

# 준직교 수열을 이용한 고대역효율 변조방식

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# A Bandwidth Efficient Modulation Scheme Using Quasi-Orthogonal Sequences

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#### **ABSTRACT**

A novel modulation scheme using quaternary quasi-orthogonal sequences [1] as M-ary modulation symbols is proposed. The controlled quasi-orthogonality between the modulation symbols allows us to increase the bandwidth efficiency with little or no degradation in the required signal-to-noise ratio per bit to achieve a given error probability compared to orthogonal modulation. An efficient hardware structure for demodulator using fast Walsh transforms is also presented.

#### I . 서 론

Orthogonal modulation schemes such as M-ary frequency shift keying  $(MFSK)^{[2]}$  and M-ary Walsh modulation[3] with noncoherent demodulation are widely used in communication systems where reliable phase recovery of the received signal is not practically feasible or undesirable. Furthermore, M-ary orthogonal modulation schemes achieve the channel capacity under AWGN channels as the modulation order  $M \rightarrow \infty$  with both coherent and noncoherent demodulation<sup>[2]</sup>. However, orthogonal modulation require bandwidth which linearly with M, rendering them unsuitable for communication systems requiring bandwidth efficiency. Thus, extension of orthogonal modulation schemes to increase the bandwidth efficiency by sacrificing the orthogonality between modulation symbols has been a topic of several previous studies, such as MFSK with nonorthogonal frequency spacing[2] and multitone MFSK<sup>[4]</sup>.

In this paper, we propose a novel modulation scheme using quaternary quasi-orthogonal sequences (QOS)<sup>[1]</sup> as *M*-ary modulation symbols, which we shall refer to as quasi-orthogonal modulation (QOM). Using the QOM, we are able to increase the bandwidth efficiency with little or no degradation in the required signal-to-noise ratio (SNR) per bit to achieve a given error probability compared to orthogonal modulation. An efficient hardware structure for demodulator using fast Walsh transforms (FWT)<sup>[5]</sup> is also presented.

The rest of this paper is organized as follows. Set of QOS is defined in Section II. In Section III, system model is presented, and an efficient hardware structure for demodulator is presented in Section IV. In Section V, performance of QOM is investigated using the union bound approach. Numerical results are presented in Section VI, and conclusions are drawn in Section VII.

# II. Quasi-Orthogonal Sequences

QOS was proposed in [1] as spreading sequences for synchronous code division multiple

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<sup>※</sup> 본 연구는 정보통신부 대학 IT 연구센터 육성・지원사업 지원에 의하여 수행되었음.

<sup>\*</sup> 본 논문은 2002년 4월 JCCI 학술대회에서 우수논문으로 선정되어 게재 추천된 논문입니다.

access (CDMA) networks. Let  $\mathbf{W}_m$  be the set of Walsh sequences of length  $N=2^m$ , f be a binary (or quaternary) sequence of length N not in  $\mathbf{W}_m$  and  $R_{\max}(f)$  be the maximum absolute value of correlation (inner product) between f and all the sequences in  $\mathbf{W}_m$ . Define  $\lambda_{\min}(N)$  to be the minimum achievable correlation value given by

$$\lambda_{\min}(N) = \min_{f} R_{\max}(f) \tag{1}$$

where f runs through all sequences of length N excluding those in  $\mathbf{W}_m$ . Then, a set  $\mathbf{F}$  of QOS of length  $N=2^m$  is defined as a set of sequences with the following properties<sup>[1]</sup>:

- (a)  $\mathbf{F}$  contains  $\mathbf{W}_m$ .
- (b) Absolute value of correlation (inner product) between any two distinct sequences in  ${\bf F}$  is less than or equal to  $\lambda_{\min}$  (N).

A third property called the window property was also included in [1] which ensures that different users in a synchronous CDMA network using spreading sequences of different lengths have minimal interference on each other. In this paper, however, we are interested in sets of modulation sequences in which every sequence is of identical length N. Since the window property reduces the family size (number of sequences in the set) of QOS and thus reduces the modulation order for a given sequence length N, we exclude the window property in this paper.

In [1], binary and quaternary QOS were presented. Since  $\lambda_{\min}(N)$  of quaternary QOS is less than or equal to that of binary QOS and the family size of quaternary QOS is greater than or equal to that of binary QOS for a given sequence length N, quaternary QOS are used in this paper in order to minimize the correlation between modulation symbols and maximize the modulation order for a given sequence length.

A set of quaternary QOS of length  $N=2^m$ 

constructed based on the Family A sequences<sup>[6]</sup> has a family size of  $M=N^2$  and can be partitioned into N nonoverlapping equal size subsets, each consisting of all the sequences in  $\mathbf{W}_m$  multiplied by a unique masking sequence of length N. Thus, any two distinct sequences within the same subset are orthogonal to each other, while the correlation (inner product) between any two sequences from different subsets is given by

$$\rho_{l,k} = \lambda_{\min} (N) e^{j\phi_{l,k}} \tag{2}$$

where l and k are indices of the two sequences and  $\phi_{l,k}$  can take one of the following four values:

$$\phi_{l,k} = a\frac{\pi}{2} + \psi \tag{3}$$

where  $a \in \{0, 1, 2, 3\}$  and  $\psi = \pi/4$  when m is odd and zero when m is even. For quaternary QOS,  $\lambda_{\min}(N) = \sqrt{N}$ . For details on the construction of QOS, readers are referred to [1].

# III. System Model

Similar to the M-ary Walsh modulation<sup>[3]</sup>,  $k = \log_2 M$  bits are used to choose one of the M quaternary QOS of length  $N=2^m=\sqrt{M}$ . Let  $c_i$  be the *i*th QOS. The M QOS are indexed such that  $c_{i\sqrt{M}}, c_{i\sqrt{M}+1}, \cdots, c_{(i+1)\sqrt{M}-1}$ are orthogonal to each other for  $i=0,1,\cdots,\sqrt{M}-1$ . Thus, we may obtain  $c_{i\sqrt{M}},\,c_{i\sqrt{M}+1},\cdots,\,c_{(i+1)\sqrt{M}-1}$  by multiplying all the sequences in  $W_m$  by a unique masking sequence b<sub>i</sub>. Since a set of QOS always contains  $\mathbf{W}_m$ , one of the masking sequences is an all-one sequence. In this paper, we assume  $b_0$  is the all-one sequence.

The receiver consists of M matched filters each matched to one of the M QOS. Without loss of generality, we assume that  $c_0$  is transmitted using

QPSK modulation. Assuming perfect synchronization and appropriate normalization at the receiver, the *l*th matched filter output can be written as

$$U_{l} = \begin{cases} e^{j\theta} + n_{0}, & l = 0\\ n_{l}, & l = 1, \dots, \sqrt{M} - 1\\ M^{-\frac{1}{4}} e^{j(\theta + \phi_{\theta}, j)} + n_{l}, & l = \sqrt{M}, \dots, M - 1 \end{cases}$$
(4)

where  $n_l$  is a zero mean complex Gaussian random variable due to AWGN with variance  $\sigma^2 = (2kE_b/N_0)^{-1}$  per dimension,  $E_b$  is the received signal energy per bit and  $N_0$  is the single-sided power spectral density of AWGN. Note that since the M matched filters are not all orthogonal to each other,  $n_0, \cdots, n_{M-1}$  are correlated. Phase rotation due to the channel,  $\theta$ , is assumed to be a uniform random variable over  $[0, 2\pi)$ .

# IV. An Efficient Hardware Structure for Demodulator

Since a set of M-ary QOS can be partitioned into  $\sqrt{M}$  subsets, with each subset consisting of all the sequences in  $\mathbf{W}_m$  multiplied by a unique masking sequence, the demodulator may first remove the masking sequence and then use a  $\sqrt{M}$ -ary FWT to demodulate the received signal. Since the demodulator does not have prior knowledge on which masking sequence was used at the transmitter, all  $\sqrt{M}$  masking sequences will have to be tried. Fig. 1 shows the demodulator structure using FWT for M=64 where  $b_i^*$  denotes the complex conjugate of  $b_i$ .

A demodulator implemented using M separate matched filters requires  $2M(\sqrt{M}-1)$  real additions for noncoherent demodulation, while the demodulator using the FWT requires  $M\log_2 M$  real additions. For example, the demodulator using FWT requires almost 60% less real additions than the demodulator using M separate matched filters for M=64.

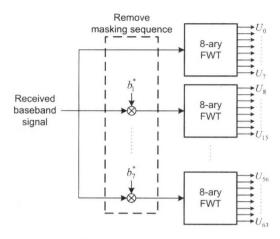


Fig. 1 An efficient hardware structure for demodulator using fast Walsh transforms: M = 64.

## V. Performance

#### 1. Symbol Error Rate

Since it is difficult to evaluate the symbol error rate (SER) of nonorthogonal modulation schemes in simple closed forms except in some very special cases [2], we resort to the union bound approach. Assume  $c_0$  is transmitted. Then, since  $c_1, c_2, \cdots, c_{\sqrt{M}-1}$  are orthogonal to  $c_0$ ,  $\Pr\left(U_l > U_0\right)$  for  $l = 1, \cdots, \sqrt{M} - 1$  is given by [2]

$$P_{\text{orth}} = \frac{1}{2} \exp\left(-\frac{kE_b}{2N_0}\right). \tag{5}$$

For  $l=\sqrt{M},\cdots,M-1$ ,  $\Pr\left(U_l>U_0\right)$  can be computed using the error probability for binary nonorthogonal signals<sup>[2]</sup>:

$$P_{\text{nonorth}} = Q_{1} \left( \sqrt{\frac{kE_{b}}{2N_{0}}} (1-\beta), \sqrt{\frac{kE_{b}}{2N_{0}}} (1+\beta) \right) - \frac{1}{2} \exp\left( -\frac{kE_{b}}{2N_{0}} \right) I_{0} \left( \frac{kE_{b}}{2N_{0}} M^{-\frac{1}{4}} \right).$$
 (6)

In (6),  $\beta = \sqrt{1 - M^{-\frac{1}{2}}}$ ,  $Q_1(a,b)$  is the first order generalized Marcum Q-function and  $I_0(x)$  is the zeroth order modified Bessel function of the first kind<sup>[7]</sup>. Then, the union bound on the SER is given by

$$P_s \le \left(\sqrt{M} - 1\right) P_{\text{orth}} + \left(M - \sqrt{M}\right) P_{\text{non orth}}. \tag{7}$$

### 2. Asymptotic Performance

As  $M \rightarrow \infty$ ,  $M^{-\frac{1}{4}} \rightarrow 0$  in (4) and it can easily be shown than the correlation between  $n_l$  goes to zero as well. Thus, the M matched filter outputs asymptotically become orthogonal to each other as  $M \rightarrow \infty$ . Hence, the asymptotic SER performance of QOM as  $M \rightarrow \infty$  should be identical to that of orthogonal signals and QOM also achieve the channel capacity as  $M \rightarrow \infty$  under AWGN.

#### 3. Bandwidth Efficiency

For a given modulation order M, sequence lengths N (number of chips) of orthogonal Walsh and OOM are  $N_i$ respectively. For simplicity, we assume that the required bandwidth W of each modulation scheme is approximately  $1/T_c$ , where  $T_c = k/(NR)$  is the chip duration and R is the transmission bit rate. Then, the bandwidth efficiency measured by the bit rate to bandwidth ratio [7] R/W is k/M for orthogonal modulation. The bandwidth efficiency for QOM is  $k/\sqrt{M}$ .

# VI. Numerical Results

Fig. 2 shows the SER of orthogonal modulation (OM) and QOM under AWGN. As expected, the SER of QOM approaches that of orthogonal modulation as M increases. For instance,  $E_b/N_0$  difference between QOM and orthogonal modulation to achieve  $P_s=10^{-4}$  is 2.3 dB for M=16 while the difference is only 0.8 dB for M=256.

Since the orthogonal modulation and QOM have different bandwidth efficiency, it is more meaningful to compare the two modulation schemes using the bandwidth efficiency versus the  $E_b/N_0$  required to achieve a given SER<sup>[7]</sup>, which we shall denote as  $E_b/N_0$ <sub>P<sub>e</sub></sub>. Fig. 3 shows the bandwidth efficiency versus  $E_b/N_0$ <sub>P<sub>e</sub> = 10<sup>-4</sup></sub>. Note

that we can increase the bandwidth efficiency with little or no degradation in  $E_b/N_0$   $_{P_s=10^{-4}}$  by employing QOM whose modulation order is 4 times larger than that of orthogonal modulation. For example, by employing 64-ary QOM instead of 16-ary orthogonal modulation, we can increase the bandwidth efficiency by 6 times while requiring only 0.1 dB higher  $E_b/N_0$   $_{P=10^{-4}}$ .

Performance of multitone MFSK is also shown in Fig. 3 where (M,m) refers to the multitone MFSK scheme in which m tones of M-ary FSK are simultaneously transmitted<sup>[4]</sup>. The multitone MFSK does provide an increase in the bandwidth efficiency over orthogonal modulation but at the cost of significant increase in  $E_b/N_0$   $_{P_s=10^{-4}}$ . For similar bandwidth efficiency, QOM requires much lower  $E_b/N_0$   $_{P_s=10^{-4}}$  than the multitone MFSK.

#### VII. Conclusions

In this paper, we proposed the quasi-orthogonal modulation (QOM) scheme in which quaternary quasi-orthogonal sequences (QOS) are used as M-ary modulation symbols. It is shown that QOM is capable of increasing the bandwidth efficiency with little or no SNR degradation compared to orthogonal modulation schemes by using

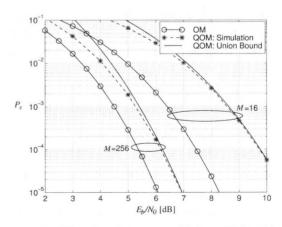


Fig. 2 SER of orthogonal modulation (OM) and quasi-orthogonal modulation (QOM) under AWGN with noncoherent demodulation.

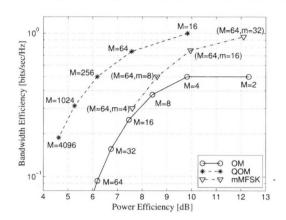


Fig. 3 Bandwidth efficiency vs.  $E_b/N_0$  required to achieve SER of  $10^{-4}$  under AWGN.

sufficiently large M. It is also shown that QOM also achieves the channel capacity as the modulation order  $M{\to}\infty$ . A hardware efficient demodulator structure using fast Walsh transforms is presented which allows us to implement QOM demodulators with large M with reasonable hardware complexity.

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