Phase Noise Effects on the Pulse Pair Spectrum Moment Estimates in a Doppler Weather Radar

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ABSTRACT

A weather radar usually extracts the necessary information from the return Doppler spectrum moment estimates. Phase stability is a very important factor in obtaining accurate and reliable information in a Doppler weather radar system since the system phase noise may seriously degrade the weather spectrum moment estimation quality. These spectrum moment estimates are commonly obtained using the pulse pair method which is simple to implement and fast enough to process an enormous amount of weather radar data in real time. Therefore, an analytical method is developed in this paper to analyze and quantify the phase noise effects on the pulse pair spectrum moment estimates in terms of the phase noise power and broadness.

I. Introduction

In the weather radar, the weather information such as windspeed and turbulence is obtained from the return Doppler spectrum moment estimates called the mean and width estimates[1]. The degree of accuracy in these estimates is important for the real-time weather surveillance system to prevent the weather-related accidents. The superimposed scattering of incident electromagnetic energy from many randomly distributed particles causes an unavoidable phase jittter effect which generally contributes to the spectrum width of the weather radar return signal which often indicates the degree of turbulenc. With a coherent pulse Doppler radar, this creates some ambiguity in determining a representative windspeed condition within a particular range cell. Any radar system phase instabilities may also contribute to this ambiguity and affect the Doppler frequency resolution as well as the dynamic range capability. There are many potential sources of system phase instability (phase noise or phase jitter) but a potential primary source is the stable local oscillator (commonly STALO) which provides transmitted and reference carriers within the radar. If the range time is adequate to decorrelate the STALO oscillator phase, any phase variation with time (jitter) may be adequate to contribute error in return Doppler spectral estimates. Furthermore, this may not be apparent when observing the weather return Doppler spectrum because of its inherent spread.

The pulse pair estimator often called a covariance estimator is widely accepted for the spectrum moment estimation method in a weather radar since it is simple to implement and fast enough to process huge amounts of data for real time mapping of the weather situation in an interested area[1][2]. Therefore, this paper provides an analysis of the effect of pulse to pulse system phase jitter uncertainty on the pulse pair spectral moment estimates. Intrapulse phase uncertainty is not considered here. An analytical development of the Doppler spectrum using a Gaussian approximation for the phase jitter spectrum is presented. Even though a power-law model is more appropriate in representing experimental results of a STALO phase noise spectrum[3], a Gaussian model is considered here for two reasons. First, it

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is reasonable to fit a Gaussian distribution to a major portion of a power-law phase noise spectrum particularly in the tails of the spectrum. Secondly, a Gaussian assumption provides mathematical tractability.

The spectral mean and width as estimated by the pulse pair method are analyzed analytically considering the effect of phase jitter. For the Gaussian model, the effect of phase jitter is computed for a particular phase jitter power level while allowing variation in both the weather spectrum width and the phase jitter spectrum width.

Ⅱ. Gaussian Phase Noise

Any phase jitter occurring at the transmitter stage will spread the original sinusoidal signal spectrum. The stable local oscillator output is modelled as

$$V(t)=V_0\,COS(\omega_c t+\Phi(t))$$

where $\Phi(t)$ is a phase jitter and the carrier is $\omega_c=2\pi f_c$. If one assumes that $\Phi(t)$ is a normal stationary process with zero mean and variance $\sigma^2=E[\ \Phi^2(t)]$, the spectral density of the transmitted signal is given by[4]

$$S_V(f)=\frac{V_0^2}{2}e^{-\sigma^2}\{\delta(f-f_c)+S_\Phi(f-f_c)+\sum_{n=2}^{\infty}\frac{1}{n!}[S_\Phi(f)^{n-1}*S_\Phi(f)]f_c\} \qquad (1)$$

where the first term inside the braces of (1) relates to the sinusoidal energy at f_c , the second term is the phase noise spectrum located at f_c and the last term includes n-1 convolutions of the phase noise spectrum with itself, followed by a translation around the carrier frequency f_c. As the mean intensity of phase noise becomes very small, the spectral density of phase jitter can be approximated by a Gaussian spectrum[5]; i.e.

$$S_\Phi(f)=W_0\exp\{\frac{-\omega^2}{\Delta\,\omega_c^{\,2}}\}$$

where the relationship between phase jitter width $\Delta\omega_c^2$ and the total phase jitter power σ^2 is represented by

$$\sigma^2=\int_{-\infty}^{\infty}s_\Phi(f)\,df=\frac{\Delta\omega_c}{2\sqrt{\pi}}W_0$$

With Gaussian phase noise , the spectral density of the transmitted signal from (1) can be obtained analytically as

$$S_v(f)=\frac{V_0^2}{2}e^{-\sigma^2}\{\delta(f-f_c)+\frac{2\sqrt{\pi}}{\Delta\omega_c}\sum_{n=1}^{\infty}\frac{\sigma^{2n}}{n!\sqrt{n}}\exp[\frac{-(\omega-\omega_c)^2}{n\Delta\omega_c^2}]\}$$

If σ is small(σ<0.5 radians), which should be true in a high quality oscillator, $S_v(f)$ can be approximately given by

$$S_v(f)=\frac{V_0^2}{2}e^{-\sigma^2}\{\delta(f-f_c)+\frac{2\sqrt{\pi}}{\Delta\omega_c}\sigma^2\exp[\frac{-(\omega-\omega_c)^2}{\Delta\omega_c^2}]\} \qquad (2)$$

It can be noted that the theoretically exact description for the spectral density is[6]

$$S_v(f)=\frac{V_0^2}{2}\int_{-\infty}^{\infty}\exp(-\frac{\omega_c^2}{2}\tau^2 I^2(\tau))\cos(\omega_c\tau)e^{-j\omega\tau}d\tau$$

where

$$I^2(\tau)=\frac{E\{[\Phi(t_k+\tau)-\Phi(t_k)]^2\}}{(\omega_c\tau)^2}$$

This theoretical expression would be useful if an approximation for the true variance $I^2(\tau)$ could be determined for all τ, but it is very difficult to obtain this experimentally except for a few discrete values of τ due to the requirement of very long observation time. Therefore, the approximation in (2) provides a realistic means of analyzing the effects of phase jitter in the Doppler radar.

The transmitted radar signal is scattered by many particles and the signal frequency will be shifted according to the particles' velocity which is the well known Doppler effect. Using the description given in (2), the scattered return signal can be approximated as

$$S_v(f)=\frac{v_0^2}{2}e^{-\sigma^2}C\{\frac{1}{\sqrt{2\pi}W}\exp(-\frac{(f-f_c-f_d)^2}{2W^2})+\sigma^2\frac{1}{\sqrt{2\pi(\Delta f_c^2/2+W^2)}}\cdot\exp(-\frac{(f-f_c-f_d)^2}{2(\Delta f_c^2/2+W^2)})\} \qquad (3)$$

where W is the spectrum width with zero transmitter phase noise and C is a constant determined by the reflectivity of target particles and some other factors (see for example[1]). If the noise is small (σ small), the second term in (3) can be ignored and the typical Doppler return signal will have a purely Gaussian spectral shape. With this assumption the autocorrelation function of the return signal after demodulation will be

$$R(\tau)=\frac{v_0^2}{2}C\gamma(\sigma,w,\tau)e^{j\omega_d\tau}+N\delta_{\tau,0} \qquad (4)$$

where

$$\gamma(\sigma,w,\tau)=e^{-\sigma^2}\{e^{-2\pi^2w^2\tau^2}+\sigma^2e^{-2\pi^2(w^2+\frac{\Delta f_c}{2})\tau^2}\}$$

The additive white noise $N\delta_{\tau,0}$ is included in the signal model to represent the usual background noises[2]. The autocorrelation function shown in (4) is obtained under the assumption that no phase noise occurs at the receiver in the process of demodulation. However, in the interest of a more complete understanding of pulse pair estimation, the effect of phase jitter occurring at the receiver should be included. That effect is analyzed in the next chapter.

Ⅲ. Phase Noise Effects on the Pulse Pair Method

In the pulse pair method[7], a two-point estimate of the complex autocorrelation is based upon processing coherent returns from pairs of transmitted pulses. Assuming that the pulse pairs are separated by T_s and repeated with period T as illustrated in Figure 1, the complex time autocorrelation function of the complex received wave form z(t) at the lag value T_s is defined as

$$R(Ts)=E\{Z^*(iT)Z(iT+T_s)\}$$

Considering the phase deviations in the process of heterodyne demodulation designated by ϕ_{iT} and ϕ_{iT+Ts}, the autocorrelation function is

$$R'(T_s)=R(T_s)E\{e^{-j\Phi_{iT}}e^{j\Phi_{iT+T_s}}\} \qquad (5)$$

where, from (4),

$$R(T_s)=S\gamma(\sigma,w,T_s)e^{j\omega_dT_s}\quad, S=\frac{v_0^2}{2}C\ . \qquad (6)$$

If one assumes that $\Phi_1=\Phi_{iT}$ and $\Phi_2=\Phi_{iT+Ts}$ and are, for example, normal random variables[3], then (5) can be rewritten as

$$R'(T_s)=R(T_s)(2\pi\overline{\Phi^2}\sqrt{1-\rho^2})^{-1}\int\int_{-\infty}^{\infty}e^{-j\Phi_1}e^{j\Phi_2}\cdot\exp[-(\Phi_1^2+\Phi_2^2-2\rho(T_s)\Phi_1\Phi_2)/(2\overline{\Phi^2}(1-\rho^2))]d\Phi_1d\Phi_2$$

where

$$\rho(T_s)=E[\Phi_1\Phi_2]/\overline{\Phi^2},\quad E[\Phi_1\Phi_2]=\overline{\Phi_1\Phi_2}\ .$$

The complex autocorrelation function can be further reduced to[8]

$$R'(T_s)=R(T_s)\exp[-(\overline{\Phi^2}-\overline{\Phi_1\Phi_2})]=R(T_s)\exp[\int_{-\infty}^{\infty}S_\Phi(f)(1-\cos\omega T_s)df] \qquad (7)$$

Similarly $S_\Phi(f)$ can be assumed to have a Gaussian noise spectrum as described in the previous section since the same oscillator signal is fed into the receiver for heterodyning. Considering the additive white noise with this assumption, (7) will become

$$R'(T_s)=R(T_s)\Psi(\sigma,T_s)+N\delta_{T_s,0} \qquad (8)$$
$$\Psi(\sigma,T_s)=\exp[-\sigma^2\{1-\exp(\frac{-\Delta\omega_c^2T_s^2}{4})\}]$$

where $R(T_s)$ is given in (6) and σ is the r.m.s. phase noise. As seen from (8), this modified autocorrelation at lag value T_s is explicitly a function of the phase spectrum width $\Delta\omega_c$ and the total phase jitter power σ^2 with the additive white background noise also included.

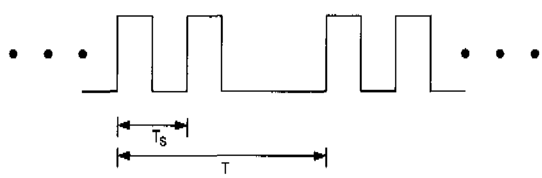

Fig. 1 Pulse pairs for the estimate of autocorrelation function

Ⅳ. Pulse Pair Estimation Errors

Considering estimation of the mean frequency with pulse pair processing[2], the estimated return spectrum mean is defined by

$$\hat{f}_d = \frac{1}{2\pi T_s} \arg\{\hat{R}(T_s)\} \quad (9)$$

where

$$\hat{R}(T_s) = \frac{1}{M} \sum_{i=0}^{M-1} Z^*(iT)Z(iT+T_s) \ .$$

As seen from (8) and (9), the estimate of the mean frequency will not be dependent upon phase jitter because it does not affect arg{R(Ts)}, i.e., only the magnitude scaling is changed by phase jitter. Thus, according to this analysis, no bias will be introduced into the mean frequency estimate because of phase jitter. The variance of the mean frequency estimate as previously published[7] is

$$\begin{aligned} VAR(\hat{f}_d) = & [8\pi^2 T_s^2 \beta^2(T_s)]^{-1}\{M^{-2}[1-\beta^2(T_s)] \\ & \cdot \sum_{m=-(M-1)}^{M-1} \beta^2(mT)(M-|m|) + \frac{N^2}{MS^2} \\ & + \frac{2N}{MS}[1-\beta(2T_s)\delta_{T-T_s,0} + \frac{\beta(2T_s)}{M}\delta_{T-T_s,0}]\} \end{aligned} \quad (10)$$

where $\beta(T_s) = \exp(-2\pi^2 w^2 T_s^2)$ and T=Ts for contiguous pulse pairs as shown in Figure 1. This expression can be rewritten considering phase jitter by replacing $\beta(T_s)$ with $\beta'(T_s)$ where

$$\beta'(T_s) = \gamma(\sigma, w, T_s)\Psi(\sigma, T_s) \ .$$

Here $\delta_{T-T_s,0}$ for T=Ts and zero otherwise, M is the number of sample pairs and S, N represent the average signal power and the background noise respectively. Analyzing the variance expression in (10), if the total phase jitter power σ^2 is held constant, one can investigate the effect of varying the spectral distribution of the phase jitter. The parameter Δfc determines the shape of this spectral distribution. As Δfc increases, the phase jitter spectrum is broadened. As can be seen from Figure 2, which is a plot of the mean frequency estimate standard deviation versus normalized Δfc for contiguous pairs, when the phase jitter spectrum is broadened, an increase in the variance of the mean frequency estimate can be expected. Also from Figure 2, as the weather spectrum width W increases, this effect on the mean frequency estimate is more pronounced.

Considering the weather spectrum width estimate using the pulse pair method, the width estimation is independent of the mean frequency[9]. If the width is sufficiently smaller than the Nyquist interval, one form of the estimator which is not dependent upon spectrum shape is given by[9]

$$\hat{W} = \begin{cases} \dfrac{1}{\sqrt{2}\pi T_s}\left|1 - \dfrac{|\hat{R}(T_s)|}{\hat{S}}\right|^{\frac{1}{2}} & \text{for } |\hat{R}(T_s)| < \hat{S} \\ \delta \quad \text{for } |\hat{R}(T_s)| \geq \hat{S} \end{cases} \quad (11)$$

where

$$\hat{S} = \frac{1}{L}\sum_{K=0}^{L-1} |Z_k|^2 - N$$

and δ is an arbitrarily small number. With the introduction of phase jitter, this estimator becomes

$$\hat{W}' = \begin{cases} \dfrac{1}{\sqrt{2}\pi T_s}\left|1 - \dfrac{|\hat{R}'(T_s)|}{\hat{S}}\right|^{\frac{1}{2}} & \text{for } |\hat{R}'(T_s)| < \hat{S} \\ \delta \quad \text{for } |\hat{R}'(T_s)| \geq \hat{S} \end{cases} \quad (12)$$

where since $|\hat{R}'(T_s)|$ from (7) replaces $|\hat{R}(T_s)|$ in (11), the stimate of spectrum width is biased. Previous results for the asymptotically unbiased variance can now be extended to yield the variance of the width estimate as

$$\begin{aligned} VAR(\hat{W}) = & [32M\pi^4(WT_s)^2\beta^2(T_s)T_s^2]^{-1}\{2 \cdot \\ & [1-(1+\delta_{T-T_s,0})\beta^2(T_s) + \delta_{T-T_s,0}\beta^4(T_s)]\frac{N}{S} \\ & + [1+(1+\delta_{T-T_s,0})\beta^2(T_s)]\frac{N^2}{S^2} \\ & + \beta^2(T_s)\sum_{m=-(M-1)}^{M-1}\{2\beta^2(mT) + \beta^2(mT)\beta^2(T_s) \\ & + \beta(mT+T_s(1-\delta_{T-T_s,0})) - 4\beta(mT+T_s) \\ & \cdot \beta(mT)\beta^{-1}(T_s)\} \cdot (1-\frac{|m|}{M})\} \end{aligned}$$

where $\beta(T_s)$ given in a previously derived result[9] is replaced by $\beta'(T_s)$ as was done in (10). The variance of the width estimate may be increased by phase jitter, but does not completely describe the goodness of the width estimator, since phase noise will bias the width estimate of spectrum as seen from (12). Thus, in this situation the mean square error defined by

$$E\{(\hat{\theta}-\theta)^2\} = VAR(\hat{\theta}) + B^2$$

where B is the bias of a biased estimation $\hat{\theta}$ of the parameter θ , will be a more representative measure of the effect of phase jitter on the pulse pair estimation of spectrum width. The bias term and the root mean square error of the width estimate for contiguous pairs are plotted in Figures 3 and 4 respectively. Figure 4 shows that the variance of width estimate is also increased due to phase jitter. In each case the normalized weather spectral width has been varied to illustrate its effect upon the estimation errors.

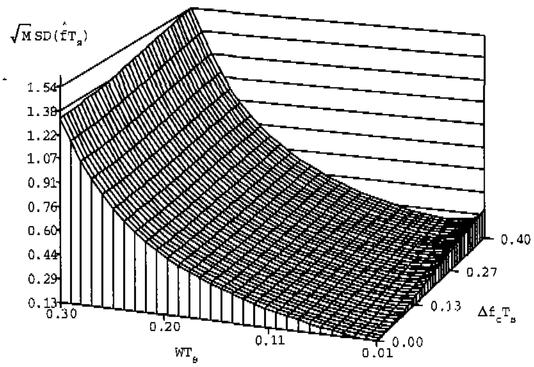

Fig. 2 Error standard deviation of the spectrum mean pulse pair estimate considering STALO phase jitter for σ=0.3 and S/N=0 dB

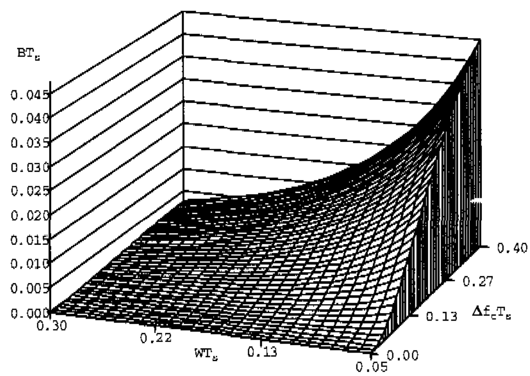

Fig. 3 Bias of the spectrum width pulse pair estimate considering STALO phase jitter for σ=0.3

As is seen in Figure 3, the bias in the width estimation is more sensitive to increasing phase jitter when the true normalized spectral width is small. This variation in sensitivity is less evident when considering the r.m.s. error in the width estimate in Figure 4. It should be noted that in each of Figures 2, 3 and 4 the curve for f_c=0 (no phase jitter) has been previously published[7]

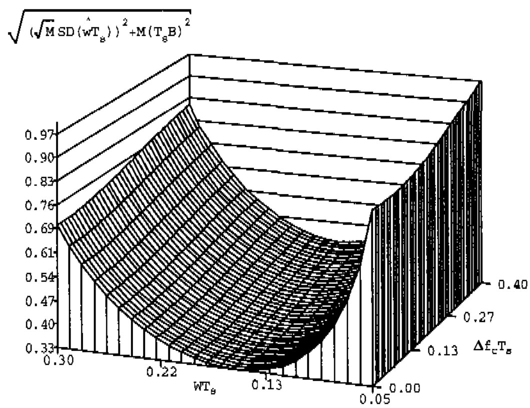

Fig. 4 R.M.S. error of the spectrum width pulse pair estimate considering STALO phase jitter for σ =0.3 and S/N=0 dB

V. Conclusions

Since the quality of spectrum estimates from the widely used pulse pair method is very important for the analysis of weather information, this paper has developed a procedure for specifying the phase stability of a Doppler radar system through definition of the relationship between pulse to pulse phase error (phase jitter) and weather spectrum parameter estimates. As seen from the results, The effect of phase jitter can seriously degrade the performance of a Doppler weather radar due to pulse pair mean and width estimation errors. Therefore, this paper provides a method to analyze and quantify this phase jitter in terms of the phase noise power and broadness.

The further research may include the phase jitter effect on the clutter cancellation capability since it can spread the clutter return resulting the increased difficulty in differentiating the weather information from the other clutter returns.

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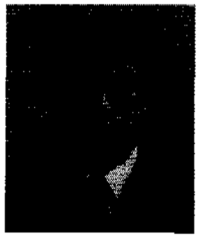

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