actual next state is  $s_k$ . In the prediction error of

type  $\[ \] \]$  , the receiver predicts the next state as  $s_{k-1}$ while the actual next state is  $s_k$ . In the prediction error of type III, the receiver predicts the next state as  $s_k$  while the actual next state is  $s_{k+1}$ . In the prediction error of type IV, the receiver predicts the next state as  $s_k$  while the actual next state is  $s_{k-1}$ . In state  $s_0$ , only the prediction errors of type I and type  $\coprod$  are possible. In state  $s_{K-1}$ , only the prediction errors of type II and type IV are possible. In the other states, all the types of prediction errors are possible. These four types of prediction errors are given by (20a), (20b), (20c) and (20d), and are also appeared in Fig.3. In these equations, the notation  $T_{b,c}^{(a)}$  is used where T denotes the error type and a, b, c denotes the current state, the actual next state, the predicted next state respectively.



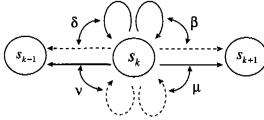


Fig. 3 Types of prediction errors.

type I: 
$$\beta_{k,k+1}^{(k)}$$
  
=  $\Pr^{n}$   $\hat{S}_{n+1} = s_{k+1}$ ,  $S_{n+1} = s_k \mid S_n = s_k$ "  
=  $\Pr^{n}$   $a_{k+1} \le \hat{\gamma}_{n+1} < a_{k+2}$ "  $t_{k,k}$ ,  
 $k = 0, 1, \dots, K-2$ , (20a)

type II: 
$$\delta_{k,k-1}^{(k)}$$
  
=  $\Pr^{n} \hat{S}_{n+1} = s_{k-1}$ ,  $S_{n+1} = s_k \mid S_n = s_k^{n}$   
=  $\Pr^{n} a_{k-1} \leq \hat{\gamma}_{n+1} \langle a_k^{n} + t_{k,k} \rangle$ ,  
 $k=1, 2, \dots, K-1$ . (20b)

type IV: 
$$\nu_{k-1,k}^{(k)}$$
  
=  $\Pr$  "  $\hat{S}_{n+1} = s_k$ ,  $S_{n+1} = s_{k-1} \mid S_n = s_k$ "  
=  $\Pr$  "  $a_k \le \hat{\gamma}_{n+1} < a_{k+1}$ " "  $t_{k,k-1}$ , (20d)

In the adaptive coding system, the prediction errors in a matrix form are given by

$$A = \begin{bmatrix} A_{0,0} & \cdots & A_{0,(K-1)} \\ \vdots & \ddots & \vdots \\ A_{(K-1),0} & \cdots & A_{(K-1),(K-1)} \end{bmatrix}$$
 (21)

where  $A_{j,k}$  is probability that the adaptive coding system presumes that channel is in state  $s_j$  while channel is actually in state  $s_k$ . Since it is assumed that a transition in each state is possible only to one of adjacent states, the value of  $A_{j,k}$  is given by

$$A_{j,k} = 0, \quad \forall |j-k| > 1, \tag{22a}$$

$$A_{(k+1),k} = \beta_{k,k+1}^{(k)} + \nu_{k,k+1}^{(k+1)}$$
 (22b)

$$A_{(k-1),k} = \delta_{k,k-1}^{(k)} + \mu_{k,k-1}^{(k-1)}$$
 (22c)

and

$$A_{k,k} = 1 - [A_{(k+1),k} + A_{(k-1),k}]$$
 (22d)

It is  $\pi_{jk}$ , bit error probability of the adaptive coding system that is calculated first, which is the probability when state  $s_j$  is predicted as the next channel state while the actual next state is in state  $s_k$ . When  $\pi_{jk}$  is computed, channel parameters related to  $s_k$  are used and code parameters related to state  $s_j$  are used. The matrix H whose elements are  $\pi_{jk}$  is given by

$$\boldsymbol{\Pi} = \begin{bmatrix} \pi_{0,0} & \cdots & \pi_{0,(K-1)} \\ \vdots & \ddots & \vdots \\ \pi_{(K-1),0} & \cdots & \pi_{(K-1),(K-1)} \end{bmatrix}. \tag{23}$$

Similarly,  $\zeta_{j,k}$  is calculated, which is throughput of the adaptive coding system when state  $s_j$  is predicted as the next channel state while the actual

next state is in state  $s_k$ . The matrix H whose elements are  $\zeta_{j,k}$  is given by

$$H = \begin{bmatrix} \zeta_{0,0} & \cdots & \zeta_{0,(K-1)} \\ \vdots & \ddots & \vdots \\ \zeta_{(K-1),0} & \cdots & \zeta_{(K-1),(K-1)} \end{bmatrix}. \tag{24}$$

Using (21), (22) and (24), the bit error probability of the adaptive coding system in state  $s_k$  is given by

$$p_{b,k} = \sum_{i=0}^{K-1} \pi_{i,k} A_{i,k}$$
 (25a)

and the throughput of the adaptive coding system in state  $s_k$  is given by

$$\eta_k = \sum_{i=0}^{K-1} \zeta_{i,k} A_{i,k}. \tag{25b}$$

Using (25a), the average bit error probability of the adaptive coding system is given by

$$P_{b,av} = \sum_{k=1}^{K-1} p_k \, p_{b,k} \tag{26a}$$

and using (25b), the average throughput of the adaptive coding system is given by

$$\eta_{av} = \sum_{k=0}^{K-1} p_k \, \eta_k \tag{26b}$$

## V. NUMERICAL RESULT

In this section, first, numerical results of performance are shown for the adaptive coding system that operates according to a received SNR, the measures of which are the average bit error probability and the average throughput of the adaptive coding system. They are computed using the method explained in section IV. Second, it is examined what effect the threshold used to demarcate channel state has on the performance of the adaptive coding system. Finally, numerical result is shown about the relation between the performance and the normalized Doppler frequency which imply the relative speed of channel variation.

The adaptive coding system considered in this paper uses punctured convolutional codes as a variable rate coding scheme. The parent code of them is (3,1,6) convolutional code. It is known that the number of states for experimental channels is varied from 2 to 4 <sup>[8]</sup>. In this section, a Rayleigh fading channel is considered as three state Markov channel model, and the maximum Doppler frequency is  $f_m = 30 \ Hz$ , which environment corresponds that vehicular speed is  $40 \ km/h$  and carrier frequency is  $800 \ MHz$ . Transmission rate is  $R_t = 20 \ kbps$ . The used code rates are 1/3, 1/2, 2/3 for the state  $s_0$ ,  $s_1$  and  $s_2$  respectively.

## Performance of the adaptive coding system.

The thresholds to demarcate the channel states are decided to get optimum performance by trial and error. In this section, the thresholds used for the Markov channel model are  $a_1 = 6.5 \, dB$  and  $a_2 = 8.5 \, dB$ .

Fig.4 shows the average bit error probability of adaptive coding system and bit error probabilities of all the codes used in the nonadaptive coding system. Fig.5 shows the average throughput of the adaptive coding system and the code rates of all the codes used in the nonadaptive coding system. In Fig.4, the average bit error probability of the adaptive coding system is lower than that of the code with code rate 1/2 over all the range of a received SNR and in Fig.5, the average throughput, of the adaptive coding system is increased with a received SNR and is equal to that of the code with code rate 1/2 at 9 dB of received SNR. When a received SNR is larger than 9 dB, the adaptive coding system achieves the lower average bit error probability and the larger throughput than those of the code with code rate 1/2. For example, when a received SNR is smaller than 12 dB, the adaptive coding system has throughput improvements of 13% and achieves the lower average bit error probability by 10 times compared to the code with code rate 1/2. Where a received SNR is smaller than 9 dB, though the adaptive coding system attains the smaller throughput compared to the code with code

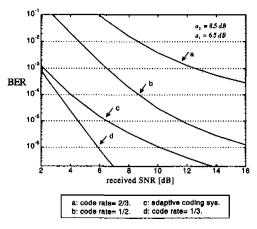


Fig. 4 The average bit error probability of the adaptive coding system.

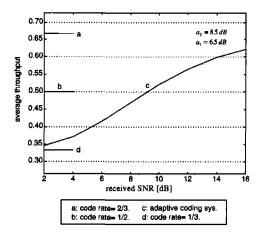


Fig. 5 The average throughput of the adaptive coding system.

rate 1/2, that achieves still lower bit error probability by 100 times compared to this. Also, compared to the code with code rate 1/3, the adaptive coding system attains the larger throughput maintaining the proper bit error probability.

### 2. The effect of threshold,

Fig.6 and Fig.7 show the effect of threshold  $a_1$  on the average bit error probability and the average throughput respectively, where the other threshold  $a_2$  is fixed at 8.5 dB. From those, it is known that the average bit error probability is decreased with the value of  $a_1$  and the average throughput is also decreased with the value of  $a_1$ . This is explained as follows. When  $a_1$  is increased with  $a_2$  fixed, the

range  $[a_0, a_1)$  is increased, and so increased is the number of times using the code assigned at that state, code rate of which is lowest of all the used codes. By this, the effect stated before is caused.

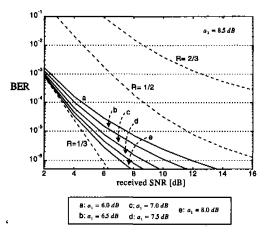


Fig. 6 Effect of threshold C<sub>1</sub> on the average bit error probability of the adaptive coding system.

To get an optimum threshold, with which the adaptive coding system achieves maximum throughput maintaining a specified average bit error probability, needed is the figures that are drawn about the average bit error probability versus  $a_1$  and average throughput versus  $a_1$  for a specific received SNR such as Fig.8 and Fig.9. If a system requires  $10^{-6}$  average bit error probability at  $10 \, dB$  of a received SNR, from 'c2' in Fig.8 it is known that the threshold  $a_1$  which satisfies the requirement

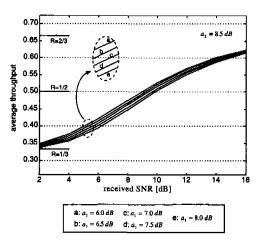


Fig. 7 Effect of threshold C<sub>1</sub> on the average throughput of the adaptive coding system.

is 6.6 dB. With this threshold, the adaptive coding system achieves average throughput of 0.52. That is, when thresholds  $a_1$ ,  $a_2$  are 6.6 dB and 8.5 dB respectively, with respect to the code with code rate 1/2, the adaptive coding gain is 7 dB and throughput gain is 4%. At this way, the optimum thresholds are attained.

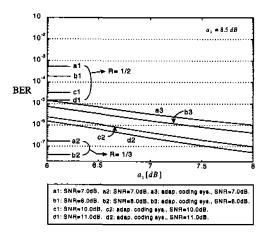


Fig. 8 Relation between the average bit error probability of adaptive coding system and threshold C<sub>1</sub>.

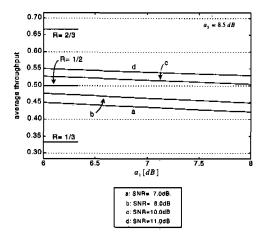


Fig. 9 Relation between the average throughput of adaptive coding system and threshold C<sub>1</sub>.

# 3. The effect of normalized Doppler frequency.

Fig.10 and Fig.11 show the effect of the normalized Doppler frequency on the average bit error probability and the average throughput of the adaptive coding system respectively. The normalized

Doppler frequency is  $f_mT$ , where  $f_m$  is maximum Doppler frequency and T is the symbol duration which is inverse of the transmission rate  $R_t$ . This value indicate the relative speed of the channel variation to the transmission rate. Generally, a channel is considered as fast fading when  $f_mT$  is larger than 0.01.

Fig. 10 shows that the average bit error probability of the adaptive coding system is increased with the normalized Doppler frequency, and Fig.11 shows that the average throughput of the adaptive coding system is nearly constant with respect to the normalized Doppler frequency. This is explained as follows. The larger  $f_m T$  is, the faster the variation of channel is. The faster the channel variation is, the more difficult it is to follow the variation of a channel and so the lower is the precision of channel prediction. By this, the average bit error probability of the adaptive coding system is increased with  $f_mT$ . Furthermore, from the fact that the average throughput is nearly constant with  $f_mT$  in Fig.11, it is more fatal to use a code with higher code rate in the bad channel state than to use a code with lower code rate in the good channel state.

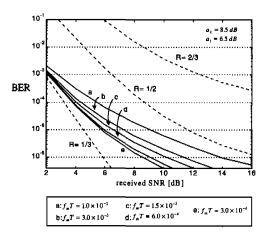


Fig. 10 Effect of the normalized maximum Doppler frequency on the average bit error probability of adaptive coding system.

Fig.12 and Fig.13 show the average bit error probability and the average throughput of the adaptive coding system versus  $f_mT$  for a specific received SNR. In Fig.12, the degradation of the

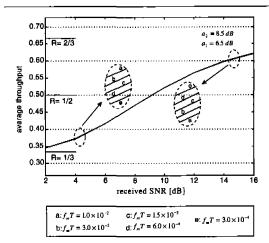


Fig. 11 Effect of the normalized maximum Doppler frequency on the average throughput of adaptive cooling system.

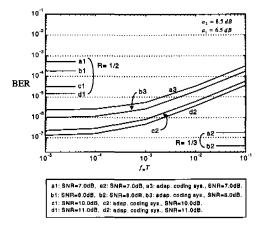


Fig. 12 Relation between the average bit error probability of adaptive coding system and the normalized maximum Doppler frequency.

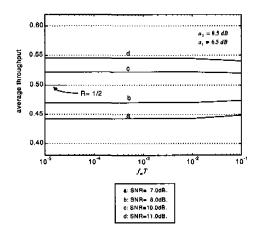


Fig. 13 Relation between the average throughput of adaptive coding system and the normalized maximum Doppler frequency.

average bit error probability is severe when  $f_mT$  is in the range of  $[10^{-3}, 10^{-1}]$ , which is due to the reason stated above. Fig.13 shows that the average throughput converges to 0.5 with larger  $f_mT$ .

### VI. CONCLUSION

In this paper, an adaptive coding system has been analyzed over a Rayleigh fading channel, which operates based on a received SNR. This system uses punctured convolutional codes as a coding scheme, which are simple to change code rates. For the analysis, a Rayleigh fading channel is modeled into a finite state Markov channel model. The criterion for demarcating channel state is a received SNR, and the value of a received SNR is divided into a finite number of intervals, each of which represents a state of Markov channel model. In each state, the probability density function was defined and then statistics of that state were calculated. Considering types of errors in the prediction process, the average bit error probability and the average throughput of an adaptive coding system have been calculated.

Numerical results have shown that an adaptive coding system has improvement compared to nonadaptive coding system when the average bit error probability and the average throughput are considered at the same time as performance measures. Specially, in the range of high SNR, the adaptive coding system achieves the larger throughput compared to the code with code rate 1/2 while an adaptive coding system maintains still lower bit error probability. Examined was the effect of threshold on the performance of the adaptive coding system, and the method to obtain the optimum threshold was also shown. Changing the value of threshold to use a code with lower code rate more often, the average bit error probability of the adaptive coding system became lower to be able to satisfy the requirement of a system while the average throughput became a little smaller. With optimum threshold, the adaptive coding system has the adaptive coding gain and throughput gain at the same time. Finally, the effect of the normalized

Doppler frequency was examined. When the channel variation is faster, the precision of the channel prediction is lower and so the average bit error probability is larger. By this reason, it is found that the adaptive coding system is proper when  $f_m T$  is lower than  $10^{-3}$ .

In this paper, there was no consideration about the estimation errors and processing delay. If more consideration about these in addition to what were considered in this paper is included, the analysis will be more realistic.

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