Erlang and Channel Capacity of Truncated Power Controlled CDMA Cellular Systems with Base Station Antenna Arrays

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ABSTRACT

We analyze the performance of a truncated power controlled CDMA(code division multiple access) cellular systems with base station antenna arrays. Erlang capacity and the channel capacity which is a maximum date rate to maintain almost error free communication are analytically derived. The numerical results show there can be a substantial increase in Erlang capacity and in channel capacity by antenna arrays incorporating with the truncated power control scheme.

Keyword : Capacity, Antenna arrays, CDMA cellular system

I . Introduction

In a cellular CDMA system, the near-far problem can be compensatedby the power control techniques. The conventional power control scheme, such as used in IS-95 systems, requires a large average transmit power to compensate for deep fading. The transmit power from mobiles especially at cell boundary increases interferences to adjacent cells since they must transmit very high power to compensate for path loss and shadowing effects. Recently, a truncated power control scheme [1], which compensates for the propagation loss above a predetermined fade depth, while below thethreshold level the transmission is suspended, is proposed to mitigate these undesirable effects; excessive power consumption of a mobile station, and large amount of interference to the adjacent base stations. It was reported the power gain is 1.3 4.5 dB with the truncated power control scheme [1].

On the other hand, one of the emerging technologies for the system capacity enhancement, without increasing in RF spectrum allocation, is the use of a spatial processing technique with cell site antenna arrays [2], [3]. By using an appropriate spatial processing technique at a cell site, we can estimate the array response vectors, and then use optimum directional beams to improve system performance and increase capacity by directing the narrow antenna beam to the desired transmitter direction.

In this paper, for better system performance and larger capacity, a combination of the truncated power control and the adaptive antenna array techniques is investigated. We first derive the performance of the proposed system in terms of the average probability of outage based on the average SIR (signal-to-interference ratio) and the transmit probability, which is the probability of the received signal strength above the predefined threshold level. Erlang capacity, which is a function of the traffic intensity and the transmit probability of the truncated power control scheme, is also compared to the capacity of the system using a conventional power control scheme. And the channel capacity with antenna array is derived in fading channels and compared with the Shannon capacity in AWGN.

This paper is organized as follows. In Section II, we describe the system and channel models with the

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This work was supported by 2003 Chongju University research grant (general project).
논문번호: 030028-0120, 접수일자 : 2003년1월 22일

truncated power control scheme. The reverse link outage probability is analyzed as a function of the number of the array elements, the transmit probability, Erlang, and outer cell interference fraction in Section III, as well as the channel capacity derived in fading channels. The analytical results and conclusions are shown in Section IV and in Section V, respectively.

II. System and channel models

A transmitted signal from a mobile station experiences various effects such as propagation loss, shadowing and fast fading, through a wireless channel. The propagation loss in a cellular system is generally accepted as proportional to the mth power of the distance between a transmitter and a receiver, where m is the path-loss exponent which is between 2 to 5.5 depending on the propagation environment [4]. Also, the shadowing caused by terrain irregularity is modeled by a log-normal probability density function. Then, the propagation gain, which is the inverse of the propagation loss, between a mobile and a base station can be written by

$$G(r,x) = r^{-m}10^{x/10} \qquad (1)$$

where r is the normalized distance between the mobile station and the base station, $r = R/D$. Where R denotes the distance between a mobile and a base station, and D is the cell radius. x is a zero mean Gaussian random variable.

In a cellular CDMA system, one of the essential prerequisites is the power control to prevent the near-far problems. In conventional power control policy, the transmitted power from a mobile station is adjusted to compensate the propagation gain $G(r,x)$ to maintain the constant received power at the base station. In [1], a truncated power control scheme is proposed to mitigate the large interference and power consumption in a mobile station; the mobile station transmits its power when the average received signal strength exceeds predetermined transmit threshold γ_0 and does not transmit when the average received strength is under the threshold. Then, we may express the truncated transmit power as

$$S_k(t) = \begin{cases} S_R(t)/G_k(r,x), & G_k(r,x) \ge \gamma_0 \\ 0, & G_k(r,x) < \gamma_0 \end{cases} \qquad (2)$$

where $S_k(t)$ is the transmit power [Watt] of a kth mobile station and $G_k(r,x)$ is the propagation gain between a kth mobile and a base station. $S_R(t)$ is the desired received power [Watt] at the base station when $G_k(r,x) \ge \gamma_0$. For the notational convenience, we use the S_k, G_k, and S_R instead of $S_k(t)$, $G_k(r,x)$, and $S_R(t)$, respectively. We assume that S_k, G_k, and S_R remain constant during a symbol interval.

The transmit probability can be obtained by [1]

$$p(\gamma_0) = P(G_k > \gamma_0) = \int_0^1 Q\left(\frac{10\log(\gamma_0 r^m)}{\sigma_c}\right)2r\,dr \qquad (3)$$

where σ_c denotes the power control error when the mobile transmit. And $Q(x)$ is given by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-\frac{x^2}{2})dx \qquad (4)$$

We assume that the random variable r and x are independent, and that the mobile's location is uniformly distributed within a cell. This leads to a probability density function of r given by

$$P(r) = \begin{cases} 2r, & 0 \le r \le 1 \\ 0, & r > 1 \end{cases} \qquad (5)$$

Fig. 1 shows the transmit probability of a mobile station with the power control error. We noticed that the transmit probability decreases with the transmit threshold γ_0 and with the power control error.

The direct sequence code division multiple access (DS-CDMA) reverse link adaptive array receiver model is shown in Fig. 2. The output of an antenna array at the base station can be represented by the $M \times 1$ vector,

$$\mathrm{y}(t) = \sum_{k=1}^{K} \sqrt{2G_k S_k} d_k(t-\tau_k) p_k(t-\tau_k) \cos[2\pi f_c(t-\tau_k)+\theta_k]\mathrm{a}_k + \mathrm{n}(t) \qquad (6)$$

where k is a number of users in a cell. M denotes the number of an array antenna elements. $d_k(t)$ and $p_k(t)$ are the data sequence and spreading sequence of the kth user, respectively. τ_k , f_c, and θ_k represents the path delay, the carrier frequency, and carrier phase for user k, respectively. a_k is the $M \times 1$ array response vector from the kth user. And $\mathrm{n}(t)$ is the zero-mean Gaussian noise vector with covariance

$$E[\mathrm{n}(t)\mathrm{n}^H(\tau)] = \begin{cases} \sigma_n^2 \mathrm{I}, & \mathrm{t} = \tau \\ 0, & \mathrm{t} \ne \tau \end{cases} \qquad (7)$$

where I is the $M \times M$ identity matrix and σ_n^2 is the system noise power.

If the desired user is $k = 1$, the antenna outputs are correlated by the desired user's spreading code

$$z_1 = \sqrt{\frac{2}{T}} \int_{\tau_1}^{\tau_1+T} \mathrm{y}(t) p_1(t-\tau_1) \cos[2\pi f_c(t-\tau_1)+\theta_1]dt$$
$$= s_I \mathrm{a}_1 + I_M \mathrm{a}_k + \boldsymbol{\eta} \qquad (8)$$

where the first term is the signal components, the 2ndterm is the interference components, and the 3rd term is the noise components.

The decision variable after optimal weighting [2] is

$$u = \boldsymbol{a}_I^H \mathbf{z}_1$$
$$= s_1 \mathbf{a}_I^H \mathbf{a}_1 + I_M \boldsymbol{a}_I^H \mathbf{a}_k + \mathbf{a}_I^H \boldsymbol{\eta}$$
$$= M s_1 + I + \mathbf{a}_I^H \boldsymbol{\eta} \qquad (9)$$

where H denotes Hermitian transpose and

$$I = I_M \mathbf{a}_I^H \mathbf{a}_k . \qquad (10)$$

The variance of the interference term in conventional DS CDMA system without antenna array is derived in [5]. If we apply this result to antenna array system, the variance can be written by

$$\mathrm{var}\{I\} = \sum_{k=2}^{K} \frac{G_k S_k T_c}{3} E\{\left|\mathbf{a}_I^H \mathbf{a}_k\right|^2\} \qquad (11)$$

The noise variance is

$$\mathrm{var}\{\boldsymbol{\eta}\} = \left(\sqrt{\frac{2}{T}}\right)^2 E[\int_{\lambda_1}^{\lambda_1+T}\int_{\tau_1}^{\tau_1+T} \mathrm{n}(t)\mathrm{n}^H(t) \\ p_1(t-\tau_1)p_1(t-\lambda_1)\cos[2\pi f_c(t-\tau_1)+\theta_1] \\ \cos[2\pi f_c(t-\lambda_1)+\theta_1]dt] \\ = \sigma_n{}^2\mathbf{I} \tag{12}$$

, therefore

$$\mathrm{var}\{\mathrm{a}_1^H\boldsymbol{\eta}\} = M\sigma_n^2 \quad . \tag{13}$$

The bit energy to interference-plus-noise density ratio can be written by

$$\frac{E_b}{(N_o)_{tot}} = \frac{M^2 G_1 S_1}{\frac{1}{3N}\sum_{k=2}^{K} G_k S_k E\{\left|\mathrm{a}_1^H \mathrm{a}_k\right|^2\} + \frac{M\sigma_n^2}{T}} \tag{14}$$

where N is the processing gain.

III. Outage probability and channel capacity

3.1 Outage probability

Now, we apply the truncated power control

$$X_k = \begin{cases} 1, & G_k \geq \gamma_o \\ 0, & G_k < \gamma_o \end{cases}, \quad 1 \leq k \leq K \quad . \tag{15}$$

Under this truncated power control scheme, (14) can be modified to

$$\frac{E_b}{(N_o)_{tot}} = \frac{X_1 P(X_1=1)}{\frac{1}{3NM^2}\sum_{k=2}^{K} X_k E\{\left|\mathrm{a}_1^H \mathrm{a}_k\right|^2\} + \frac{\sigma_n{}^2}{S_R TM}} \tag{16}$$

where we assumed the received power is perfect power controlled, $G_1 S_1 = G_k S_k = S_R$, when $G_k \geq \gamma_0$.

If we define $(E_b/N_0)_o$ is the $E_b/(N_0)_{tot}$ value required to achieve the level of adequate performance (i.e., for the reverse link to keep the $BER < 10^{-3}$, the required $(E_b/N_0)_o$ is 7 dB [4]), then the outage probability is

$$P_{out} = P\{E_b/(N_0)_{tot} < (E_b/N_0)_o\} \\ = P\{\frac{X_1 P(X_1=1)}{\frac{1}{3NM^2}\sum_{k=2}^{K} X_k E\{\left|\mathrm{a}_1^H \mathrm{a}_k\right|^2\} + \frac{\sigma_n^2}{S_R TM}} < (E_b/N_0)_o\} \\ = P\{\sum_{k=2}^{K} X_k E\{\left|\mathrm{a}_1^H \mathrm{a}_k\right|^2\} > \varsigma\} \tag{17}$$

where,

$$\varsigma = 3NM^2\{\frac{1}{(E_b/N_0)_o} - \frac{\sigma_n^2}{MTS_R}\} \quad . \tag{18}$$

In [6], the expected value of the inner product between two array vectors is given by

$$E\{\left|a_1^H a_k\right|^2\} = \sum_{n=0}^{M-1}\sum_{m=0}^{M-1} J_0^2\{\beta d(n-m)\} \tag{19}$$

where $J_0(\bullet)$ is the 1st kind 0th order Bessel function, $\beta = 2\pi/\lambda$, λ is the wave length, and d denotes the distance between two antenna elements.

If we assume that call arrivals occur randomly at Poisson-distributed intervals with an average rate of λ calls/sec and that the average call duration is $1/\mu$ sec, then K is a Poisson-distributed random variable with the occupancy rate λ/μ. Also the random variable $\sum_{k=1}^{K} X_k$ is a Poisson-distributed random variable with the occupancy rate $\lambda p(\gamma_0)/\mu$ [1], [7]. The outage probability can be obtained by approximating the Poisson by a Gaussian variable with the same mean and variance for large ς [8], [9]

$$P_{out} \approx Q(\frac{\varsigma - E[v]}{\sqrt{\text{var}(v)}}) \tag{20}$$

where

$$v = \sum_{k=2}^{K} X_k E\left\{\left|\mathrm{a}_1^H \mathrm{a}_k\right|^2\right\} \tag{21}$$

and the expectation of v can be written by

$$E[v] = E\left[\sum_{k=2}^{K} X_k E\left\{\left|\mathrm{a}_1^H \mathrm{a}_k\right|^2\right\}\right] = (\lambda/\mu)p(\gamma_0)(1+f)E\{\left|a_1^H a_k\right|^2\}e^{(\beta\sigma_c)^2/2} \tag{22}$$

The variance of v can be obtained by

$$\text{var}(v) = (\lambda/\mu)p(\gamma_0)(1+f)E\{\left|a_1^H a_k\right|^2\}e^{2(\beta\sigma_c)^2} \tag{23}$$

where f is the other cell interference rise [10], and σ_c is the power control error in dB. For further details of this derivation, see [9]. The Erlang capacity of a CDMA cellular system is the maximum number of usersthat does not exceed a given value of outage probability of error in (17) [9].

3.2 Channel capacity

It is generally accepted the maximum channel capacity under AWGN is given by Shannon's capacity. However, when the channel receives the fading, the maximum channel capacity is reduced.

The Shannon's capacity is given by

$$C = B\log_2(1+SNR) \tag{24}$$

where C denotes the maximum capacity in bit/sec, SNR is the signal-to-noise ratio. However, when the received SNR is experienced by the fading, the channel capacity is modified to

$$\overline{C} = B\int \log_2(1+SNR)f(SNR)d(SNR) \tag{25}$$

where $\overline{C}$ is the average channel capacity and $f(SNR)$ is the probability density function of SNR. In DS-CDMA cellular system, the despreaded SNR is equal to the E_b/N_0. Therefore the average channel capacity is obtained by substituting (16) in (22). An empirical data shows the received $E_b/(N_0)_{tot}$ has a log-normal density function [9]. Therefore the average capacity can be written by

$$\overline{C} = B\int_{-\infty}^{\infty}\log_2(1+\frac{X_1 P(X_1=1)}{\frac{1}{3NM^2}\sum_{k=2}^{K} X_k E\left\{\left|\mathbf{a}_1^H \mathbf{a}_k\right|^2\right\} + \frac{\sigma_n^2}{S_R TM}}) \times \frac{1}{\sqrt{2\pi}\sigma_c}\exp(-\frac{(x-\mu)^2}{2\sigma_c^2})dx \tag{26}$$

where μ is the mean of $E_b/(N_0)_{tot}$ and x is the *SNR* in dB.

Ⅳ. Numerical results

For the numerical analysis we assume followings; processing gain N=128, $(E_b/N_0)_o = 7\,dB$, $S_R T/\sigma_n^2 = 30\,dB$, other cellinterference rise $f = 0.55$, path-loss exponent m=4, shadowing standard deviation $\sigma_s = 8\,dB$, and the spacing between antenna elements of the linear antenna array is half-wavelength.

Fig.3 shows the Erlang capacity as a function of the transmit threshold γ_0 with the array antenna elements M=4. It is noticed when the transmit threshold γ_0 becomes larger, the transmit probability decreases and the MAI is decreased, but the Erlang capacity increases.

For the capacity calculation we adapt the outage probability of 1×10^{-3} in (20). The capacity is increased with the transmit threshold. An increase of the transmit threshold means a decrease of thetransmit probability. It is also noticed that the power control error is sensitive to the traffic intensity. When the power control error σ_c is 0, 2, 4 dB with the transmit threshold γ_0=1, the capacity increases 4.4, 5.0, and 6.6 times, respectively, compared to the case of the single receiving antenna at base station ; under the same condition, the capacity with single receive antenna at base station is 35, 29, 17 Erlangs, respectively. If the power control is perfect, the condition of γ_0=1 means the mobile at cell edge always transmit. However, in the case of imperfect power control, the transmission from a mobile is governed by truncated power control in (2). It is also noticed that the Erlang capacity increases with the transmit threshold which means less transmit probability as shown in Fig.2. Decrease of the transmit means decrease of MAI and increase of the system capacity.

The transmit outage probability is shown in Fig. 4 with power control error $\sigma_c = 2.5$ dB and $\gamma_0 = 1$. Analytical results show there can be a substantial increase in Erlang capacity by antenna arrays at the base station incorporating truncated power control scheme. For the transmit outage probability of 1×10^{-3}, the capacity is increased to 4.7, 9.1 times for M=4 and M=8, respectively, compared to the capacity of M=1.

The normalized channel capacity [bits/Hz] is shown in Fig. 5. The curve with γ_0=0, which means no truncated power control case, denotes the normalized Shannon capacitywith M=1 in AWGN. The other curves show the truncated power control cases with the power control error of 2.5 dB and M=4. It is noticed the channel capacity is increased with the transmit threshold γ_0. When γ_0 is 0, 1, and 10 with 150 Eralngs, the normalized channel capacity is 1.8, 3.4, and 4.7, respectively. As we expected, an increase of the transmit threshold reduces the MAI, and increases the channel capacity.

In (23), we can easily expect that the channel capacity increases with the number of antenna elements M.

Ⅴ. Conclusions

We considered the truncated power control conjunction with an antenna array system. The analytical results show there can be a substantial increase in system capacity by antenna arrays at the base station incorporating truncated power control scheme.

We also noticed the channel capacity increases with the transmit threshold and the number of array antenna elements. This result can be used to the capacity design of a cellularsystem with base station antenna arrays.

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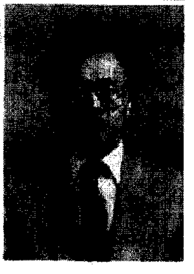

He received the MS and Ph.D. degrees in electronics engineering from the Younsei University, Seoul,Korea, in 1983 and 1991, respectively. From 1986 to 1994, he was with the Electronics and Telecommunications Research Institute (ETRI), where he developed a DS-CDMA cellular radio system as a head of radio technology division of ETRI. And he was a visiting researcher at the Bell-Northern Research (BNR), Dallas, Texas in 1991. Since 1994, he has been with the Department of Information and Communication Engineering at the Chongju University, Chongju, Korea. From January 2002 to January 2003, he was a visiting scholar at New Jersey Institute of Technology (NJIT), Newark, NJ. His research interests include spread spectrum communications, RF system design, and MIMO system.

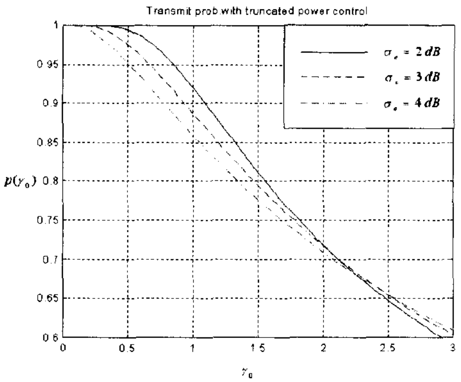

Fig. 1 Transmit probability of a mobile station (m=4).

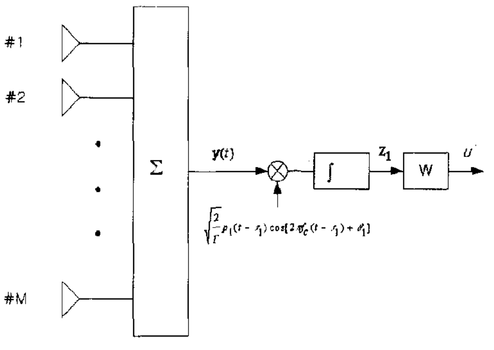

Fig. 2 CDMA reverse link adaptive array receiver model.

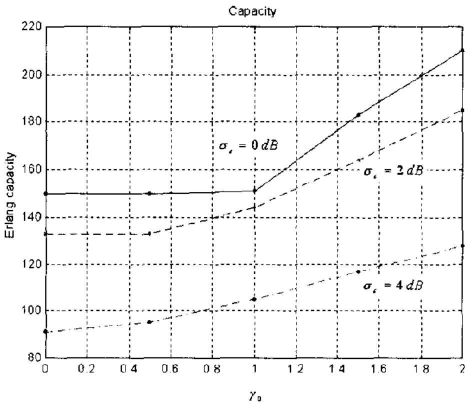

Fig.3 Erlang capacity as a function of the transmit threshold γ_0 (M=4).

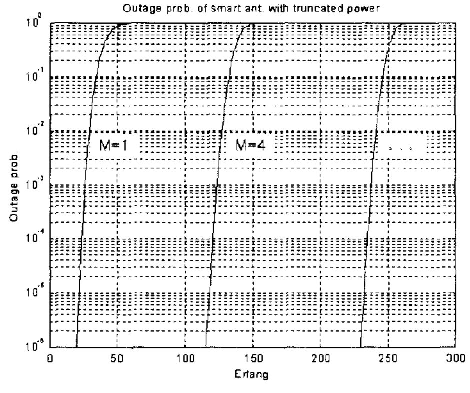

Fig. 4 Outage probability versus Erlang capacity ($\sigma_c = 2.5$ dB, $\gamma_0 = 1$).

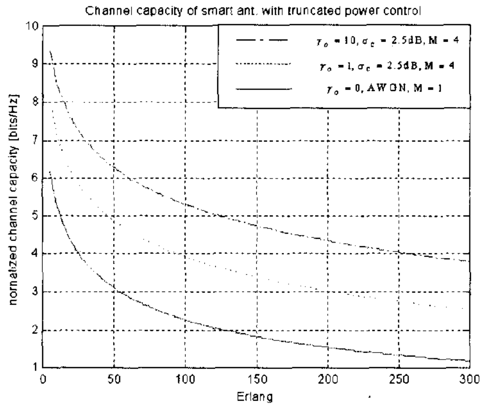

Fig. 5 Normalized channel capacity.