Performance Analysis of a Finite-Buffer Discrete-Time Queueing System with Fixed-Sized Bulk-service

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ABSTRACT

We consider a finite-buffer discrete-time queueing system with fixed-size bulk-service discipline: $Geo/G^B/1/K+B$. The main purpose of this paper is to present a performance analysis of this system that has a wide range of applications in Asynchronous Transfer Mode (ATM) and other related telecommunication systems. For this purpose, we first derive the departure-epoch probabilities based on the embedded Markov chain method. Next, based on simple rate in and rate out argument, we present stable relationships for the steady-state probabilities of the queue length at different epochs: departure, random, and arrival. Finally, based on these relationships, we present various useful performance measures of interest such as the moments of number of packets in the system at three different epochs and the loss probability. The numerical results are presented for a deterministic service-time distribution - a case that has gained importance in recent years.

Key Words: Discrete-time queue; Finite buffer; Fixed-size bulk service; ATM network

I. Introduction

Recently, B-IDSN (Broadband Integrated Services Digital Network) based on the ATM (Asynchronous Transfer Mode) technology has received considerable attention due to its capability of providing a common interface for multimedia service including video, data, and voice. The ATM is a multiplexing and switching technology that consists of transferring information units through the network in fixed size units, called cells, each containing 48 bytes of user information and 5 bytes of header (see references [10, 15], for examples). In ATM statistical multiplexer, all event, such as arrivals and departures of packets, are allowed only at regularly spaced points in time. Modeling and performance issues in this system necessitate *discrete-time queueing models.*

In recent years, there has been a growing interest in the analysis of discrete-time queueing models due to their numerous applications in the performance analysis of ATM and various related telecommunication systems. Readers are referred to Bruneel and Kim [2] and takagi [19] Tran Gia et al. [20], Miyazawa and Takagi [14] and Perros et al. [16] for extensive treatments on various types of discrete-time queueing models and their applications to various telecommunication systems.

In many modern digital telecommunication systems, it is frequently observed that packets are served in groups. For example, consider an ATM multiplexer where superposition of numerous individual packets arrives, is stored, and where a fixed limited number of packets are transmitted during a slot. This system can adequately be modeled by *discrete-time queueing model with*

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bulk service. We adopt the Bernoulli process for modeling the arrival process of this system for the following reasons: (i) A Bernoulli arrival process describes the superposition of numerous individual packet arrivals sufficiently well as long as the none of the individual precesses dominates and self-similarity effects are not present (see Section 2.3 in Kuehn [10], for details). (ii) Queues with Bernoulli arrival process are analytically and computationally tractable. We choose the *fixed-size bulk-service policy* for modeling the service mechanism of the above system because it unifies both single service and bulk service discipline. Finally, we assume that the service times (or transmission time) of groups of packets are generally distributed because this assumption leads to a great flexibility in modeling the real transmission times of above system. Besides applications in ATM multiplexer and related telecommunication systems, discrete-time bulk service queueing models have applications in various other areas such as traffic, transportation, manufacturing and other related systems. The readers are referred to Powell [17], Powell and Humblet [18] and Dshalalow [8, pp.243-244], [7, pp.61-116] and the references therein for an extensive list of references and applications on bulk-service models.

In reality, most ATM multiplexers have finite buffer. In this type of finite-buffer queues, one of the main concerns of a system designer is to provide sufficient buffer so that the loss probability of packets is minimized. To this end, it is crucial for a system designer to calculate the loss probability accurately. Also it is widely recognized that the results of the infinite-buffer queueing models can be obtained form the corresponding finite-buffer queueing models by taking the finite-buffer parameter sufficiently large. In view of this, we assume that the buffer size of our system is finite.

In this paper, we consider *a finite-buffer discrete-time queueing model with fixed-size bulk-service discipline*: $Geo/G^B/1/K+B$. Readers are referred to Section II for more details. One special case of our model is $Geo/D^B/1/K+B$, where each arriving packet requires a deterministic service time that could be greater than one slot. This queueing model has gained importance in recent years in view of a number of practical applications in telecommunication systems, such as ATM switching elements, circuit-switched TDMA systems and traffic concentrators (see Bruneel and Wuyts [3] for details on the applications of deterministic service time queueing models). The main purpose of this article is to discuss both analytically and computationally $Geo/G^B/1/K+B$ queueing model that has applications in ATM multiplexer and various other related telecommunication systems as mentioned above. For this purpose, we first derive the departure-epoch probabilities based on the embedded Markov chain method. Next, based on simple rate-in and rate-out argument, we present relationships for the steady-state probabilities of the queue length at different epochs: departure, random, and arrival. Finally, based on the relationships, we present various useful performance measures of interest such as moments of number in the system at three different epochs and the loss probability. A variety of numerical results have been obtained for several service time distributions, but only a deterministic service time distribution, a case that has gained importance in recent years, is presented.

In discrete-time queueing models, the time axis is divided into a sequence of fixed-length intervals of unit duration, called *slots*. It is always assumed that interarrival and service times are integer multiples of unit duration (without loss of generality, unit duration is assumed to be unit time). The state of the system can change only at a slot boundary $n=0,1,2,\cdots$. Under this setting, it should be remarked that an arrival and a service completion can take place simultaneously at a slot boundary. Therefore, it is required to make an assumption regarding the oder of these simultaneous events, and there have been two typical assumptions: either an arrival is assumed to have precedence over such a service completion or vice versa. The former is referred to as *Late Arrival System with Delayed Access* (LAS-DA [2, 19]) and the latter as *Early Arrival System* (EAS [9]). Under LAS-DA/EAS, an arrival/a service completion can occur in the interval $(n-, n)$ while a service completion/an arrival can occur in the interval $(n, n+)$, where $n-$ and $n+$ represent the time points immediately before and after the slot boundary n, respectively. It is widely recognized that telecommunication system is better fitted by LAS (see Bruneel and Kim [2, p.2] and Takagi [19] for more details). In view of this, we adopt the LAS-DA.

There have been many studies that dealt with analytic and computational aspects of various types of discrete-time queueing models. Researches on this topic can be found in references [2-3, 6, 9, 11-12, 20], to name a few. Excellent overview on various types of discrete-time queues can be found in Bruneel and Kim [2] and Takagi [19] and the references therein.

The rest of the paper is organized as follows: Section Ⅱ presents the assumptions of the $Geo/G^B/1/K+B$ queue. Section Ⅲ derives the steady-state probabilities of the queue length at a departure-epoch. In Section Ⅳ we derive the relationships among departure-, random- and arrival-epoch probabilities. A sample numerical example and some comments are provided in Section Ⅴ. Finally, Section Ⅵ concludes this paper, followed by References.

Ⅱ. Model and Assumptions: $Geo/G^B/1/K+B$

We consider a finite-buffer discrete-time queueing system with fixed-size bulk-service discipline: $Geo/G^B/1/K+B$. We adopt *LAS-DA* where packets arrive late during a slot and get delayed access to the server if they arrive to find the system empty. Readers are referred to Bruneel and Kim [2, p.2], Hunter [6] and Takagi [19, p.4] for more details on LAS-DA.

This paper considers the system that satisfies the following assumptions.

Assumptions

Packets arrive at the system according to a Bernoulli process with mean interarrival time $1/\lambda$. The service times of groups of packets (of size B) are independent and identically distributed (iid) random variables in accordance with a general probability mass function s_k, $k=1,2,\cdots$, finite mean $E(S)$, and PGF $S(z)$. The packets are served on a first-come, first-served basis by a single server in groups of fixed-size B $(B\geq 1)$. If the number of packets in the system at the beginning of a service is less than B, the server waits till the number of customers in the system becomes B, and then restarts service. The service times of groups of packets are independent of the arrival process and the numbers served. The buffer has finite capacity K, so that not more than $K+B$ can be present in the system at any time, K customers waiting for service and B customers being served. We assume $B\leq K$. We define $\rho=\lambda E(S)/B$ as the offered load of the system.

Ⅲ. The Departure-epoch Probabilities

In this section, we derive a set of equations for the steady-state probabilities of the queue length at an arbitrary departure-epoch of a service group (of size B) in the $Geo/G^B/1/K+B$ queueing model.

We adopt the departure-time embedded Markov chain method to derive a set of equations for the departure-epoch probabilities. Define N_n^+ be the number of customers in the queue right after a completion of n^{th} service (of a group of size B). Let us define

$$P_k^+ \equiv \lim_{n\to\infty} P(N_n^+ = k),\ \ 0 \le k \le K.$$

If we denote k_j be the probability that the number of customers arriving during a service time (of a group) is equal to j, then it is given by

$$k_j = \sum_{k=1}^{\infty} P(A=j \mid service\ time=k)\, s_k = \sum_{k=1}^{\infty} \binom{k}{j} \lambda^j (1-\lambda)^{k-j} s_k,\ \ j=0,1,2,\cdots.$$

where A denotes the number of customers arriving during a service time (of a group).

Now it can be seen that $\{N_n^+\}$ is a discrete-time Markov chain. Let $p_{ij} \equiv P(N_{n+1}^+ = j \mid N_n^+ = i)$. Then, it can be shown that $P_j^+ = \sum_{i=0}^{K} P_i^+ p_{ij}$ can be written as follows:

For $j=0$,

$$P_0^+ = P_0^+ \cdot k_0 + P_1^+ \cdot k_0 + P_2^+ \cdot k_0 + \cdots + P_{B-1}^+ \cdot k_0 + P_B^+ \cdot k_0. \tag{1}$$

For $j=1,2,\cdots,K-1$,

$$\begin{aligned} P_j^+ &= P_0^+ \cdot k_j + P_1^+ \cdot k_j \\ &+ \cdots + P_{B-1}^+ \cdot k_j + P_B^+ \cdot k_j \\ &+ P_{B+1}^+ \cdot k_{j-1} + \cdots + P_{K-1}^+ \cdot k_{j-(K-1-B)} \\ &+ P_K^+ \cdot k_{j-(K-B)}. \end{aligned} \tag{2}$$

For $j=K$,

$$\begin{aligned} P_K^+ &= P_1^+ \cdot \left[\sum_{v=K}^{\infty} k_v\right] + P_2^+ \cdot \left[\sum_{v=K}^{\infty} k_v\right] + \cdots \\ &+ P_{B-1}^+ \cdot \left[\sum_{v=K}^{\infty} k_v\right] + P_B^+ \cdot \left[\sum_{v=K}^{\infty} k_v\right] \\ &+ P_{B+1}^+ \cdot \left[\sum_{v=K-1}^{\infty} k_v\right] + \cdots \\ &+ P_{K-1}^+ \cdot \left[\sum_{v=B+1}^{\infty} k_v\right] + P_K^+ \cdot \left[\sum_{v=B}^{\infty} k_v\right]. \end{aligned} \tag{3}$$

(Note: $P_j^+ = 0$, if $j<0$ and $k_j = 0$, if $j<0$ in (1), (2) and (3).)

It can be verified that equation (3) is redundant and will not be considered henceforth. Using the software packages such as Mathematica or even otherwise, we can solve (1), (2) and an equation for normalization condition ($\sum_{j=0}^{K} P_j^+ = 1$) simultaneously to compute departure-epoch probabilities.

The evaluation of k_j's for various service time distributions in (1) and (2)

The PGF of k_j, $K(z)$, is given by (see Takagi [19, p.5], for example)

$$K(z) = S(1-\lambda+\lambda z),$$

where $K(z)$ denotes the number of packets that arrive during the service time of a group of packets (of side B). We can invert the PGF $K(z)$ using Mathematica by expanding a polynomial depending on a particular service time distribution to get the probabilities $\{k_j\}$. For more details, see Section Ⅴ. The expressions for $K(z)$ of some service time distributions are as follows:

(1) Geometric

When the service time distribution is geometric with parameter μ, i.e. $s_n = (1-\mu)^{n-1}\mu$, we have

$$K(z) = \frac{\mu u}{1-(1-\mu)u} \Big|_{u=1-\lambda+\lambda z}.$$

(2) Negative-binomial

Here, the service time is convolution of k geometric distributions. In this case, we have

$$K(z) = \left(\frac{\mu u}{1-(1-\mu)u}\right)^k \Big|_{u=1-\lambda+\lambda z}.$$

(3) Deterministic

In this case, the service time has a constant value d, so we have

$$K(z) = u^d \Big|_{u=1-\lambda+\lambda z}.$$

(4) Mixture of M geometric

In this case, the service time is a mixture of M geometric distributions such that

$$s_n = \sum_{l=1}^{M} \sigma_l (1-\mu_l)^{n-1}\mu_l, \qquad n=1,2,\cdots,$$

$\sum_{l=1}^{M} \sigma_l = 1.$

From this we get

$$K(z) = \left\{ \sum_{l=1}^{M} \sigma_l \frac{\mu_l u}{1-(1-\mu_l)u} \right\} \Big|_{u=1-\lambda+\lambda z}.$$

No table showing numerical work for (1), (2) and (4) cases is attached, though the numerical work can be done similarly. Note that all the above functions can be easily inverted using Mathematica or some other software packages.

Once we obtain departure-epoch probabilities, the performance measures such as the mean and the variance of number of packets in the system at a departure-epoch can be obtained and are given, respectively, by $L^+ = \sum_{n=0}^{K} nP_n^+$ and $Var^+ = \sum_{n=0}^{K} n^2 P_n^+ - L^{+2}$, where L^+ and Var^+, respectively denote the mean and the variance of number of packets in the queue at a departure-epoch.

Ⅳ. Relationships between Random-epoch and Departure epoch Probabilities

In this section, we derive stable relationships between random-epoch and departure-epoch probabilities.

Let $\{P_j^-,\ j=0,1,\cdots,K+B-1,K+B\}$, $\{P_j,\ j=0,1,\cdots,K+B-1,K+B\}$, and $\{P_j^+,\ j=0,1,\cdots,K-1,K\}$ be the arrival-epoch, random-epoch and departure-epoch probabilities, respectively.

Without loss of generality, we assume that packets within each departing group (of size B) are randomly ordered and that they leave the system one by one instantaneously according to their order. Under this assumption, let $\{P_j^D,\ j=0,1,\cdots,K+B-2,K+B-1\}$ be the probability that an accepted individual packet leaves behind j packets in the system (including served packets, if any, who follow it) just after its departure. Note that $P_{K+B}^D=0$ by definition.

It is well-known that in models where the arrivals follow a Bernoulli process, the arrival-epoch probabilities $\{P_j^-,\ j=0,1,\cdots,K+B-1,K+B\}$ is identical to the random-epoch probabilities $\{P_j,\ j=0,1,\cdots,K+B-1,K+B\}$ (See Boxma and Groenenndijk [1] for details). Consequently, the results in this section apply to both random-epoch and arrival-epoch probabilities.

Lemma 1. *The steady-state probabilities* $\{P_j^-\}$ and $\{P_j^D\}$ *are related by*

$$P_j^- = (1-P_{K+B}^-)P_j^D,\quad j=1,2,\cdots,K+B-1.$$
(4)

Proof: If we only consider the situation where the system is not fully occupied, the probability distribution $\{P_j^-\}$ for the number of packets in the system immediately before an arrival is proportional to the probability distribution $\{P_j^D\}$ for the number of packets in the system immediately after a departure of a packet. From this, we get

$$P_j^- = cP_j^D,\quad j=1,2,\cdots,K+B-1. \qquad (5)$$

where c is a proportionality constant which can be determined as follows: Summing (5) over for $j=1,2,\cdots,K+B-2,K+B-1$, we get

$$c = 1-P_{K+B}^-. \qquad (6)$$

Combining (5) and (6) gives (4). ■

Next, we characterize the relationships between $\{P_j^D,\ j=0,1,\cdots,K+B-2,K+B-1\}$ and $\{P_j^+,\ j=0,1,\cdots,K-1,K\}$ using the rate-in and rate-out argument in the following Lemma 2.

Lemma 2. *The steady-state Probabilities* $\{P_j^+\}$ *and* $\{P_j^D\}$ *are related by*

$$P_j^D = \frac{1}{B}\sum_{i=0}^{B-1} P_{j-i}^+, \quad j=0,1,2,\cdots,K+B-1.$$
(7)

Proof: We view a departure in two respects: a departure of group (of size B), and a departure of a packet. First, we consider the transition rate into the state j by the departures of individual packets. As stated in Section II, λ represents the mean arrival rate of individual packets. Then it can be easily seen that $\lambda(1-P_{K+B}^-)$ is the effective input rate of individual packets. Note that in steady-state, $\lambda(1-P_{K+B}^-)$ is also equal to the effective output rate of individual packets (i.e. the expected number of individual packet departures per slot). Note further that the probabilities $\{P_j^D\}$ can be interpreted as the long-run fraction of individual packet departures that leave behind the system with state j. Then, it is clear that the transition rate into the state j (from the state $j+1$) by the departures of individual packets is given by $\lambda(1-P_{K+B}^-)P_j^D$. Second, we consider the transition rate into the state j by the departures of groups (of size B). Since $\lambda(1-P_{K+B}^-)$ is equal to the mean departure rate of individual packets, it can be seen that $\lambda(1-P_{K+B}^-)/B$ is the mean departure rate of groups (of size B). By the similar reasoning as above, the transition rate into the state j from the state $j+k$, $k=1,2,\cdots,B-1,B$, by the departures of groups is given by $\frac{\lambda(1-P_{K+B}^-)}{B}\sum_{k=1}^{B} P_{j+k-B}^+$. In summary, we have

Transition rate into state j by departures of individual packets = $\lambda(1-P_{K+B}^-)P_j^D$. (8)

Transition rate into state j by departures of groups = $\frac{\lambda(1-P_{K+B}^-)}{B}\sum_{k=1}^{B} P_{j+k-B}^+$. (9)

(Note: $P_j^D=0$ if $j<0$ in (8), and $P_j^+=0$, if $j<0$ in (9).) Equating (8) and (9), and algebraic simplification gives (7). ■

Theorem 1. *The steady-state probabilities $\{P_j^-\}$ and $\{P_j^+\}$ are related by*

$$P_j^- = \frac{1}{\lambda E(S)-Q+B}\sum_{i=j-B+1}^{j} P_i^+,$$
$$j=0,1,\cdots,K+B-1, \tag{10}$$

and $$P_{K+B}^- = \frac{\lambda E(S)-Q}{\lambda E(S)-Q+B}, \tag{11}$$

where Q is as defined in (15).

Proof: Combining Lemmas 1 and 2, we get

$$P_j^- = \frac{1}{B}(1-P_{K+B}^-)\sum_{i=0}^{B-1} P_{j-i}^+,$$
$$j=0,1,\cdots,K+B-1. \tag{12}$$

Using the argument that, in steady state, the effective input rate of individual packets is equal to the effective output rate of individual packets, we get

$$\lambda(1-P_{K+B}^-) = P(server\ is\ busy)\frac{B}{E(S)}$$
$$= \sum_{j=B}^{K+B} P_j^- \frac{B}{E(S)}. \tag{13}$$

Substituting (12) into (13), we have

$$\lambda(1-P_{K+B}^-) = P(server\ is\ busy)\frac{B}{E(S)}$$
$$= (1-P_{K+B}^-)\sum_{j=B}^{K+B-1}\sum_{i=0}^{B-1} P_{j-i}^+ \frac{1}{E(S)} \tag{14}$$
$$+ \frac{B}{E(S)} P_{K+B}^-.$$

Let $Q = \sum_{j=B}^{K+B-1}\sum_{i=0}^{B-1} P_{j-i}^+ = \sum_{j=1}^{B-1} jP_j^+ + \sum_{j=B}^{K} BP_j^+$. (15)

Combining (14) and (15), we get

$$P_{K+B}^- = \frac{\lambda E(S)-Q}{\lambda E(S)-Q+B}. \tag{16}$$

Substituting (16) into (12) and algebraic simplifications give (10). This completes the proof. ■

Remark 1. Once we obtain $\{P_j^+\}$ probabilities from Section III, we can also obtain $\{P_j^-\}$ and $\{P_j\}$ probabilities using Theorem 1.

Remark 2. Since the right-hand sides of the above relationships are nonnegative, the computations produce stable results.

Remark 3. The derivation of Theorem 1 shows that the relationships hold for the continuous-time $M/G^B/1/K+B$ queue.

Once we obtain departure-epoch probabilities $\{P_j^+\}$, we can also obtain random-epoch probabilities $\{P_j\}$ (or arrival-epoch probabilities $\{P_j^-\}$) by using Theorem 1. We can also get useful performance measures such as the mean and the variance of the number in the system at a random-epoch (or at an arrival-epoch), and the loss probability P^* as follows:

$L=\sum_{k=1}^{K+B} kP_{k,1}$, $Var=\sum_{k=1}^{K+B} k^2 P_{k,1}-L^2$ and $P^*=P^-_{K+B}(=P_{K+B})$, where L, Var and P^*, respectively, denote the mean number of packets in the queue at a random-epoch, the variance of number of packets in the queue at a random-epoch, and the loss probability.

V. Sample Numerical Example

In this section, we provide one sample example to explain the procedure developed in the previous sections. Consider an ATM multiplexer where superpositions of numerous individual packets arrive, and stored, and where only five packets can be served during a transmission time. For this system, it is realistic to assume that the transmission time of groups is deterministically distributed (see Bruneel and Kim [2], for example). Suppose that the transmission time of groups is 5 and buffer size is 10. Then, this system can be adequately modeled by discrete-time $Geo/D^5/1/10+5$ queue. We consider $Geo/D^5/1/10+5$, $\lambda=1/30$, $E(S)=15$. In this case, $S(z)$ and $K(z)$ are given, respectively, by

$$S(z)=z^5,\ K(z)=\left\{\frac{1}{30}z+\frac{29}{30}\right\}^5, \quad (17)$$

To solve (1), (2) and an equation for normalization condition ($\sum_{j=0}^{K} P_j^+=1$) simultaneously, we, first, have to obtain the distribution $\{k_j\}$ contained (1) and (2). Inverting the PGF $K(z)$ given in (17) using Mathematica, we get <Table 1>.

<Table 1> The distribution $\{k_j\}$ for $\lambda=1/30$, $E(S)=15$, $\rho=\lambda E(S)/B=0.100000$.

$\{k_j\}$	ρ=0.100000
k_0	0.844080
k_1	0.145531
k_2	0.010037
k_3	0.000346
k_4	0.000005
k_5	0.000000
k_6	0.000000
k_7	0.000000
k_8	0.000000
k_9	0.000000
k_{10}	0.000000

Note that we only present k_j, for $0\le j\le 10$, which are needed to solve departure-epoch probabilities (see equations (1) and (2) in Section III, for details). Using <Table 1>, we solve equations (1), (2), and the equation for the normalization condition simultaneously to compute departure-epoch probabilities for $\lambda=0.1$ and $\rho=\lambda E(S)/B=0.1$ (see <Table A1> in Appendix). Finally, using Theorem 1, we compute random-epoch probabilities (or arrival-epoch probabilities) and loss probabilities. For details, see <Table A2> in Appendix.

Ⅵ. Conclusions

We have successfully discussed performance analysis of $Geo/G^B/1/K+B$ queueing model that has a wide range of applications in ATM multiplexer and various other related telecommunication systems. We present steady-state probabilities and moments of the number of customers in the system at three different epochs: departure, random and arrival. Various useful performance measures such as the loss probability and the moments of the queue length at three different epochs have also been obtained both analytically and computationally. Further, we present stable relationships for the steady-state probabilities of the queue length at different epochs: departure, random, and arrival. Based on the results of this paper, further researches are required to extend the approach presented in this paper to more general queueing models, such as bulk-service queue with variable server capacity and bulk queues with vacations. We're currently working on this direction [4-5].

The results and approach presented in this paper would be beneficial not only to researchers who study related research topics but also to practitioners who try to evaluate the performance of their systems.

Appendix. Sample Numerical Results

Extensive numerical work has been carried out for the model under discussion. Though many tables have been produced, only a few are presented here. The selection has been made in such a way that by looking at them one gets a feel and appreciation of the general applicability of the numerical results. In view of this, attaching only three tables for some specific example queue is satisfactory.

All the computations (such as probabilities, means and variances) were carried out in double precision, but the results are presented after rounding up after the sixth decimal point. It can be easily noticed that the loss probability and mean number of packets in the system at a departure-epoch, a random-epoch and an arrival-epoch increase as the offered load (traffic intensity) increases. This matches well with our intuitive prediction.

<Table A1> Departure-epoch probabilities of $Geo/D^5/1/10+5$ where $\rho=\lambda E(S)/B$, $B=5$ and $E(S)=15$.

	ρ=0.1	ρ=1.0	ρ=2.0
P_0^+	0.844080	0.131687	0.000034
P_1^+	0.145531	0.329218	0.000386
P_2^+	0.010037	0.329218	0.001939
P_3^+	0.000346	0.164609	0.005788
P_4^+	0.000005	0.041153	0.012309
P_5^+	0.000000	0.004115	0.022307
P_6^+	0.000000	0.000000	0.039274
P_7^+	0.000000	0.000000	0.069615
P_8^+	0.000000	0.000000	0.123608
P_9^+	0.000000	0.000000	0.202492
P_{10}^+	0.000000	0.000000	0.522246
Sum	1.000000	1.000000	1.000000
Mean	0.166667	1.666670	8.939116
Variance	0.161111	1.111111	2.269537
Q	0.166667	1.666670	4.968586

<Table A2> Arrival-epoch (=Random-epoch) probabilities of $Geo/D^5/1/10+5$ **where** $\rho=\frac{\lambda E(S)}{B}$, $B=5$ and $E(S)=15$.

	ρ=0.100000	ρ=1.000000	ρ=2.000000
P_0^-	0.158265	0.015803	0.000003
P_1^-	0.185552	0.055309	0.000042
P_2^-	0.187434	0.094815	0.000235
P_3^-	0.187499	0.114568	0.000812
P_4^-	0.187500	0.119506	0.002039
P_5^-	0.029235	0.104198	0.004260
P_6^-	0.001948	0.064691	0.008136
P_7^-	0.000066	0.025185	0.014883
P_8^-	0.000001	0.005432	0.026628
P_9^-	0.000000	0.000494	0.045587
P_{10}^-	0.000000	0.000000	0.095424
P_{11}^-	0.000000	0.000000	0.091509
P_{12}^-	0.000000	0.000000	0.084569
P_{13}^-	0.000000	0.000000	0.072247
P_{14}^-	0.000000	0.000000	0.052061
P_{15}^- (Loss prob.)	0.062500	0.400000	0.501566
Sum	1.000000	1.000000	1.000000
Mean	2.968750	8.200000	12.97592
Variance	11.675928	32.693333	6.250737

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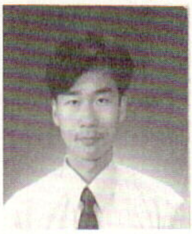

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