Effects of Non-Uniform Traffic Distribution on the Capacity of Reverse Link CDMA System

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ABSTRACT

In this paper, we analyzed the other-cell interference characteristics for various non-uniform traffic distributions and their effects on the capacity of multi-cell CDMA system. We consider three different traffic distributions, i.e., linear, exponential and Gaussian traffic distribution with distribution parameter s. Changing the distribution parameter, we can obtain the center-focused distributions or uniform distributions for each model. From the results of other-cell interference calculation we can see that the other-cell interference decreases, as the user concentrates on the base station. Also using frequency reuse efficiency indicating the capacity reduction of a multi-cell system when compared to a single cell system, we evaluate the effect of traffic distribution on the reverse link CDMA capacity. For linear case, the capacity of multi-cell system is reduced to 0.637 ~ 0.867 times that of single cell system. On the other hand, for both exponential and Gaussian cases, the capacity under a multi-cell environment is equal to 70~100% of that under a single cell. Therefore, we conclude that the average capacity of multi-cell CDMA system are increased when users are likely to be at near the cell base station due to reduced total other-cell interference and decreased when users exist at near the cell edge regardless of traffic distribution models.

I. Introduction

In CDMA cellular systems, power control is a vital issue to the system capacity. Since different users occupy the same frequency band at the same time, signal of a user may be overwhelmed by strong signals from other users. Based on the other-cell and same-cell interference, an optimum power control function is found to maximize the system capacity. This interference depends on the numbers of users in the cell of interest and in the adjacent cells, and the bit energies used by all of these users. So, the knowledge of other-cell interference characteristic is essential for evaluation of system performance. There is a large amount of papers related to performance analysis and capacity evaluation of DS/CDMA cellular system. But in most analytical studies, the user density is assumed to be uniform across the entire coverage area[1-2]. That is, the total interference calculation has been done under the assumption as follow:

(a) users in a cell are uniformly distributed and cells in coverage area are uniformly loaded, i. e., the average number of users in all the cells are the same, (b) the interference caused by the users in the other cells can be accounted for by artificially increasing the load in the cell of interest by a factor *(1+f)*, where *f* is the ratio of total other-cell interference to reference cell interference.

But in the real world, equal load in all cells and the uniformly offered traffic spatial distribution inside cells are very uncommon. The knowledge related to the impact of spatial user non-uniformity on the cellular system's performance are important for the network planning stage and for the study cellular radio networks control algorithms (handoff, power control, admission control, etc).

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When a service area is consisting of many cells, the channel capacity per CDMA cell decreases as a result of the excessive interference from other cells, resulting in the number of traffic channels per cell smaller than what would be available for a single isolated cell case. In general, the capacity per cell in the multi-cell CDMA environment is limited by the interference. The amount of out-of-cell interference determines the frequency reuse efficiency of a CDMA cellular system indicating the capacity reduction of a multi-cell system when compared to a single cell network, such that

$$F=\frac{\text{total same cell power}}{\text{total same plus other cell power}} \tag{1}$$

In this study, we consider the interference analysis and capacity evaluation problem of a multi-cell CDMA system. Specifically, we look at the dependence of other-cell interference on the location probability of mobile user and its impact on the reverse link capacity of CDMA system using the behavior of the reuse efficiency as a function of the traffic distribution.

Ⅱ. Other-cell interference analysis by circular cell approximation

To calculate other-cell interference, we will use a circular cell having the same area as the hexagonal cell as Figure 1. So, the radius of the circle R_c will have the following relation with the radius of the hexagonal cell R_h

$$\pi R_c^2=\frac{3\sqrt{3}}{2}R_h^2 \Rightarrow R_c=0.9094R_h \tag{2}$$

For a mobile locating at a point K from its home base station (BS), the distance between the base station of the surrounding cell and given mobile will be given by

$$d(r,\theta)=\sqrt{D_i^2+r^2-2D_i r\cos\theta} \tag{3}$$

where, $0\le r\le R_c$, $0\le\theta\le 2\pi$ and D_i is the distance between the BS in the interfering cell i

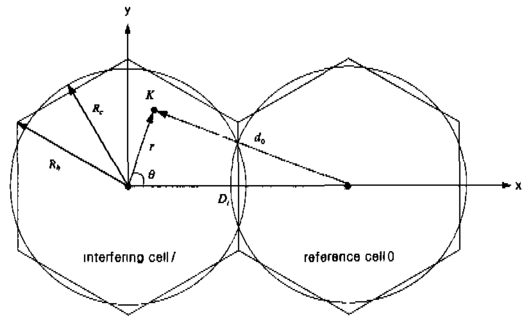

Fig 1. The interference caused by a mobile in interfering cell

and that of reference cell 0. D_i is related to the cell radius R_h with the following expression

$$D_i=\sqrt{3L_i}R_h \tag{4}$$

where $L_i=u_i^2+u_iv_i+v_i^2$ and (u_i,v_i) is the displacement in the u and v directions from the reference cell to the interfering cell[3].

If a perfect power control scheme of ensuring equal received power at the base station is assumed, the interference caused by a mobile in the interfering cell to the reference cell will be given by

$$\begin{aligned} I(r,\theta) &= P_R\left(\frac{r}{d_0}\right)^{\alpha} \\ &= \frac{P_R r^{\alpha}}{[D_i^2+r^2-2D_i r\cos\theta]^{\alpha/2}} \end{aligned} \tag{5}$$

where P_R is the constant received power at the interfering base station and α is the propagation exponent.

We assume that the mobiles are evenly distributed over the circular cell. The probability distribution function of a user in the circle will be given by

$$f(r,\theta)=\begin{cases} \frac{1}{\pi R_c^2} & \text{for } 0\le r\le R_c,\ 0\le\theta\le 2\pi \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

The average interference from a mobile in the other cell i to the reference cell will be given by

$$\begin{aligned} \overline{I_i} &= E[I(R,\Theta)]=\int\int I(r,\theta)f(r,\theta)\,r\,dr\,d\theta \\ &= \frac{P_R}{\pi R_c^2}\int_0^{R_c}\int_0^{2\pi}\frac{r^{(\alpha+1)}dr\,d\theta}{[D_i^2+r^2-2D_i\cos\theta]^{\alpha/2}} \end{aligned} \tag{7}$$

If we use propagation exponent $\alpha=4$ and the equation $\int_0^{2\pi}\frac{dx}{(a+b\cos x)^2}=\frac{2\pi a}{(a^2-b^2)^{3/2}}$ then we get

$$\overline{I}_i=\frac{2P_R}{R_c^2}\int_0^{R_c}\frac{r^5(D_i^2+r^2)}{(D_i^2-r^2)^3}\,dr \tag{8}$$

We can solve the integral in equation (8) the following equation

$$\begin{aligned}&\int_0^R\frac{d^2r^5+r^7}{(d^2-r^2)^3}\,dr\\&=\int_0^R -r-\frac{4d^2r^5-3d^4r^3+d^6r}{(d^2-r^2)^3}\,dr\\&=\int_0^R\left(-r+\frac{4d^2r}{(d^2-r^2)}-\frac{5d^4r}{(d^2-r^2)^2}+\frac{2d^6r}{(d^2-r^2)^3}\right)dr\\&=\left[-2d^2\ln(d^2-r^2)-\frac{r^6-2d^2r^4-4d^4r^2+4d^6}{2(d^2-r^2)^2}\right]_0^R\\&=-2d^2\ln\left(1-\frac{R^2}{d^2}\right)-\frac{(R^6-6d^2R^4+4d^4R^2)}{2(d^2-R^2)^2}\end{aligned} \tag{9}$$

and if we consider the interference caused by the first tier cells only, then a closed-form average interference is 0.0633 P_R

So far in our analysis, we have used regular hexagonal and circular cell structure. Indeed, the analytical result can be generalized and applied to irregular and more realistic cell shape. In a practical cellular radio system, the cell layout will not follow regular hexagonal structure. For a particular cell i with arbitrary cell shape and away from a reference cell by D_i, we can approximate the interference from this cell by using a circle with equal area and radius R_i to approximate this cell. Applying equation (8) and (9) and letting $k=R_i/D_i$, we can obtain the average other cell interference for irregular cell structur

$$\overline{I}_{irr}=P_R\left[-\frac{4-6k^2+k^4}{(1-k^2)^2}-\frac{4}{k^2}\ln(1-k^2)\right] \tag{10}$$

The total average interference from a mobile in the other cells to the reference cell will be given

$$\overline{I}_{oc}=\sum_{i=1}^{C}\overline{I}_i \tag{11}$$

where C is the total number of adjacent cell in the service area and $\overline{I}_i$ is average interference from cell i given as the equation (8).

If we assume that the average number of users in all the cells are the same and further there are M users communicating with base station in each cell, the total interference received at the base station from home cell and adjacent cells will be given by

$$\overline{I}_{total}=(M-1)P_R+\sum_{i=1}^{C}M\,\overline{I}_i \tag{12}$$

To calculate the total other-cell interference, we have to determine the distance D_i defined in equation(4). The distances between any base stations in the first tier and the reference cell base station are identical ($D_i=\sqrt{3}R_h$ $_{i=1,\cdots,6}$ where R_h is the radius of the hexagonal cell). The second tier of cells contains 12 cells, of which six have the distance($D_i=2\sqrt{3}R_h$) from the reference cell base station and the other six have the distance($D_i=3R_h$) as shown in table 1.

Table. 1 The distance between the reference cell base station and any other interfering cell base station according to their position.

Tier	Total cells	$D_i/R_h=\sqrt{3K_i}\times$ vmberofcells
1	6	$\sqrt{3}\times6$,
2	12	$\sqrt{12}\times6, \sqrt{9}\times6$
3	18	$\sqrt{27}\times6, \sqrt{21}\times12$
4	24	$\sqrt{48}\times6, \sqrt{39}\times12, \sqrt{36}\times6$
5	30	$\sqrt{75}\times6, \sqrt{63}\times12, \sqrt{57}\times12$
6	36	$\sqrt{108}\times6, \sqrt{93}\times12, \sqrt{84}\times12, \sqrt{81}\times6$
7	42	$\sqrt{147}\times6, \sqrt{129}\times12, \sqrt{117}\times12, \sqrt{111}\times12$
8	48	$\sqrt{192}\times6, \sqrt{171}\times12, \sqrt{156}\times12, \sqrt{147}\times12, \sqrt{144}\times6$
9	54	$\sqrt{243}\times6, \sqrt{219}\times12, \sqrt{201}\times12, \sqrt{189}\times12, \sqrt{183}\times12$
10	60	$\sqrt{300}\times6, \sqrt{273}\times12, \sqrt{252}\times12, \sqrt{237}\times12, \sqrt{228}\times12, \sqrt{225}\times6$

Figure 2 shows the graph of total other-cell interference from the surrounding cells against the number of tiers. From this Figure, we can see that the other-cell interference from the adjacent cells is mainly contributed from the first few tiers of cells. The interference from cells away from the ninth tier becomes insignificant and can be neglected. So the total other-cell interference from all the adjacent cells is about 0.431 M can be approximated about 0.431 $M P_R$

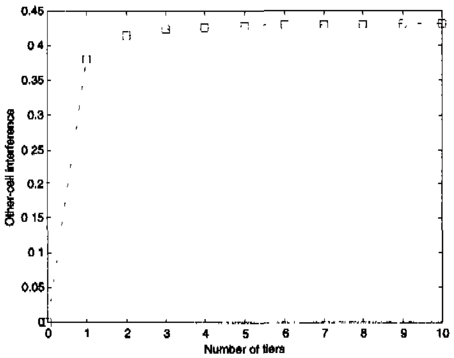

Fig. 2 Total interference from adjacent cells against the number of tiers

Ⅲ. Effect of non-uniform traffic distribution on the other-cell interference.

The availability of radio resources such as transmission powers, rates, and channels in cellular networks is location dependent and time dependent due to the effects of signal propagation and fading. That is, mobile users at different locations in a cell consume different amounts of radio resources by introducing different levels of interference into the system. But in most analytical studies for CDMA interference and capacity estimation, the user density is assumed to be uniform across the entire coverage area. Ganesh *et al.*, addresses the effect of non-uniform traffic distribution on the system performance, but they only focus on the viewpoint of the whole service area[4].

In this section, we focus on the location dependent aspect of the radio resources of a cellular CDMA system in a non-fading environment and present the analytical results of the interference with non-uniform user traffic distribution. Three different non-uniform user density models are considered and their impacts on the total interference are determined and compared to typical uniformly distributed CDMA system. Three non-uniform user traffic density models considered here are linear, exponential and Gaussian, represented by equations (13), (14) and (15), respectively[4]. We assume that a mobile with same distance from its base station has identical distribution independent of angle. It is assumed also that all cells in the whole coverage have same user distribution function and same number of active users M.

$$f(r,\theta)=\begin{cases} K\left(1-\frac{(1-1/s)r}{R_c}\right) & 0\le r\le R_c,\ 0\le\theta\le 2\pi \\ 0 & otherwise \end{cases} \quad (13)$$

$$f(r,\theta)=\begin{cases} Ee^{(-r/s)} & 0\le r\le R_c,\ 0\le\theta\le 2\pi \\ 0 & otherwise \end{cases} \quad (14)$$

$$f(r,\theta)=\begin{cases} Ge^{-r^2/2s^2} & 0\le r\le R_c,\ 0\le\theta\le 2\pi \\ 0 & otherwise \end{cases} \quad (15)$$

The constraint used in determining the parameters K, E, and G in each model is given in equation (16). The constraint states that the total user in a cell M is fixed and distributed across a cell according to the selected non-uniform user density function.

$$\int_0^{R_c} 2\pi r f(r,\theta)dr = M \quad (16)$$

Using above equation, the parameters K, E, and G normalized by M are obtained according to their non-uniformity parameter s.

$$K=\frac{3}{\pi R_c^2(1+2/s)},\quad E=\frac{1}{2\pi s^2\{e^{-R_c/s}(-R_c/s-1)+1\}},\quad G=\frac{1}{2\pi s^2(1-e^{-R_c^2/2s^2})} \quad (17)$$

Assuming the distribution parameter s is positive value only, we analyzed the characteristic of three different non-uniform density distribution models. For the linear density model depicted in Figure 3, when $s=1$, $f(r,\theta)$ is constant across the cell indicating a uniform distribution of users. When $s>1$, $f(r,\theta)$ is non-uniform leading to a correspondingly higher user density near the base station (center-focused distribution) and when s is increased to infinite, $f(r,\theta)$ becomes a triangular distribution with zero users at the cell radius R_c. On the other hand, when $s>1$, many users concentrate on near the boundary of the cell (edge-focused distribution).

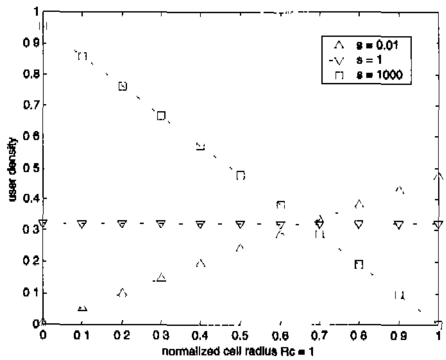

Fig. 3 Linear user density model according to distribution parameter s.

Similarly, for both the exponential and Gaussian density models plotted in Figure 4, $f(r,\theta)$ is very high at the base station when s converges to zero, and it follows a uniform user density distribution as s increased to infinity. But in two cases, edge-focused distribution is not occurred because s is considered only positive values.

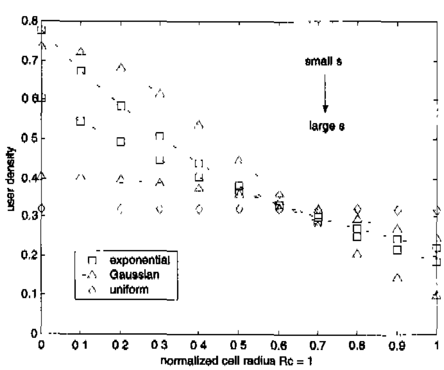

Fig. 4 Gaussian and exponential user density model according to parameter s.

Using equation (7), we can obtain the other-cell interference for three different non-uniform user distribution function. Figure 5~6 shows the other-cell interference as a function of distribution factor s, when the path loss coefficient is assumed by 4. Note that the other-cell interference decreases, as the mobile concentrates on the base station. For linear case, as the s becomes less than 1, the other-cell interference becomes high due to the characteristic of the edge-foused distribution. And when s is positive infinity, the minimum other-cell interference is obtained due to the characteristic of the proposed linear model, i. e, triangular distribution.

For both exponential and Gaussian cases plotted in Figure 6, the more smaller s is (center-focus distribution), the lower other-cell interference is incurred. But in Figure 6, we selected arbitrary values of s just in order to show the other-cell interference patterns according to the different two models. Also, we can see that the other-cell interference is matched to that of uniform distribution as distribution parameter s is increased.

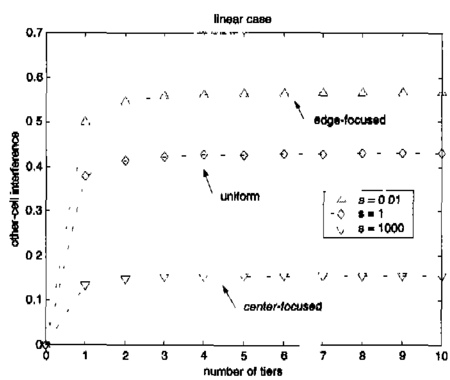

Fig. 5 Other-cell interference for linear density model.

For more fair comparisons of the effect on the system performance, we choose the non-uniformity parameter s satisfying the condition that user density $f(r,\theta)$ at $r=R_c$ is equal to p percent of user density at $r=0$. In this section, two values are considered for the fraction p: (a) $p=1\%$ and (b) $p=50\%$. Three user density models satisfying the above values are shown in Figure 7. From the Figure 7~8, we can conclude that when

p=50%, three user density models have almost same distribution and identical other-cell interference over the distance from the base station to its cell boundary. But in case of p=1%, other-cell interference varies according to their distribution models. Also, the other-cell interference for three models converges to that of uniform distribution as the fraction value p increased to 100%.

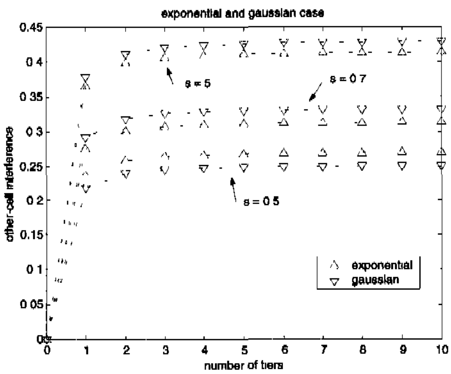

Fig. 6. Other-cell interference for exponential and Gaussian cases.

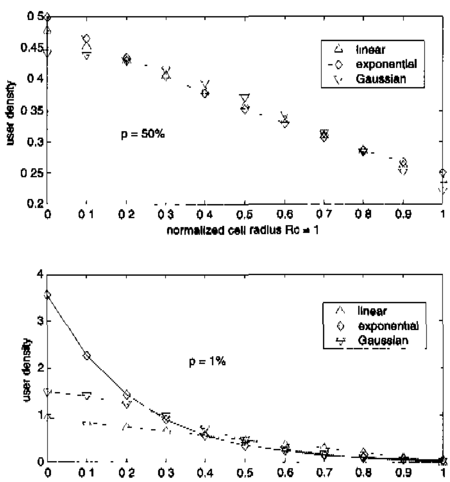

Fig. 7 User density models satisfing fraction value p.

Ⅳ. Performance evaluation and conclusion

In this section, the performance of the cellular CDMA is analyzed in a multi-cell environment with non-uniform traffic distribution using

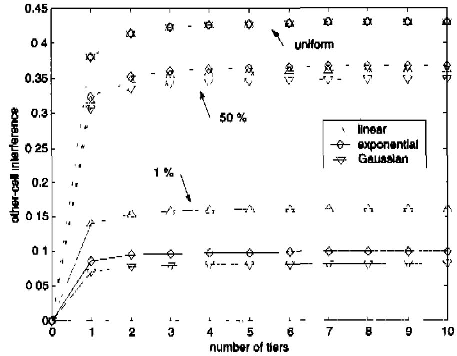

Fig. 8 Other-cell interference according to fraction parameter p.

frequency reuse efficiency. The performance of a CDMA system is interference-limited. This means that the capacity and quality of the system are limited by the amount of interference power. Capacity is defined as the total number of simultaneous users that the system can support, and quality is defined as the perceived condition of a radio link assigned to a particular user; this perceived link quality is directly related to the probability of bit error. The actual capacity of a CDMA system depends on many different factors such as receiver demodulation, power control accuracy, and actual interference power introduced by other users in the same cell and in neighboring cells.

The signal to interference ratio (SIR) of a mobile user j in the reference cell is

$$\gamma_j = \frac{GP_{R,j}}{\sum_{k\neq j,k=1}^{M} P_{R,k} + I_{oc} + N} \qquad (18)$$

where $G = W/R$ is the processing gain and $P_{R,j}$ is the received power of user j, I_{oc} is total other-cell interference and N is the background noise power. With perfect power control and a interference-domain system (neglecting N) for a fixed SIR target γ_{min}, we have the uplink capacity of multi-cell CDMA system, i. e., the number of simultaneous users that reference cell can support as the following:

$$M \le \frac{\frac{G}{\gamma_{min}} + 1}{1 + I_{oc}'/P_R}, \quad where\ I_{oc}' = \sum_{i=1}^{C} \overline{I_i} \qquad (19)$$

where C is total number of cells in the coverage and M is the number of users in a cell. Using equation (19), the ratio of total multi-cell capacity to that of single cell equals to the frequency reuse efficiency defined in equation (1) and it can be estimated as

$$F = \frac{MP_R}{MP_R + M\sum_{i \neq j}^{C} I_i} = \frac{1}{1+\frac{1}{P_R}\sum_{i \neq j}^{C} I_i} = \frac{M_m}{M_s} \quad (20)$$

where I_i is the other-cell interference by a mobile from cell j, M_s is capacity of single cell system and M_m is capacity of multi-cell system. The interference I_i can be calculated as a function of the propagation parameters and of the traffic density as given equation (7).

Figure 9 depicts the frequency reuse efficiency of the various non-uniform user distribution model according to distribution parameter s, using the system parameters similar to IS-95 with assumption of system bandwidth W=1.2288 MHz, data rate R = 9.6Kbps and SIR target $\gamma_{\min}$=7 dB. For linear traffic distribution, mobiles concentrate on near the cell boundary, the other-cell interference is increased and the capacity of multi-cell system is reduced maximally to about 0.637 times that of single cell. On the other hand, when s is increased to infinity, the total capacity of multi-cell system is converged to 0.867 times that of single cell, because the user density becomes triangular distribution. From this curve, we can conclude that the center-focused linear traffic distribution can guarantee the higher reuse efficiency.

For both exponential and Gaussian density models, the capacity of multi-cell system increased to that of single cell system, as s is converges to zero and the capacity ratio of multi-cell to single cell system follows the uniformly distributed system, as s increased to infinite. That is, for both cases, the capacity under a multi-cell environment is equal to 70 ~ 100% of the capacity under a single cell. Therefore, we conclude that the average capacity of multi-cell CDMA system are increased when users are likely to be at near the cell base station and decreased when users exist at near the cell edge regardless of traffic distribution models.

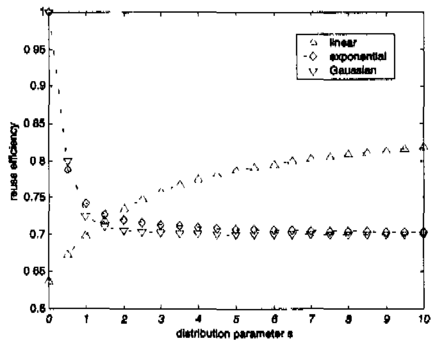

Fig. 9 Reuse efficiency for different user density distribution models

Figure 10 demonstrates the effect of the traffic distribution on the reverse link CDMA capacity. The capacity reduction caused by other-cell interference is almost same for three different traffic distribution models when p approaches to 100%, because three models have uniform traffic density distribution. And the CDMA capacity under a multi-cell environment is equal to 70% of that under a single-cell environment. But, generally the capacity of system for Gaussian distribution model is more superior to those of the other two models over all fraction value p.

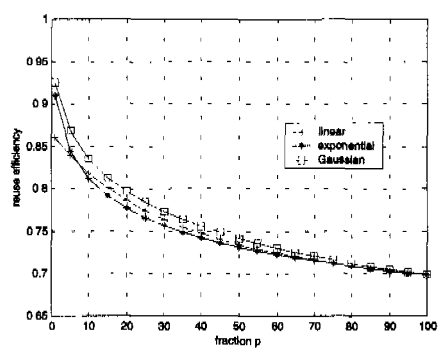

Fig. 10 Reuse efficiency against the fraction p.

In this study, the effects of the traffic distribution on the reverse link CDMA capacity is analyzed. The results indicate that the cell center-focused user density is always beneficial to system performance since it causes the lower other-cell interference. On the other hand, the cell edge-focused user distribution is detrimental to system performance regardless of any traffic distribution types. Further study on the other-cell interference calculation for multi-rate CDMA according to user density distribution and performance evaluation is necessary to efficiently use the radio resources in call admission control stage.

References

[1] A. M. Viterbi and A. J. Viterbi, "Erlang capacity of a power controlled CDMA system," *IEEE* J. Select. Areas Commun., vol. 11, pp. 892-899, Aug. 1993.

[2] K.S.Gilhouse, J.M.Jacob, R.Padovani, A.J. Viterbi, L.A.Weaver, C.E.Wheatly III, "On the Capacity of a Cellular CDMA System," *IEEE* Trans. Commun., vol. COM-40, pp. 302-312, Aug. 1991.

[3] J. S. Lee and L. E. Miller, CDMA systems engineering handbook, Boston: *Artech House*, pp. 279-282, 1998.

[4] R. Ganesh and K. Joseph, "Effect of mobile distributions on performance of a cellular CDMA systems", Proc. *IEEE ICUPC'97*, San Diego, CA, pp. 598-602, Oct. 1997

[5] K. I. Kim, "CDMA cellular engineering issues," *IEEE* Trans. VT., vol. 42, pp. 345-350, Aug. 1993.

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