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Performance analysis of asynchronous DS-CDMA system with MRC diversity in fading channels

Seok Seo* and Chan-kil Lee* ***Regular Members***

ABSTRACT

This paper presents and analyses the closed-form expression of the average bit error rate (BER) for an asynchronous direct-sequence code division multiple access (DS-CDMA) system with coherent binary phase shift keying (BPSK) modulation scheme using a maximal ratio combining (MRC) diversity over a Rician fading channel. In addition to the average BER, outage probability, and user capacity of system are estimated as performance measures. The results are general enough so that it includes Rayleigh fading and nonfading channel with zero and infinite Rician factor, respectively, as special cases. The effects of various channel models, processing gains, and diversity orders on the system performances are also considered for the typical multipath delay profiles characterized by Rician fading channel.

key Words : BER, outage probability, DS-CDMA; MRC diversity; Rician fading channel.

Ⅰ. Introduction

An asynchronous DS-CDMA system is a multiple-access interference-limited system, and its performance degrades as the number of users increase. In a land-mobile system, the error rate performance of each user is also affected by the multipath fading statistics and AWGN of the received signal. Multipath propagation can be resolved at the receiver of a DS-CDMA system that uses a bandwidth much larger than the coherence bandwidth of the channel. Use of diversity reception is one of the effective techniques to combat the detrimental effects of channel fading on digital transmissions. As is well known, MRC diversity is the optimal linear combining technique and provides the maximum possible improvement that a diversity system can attain through a fading channel [1]. Rician fading is observed in microcellular urban and suburban land mobile channels, picocellular indoor and factory environment, and also in satellite radio link [2].

The closed-form of the average BER expression over a Rayleigh fading channel can be easily derived due to the relatively simple form of a fading distribution function [3],[4]. However, semi-analytic simulation techniques have been employed to evaluate the BER estimate over a Rician fading channel [5]. The disadvantage of this approach is that the computational accuracy is dependent on the Rician parameters of the channel in addition to computational complexity.

In this paper, we further extend the work in [6] and derive a simple and accurate BER expression of DS-CDMA system with MRC diversity over the Rician multipath fading channel. Based on the derived expressions, average BER, outage probability, and user capacity of the proposed system are estimated in Rician fading channel with a various fading parameters encountered in a typical mobile environment. Throughout an analysis, we assume that perfect carrier phase recovery and channel estimation are feasible at the receiver.

Section Ⅱ describes transmitter, multipath fading channels, and coherent receiver model. Based on the system model, the analysis and numerical

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* Department of Electrical and Computer Engineering, Hanyang University, Ansan, Kyoungki-do (seok@dcs.hanyang.ac.kr)

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results for the average BER, outage probability, and user capacity of the proposed system are presented in Section III and IV, respectively.

Ⅱ. System Model

1. Transmitter model

In DS-CDMA system with BPSK modulation, each of *K* users is assigned a code sequence that modulates the phase of the carrier along with the data sequence. The transmitted signal for the *k*-th user is a biphase-coded carrier, which may be written as

$$s_k(t) = A a_k(t) b_k(t) \cos(w_c t + \theta_k) \tag{1}$$

where A is the signal amplitude, w_c is the common carrier frequency, and θ_k is the carrier phase for the *k*-th user. The carrier phase θ_k is assumed to be uniformly distributed in $[0, 2\pi)$. In eq. (1), a code waveform assigned to *k*-th user $a_k(t)$ can be represented by

$$a_k(t) = \sum_{i=-\infty}^{\infty} a_i^k P_{T_c}(t - iT_c) \tag{2}$$

where $a_i^k \in \{+1, -1\}$ represents the *i*-th chip value of the *k*-th user and $P_{T_c}(t)$ is a rectangular pulse with chip duration of T_c. And $b_k(t)$ is the information signal of the *k*-th user and may be expressed as

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_j^k P_T(t - jT) \tag{3}$$

where $b_j^k \in \{+1, -1\}$ represents the *k*-th user data at the *j*-th symbol timing interval and $P_T(t)$ is a rectangular pulse with bit duration of *T*. It is assumed that user code sequence has a period of $N = T/T_c$, and N also means processing gain of the direct-sequence spread-spectrum system.

2. Channel model

The mobile radio channel is usually modeled as a slow fading, time-invariant, and discrete multipah fading channel. The low-pass equivalent multipath channel impulse response for *k*-th user may be represented as

$$h_k(t) = \sum_{l=1}^{L} \beta_{k,l} \delta(t - \tau_{k,l}) \exp(j\phi_{k,l}) \tag{4}$$

where $\beta_{k,l}$, $\phi_{k,l}$, and $\tau_{k,l}$, denote the gain, phase, and time delay of the *l*-th path, and are assumed as a Rician, uniform in $[0, 2\pi)$, and uniform in $[0, T)$ random variable, respectively.

3. Receiver model

When the channel output is corrupted by *K*-1 user interferers with the resolvable paths *L* and $n(t)$ which is AWGN with a two-sided spectral density $N_0/2$, then the received signal can be expressed by

$$\begin{aligned} r(t) = {} & A \sum_{l=1}^{L} \beta_{1,l} a_1(t - \tau_{1,l}) b_1(t - \tau_{1,l}) \\ & \times \cos(w_c t + \phi_{1,l}) \\ & + A \sum_{k=2}^{K} \sum_{l=1}^{L} \beta_{k,l} a_k(t - \tau_{k,l}) \\ & \times b_k(t - \tau_{k,l}) \cos(w_c t + \phi_{k,l}) \\ & + n(t) \end{aligned} \tag{5}$$

The desired receiver (*k*=1) is assumed to ideally lock onto the *j*-th path. After the correlation operation and demodulation process, a signal sample at the receiver, the low-pass filter output can be expressed as

$$\xi_j = \int_0^T r(t) a_1(t - \tau_{1,j}) \cos(w_c t + \phi_{1,j}) \, dt \tag{6}$$

where $a_1(t)$ is the regenerated PN code at the receiver, $\tau_{1,j}$ and $\phi_{1,j}$ are the compensated time delay and phase for a desired user's *j*-th path, respectively. Then the *j*-th decision variable for detection is obtained by substituting eq. (5) into eq. (6) as follows:

$$\begin{aligned}\xi_j = {} & b_0^1 \frac{AT}{2}\beta_{1,j} \\ & + \frac{A}{2}\sum_{l=1,l\neq j}^{L}\beta_{1,l}\cos(\phi_{1,l}-\phi_{1,j}) \\ & \times[b_{-1}^1 R_{1,1}(t_{1,l}) + b_0^1\hat{R}_{1,1}(t_{1,l})] \\ & + \frac{A}{2}\sum_{k=2}^{K}\sum_{l=1}^{L}\beta_{k,l}\cos(\phi_{k,l}-\phi_{1,j}) \\ & \times[b_{-1}^k R_{k,1}(t_{k,l}) + b_0^k\hat{R}_{k,1}(t_{k,l})] + \nu\end{aligned} \tag{7}$$

where b_0^1 is the information bit to be detected, b_{-1}^1 is the preceding bit due to the path delay, $t_{k,l} = \tau_{k,l} - \tau_{1,j}$, ν is random variable with Gaussian distribution, and

$$R_{k,1}(t_{k,l}) = \int_0^{t_{k,l}} a_k(t-t_{k,l})\,a_1(t)\,dt \tag{8}$$

$$\hat{R}_{k,1}(t_{k,l}) = \int_{t_{k,l}}^{T} a_k(t-t_{k,l})\,a_1(t)\,dt \tag{9}$$

In eq. (7), the first term represents the desired signal to be detected, the second term is the self-interference (SI) due to multipath components of a desired user's signal, and the third term is the multiuser interference (MUI) from the K-1 other simultaneous users.

Ⅲ. Performance Analysis

1. Average BER

We treat all interferences in eq. (7) as a Gaussian process. In eq. (7), the first term is the desired signal with average power $\beta_{1,j}^2 A^2 T^2/4$ for a fixed $\beta_{1,j}$ and the last term, v, is a Gaussian random variable with zero mean and average power of $P_{noise} = N_0 T/4$. The remaining terms are considered to be all mutually independent. Hence, the interference powers can be expressed as

$$\begin{aligned}P_{SI} = {} & \frac{A^2}{4}\sum_{l=1,l\neq j}^{L} E(W_{1,l}^2) \\ & \times E[(\beta_{1,l}\cos(\phi_{1,l}-\phi_{1,j}))^2]\end{aligned} \tag{10}$$

$$\begin{aligned}P_{MUI} = {} & \frac{A^2}{4}\sum_{k=2}^{K}\sum_{l=1}^{L} E(W_{k,l}^2) \\ & \times E[(\beta_{k,l}\cos(\phi_{k,l}-\phi_{1,j}))^2]\end{aligned} \tag{11}$$

where P_{SI} and P_{MUI} denote the power of self-interference and multi-user interference, respectively and

$$W_{k,l} = b_{-1}^k R_{k,1}(t_{k,l}) + b_0^k \hat{R}_{k,1}(t_{k,l}) \tag{12}$$

For Gold codes with sequence length N, Pursely [7, 8] has shown that

$$E(W_{k,l}^2) = 2T^2/(3N), \text{ for all } k. \tag{13}$$

Thus, assuming that the L multipaths have equal power, P_{SI} and P_{MUI} can be represented as:

$$P_{SI} = \left(\frac{AT}{2}\right)^2 \frac{1}{3N}(L-1)E(\beta_{1,l}^2) \tag{14}$$

$$P_{MUI} = \left(\frac{AT}{2}\right)^2 \frac{1}{3N} L(K-1)E(\beta_{k,l}^2),\ k\neq 1 \tag{15}$$

Consider the M-th order MRC diversity, $1 \leq M \leq L$. Then the SNR at the output of the maximal-ratio combiner is given by

$$\begin{aligned}\gamma & = \frac{1}{2}\frac{P_{signal}}{P_{noise}+P_{SI}+P_{MUI}} \\ & = \frac{\sum_{j=1}^{M}\beta_{1,j}^2}{\left(\frac{E_b}{N_0}\right)^{-1} + \frac{2(L-1)}{3N}E(\beta_{1,l}^2) + \frac{2L(K-1)}{3N}E(\beta_{k,l}^2)} \\ & = \alpha H\end{aligned} \tag{16}$$

where $\alpha = \sum_{j=1}^{M}\beta_{1,j}^2$ and H denotes the remaining term. Assuming an independent multipath fading channel, it is apparent that α has the noncentral chi-square distribution with 2*M* degree of freedom and with the noncentrality parameter *P*, and the pdf of α is given by [6]

$$p(\alpha) = \frac{1}{2\sigma^2}\left(\frac{\alpha}{P}\right)^{\frac{M-1}{2}} \exp\left(\frac{\alpha+P}{2\sigma^2}\right) \times I_{M-1}\left(\frac{\sqrt{\alpha P}}{\sigma^2}\right) \tag{17}$$

where $2\sigma^2$ is the common mean squared value of the scatter component and $P = \sum_{m=1}^{M} s_j^2$, s_j is the strength of the fixed component in the *j*-th path.

For coherent BPSK, the conditional bit error probability is given by

$$P_{b,M|\alpha} = \frac{1}{2} erfc\left(\sqrt{\alpha H}\right) \tag{18}$$

To obtain average BER, we must average $P_{b,M|\alpha}$ over the pdf of α given by eq. (17). Hence, average BER may be estimated by solving the integral

$$P_{b,M} = \int_0^\infty P_{b,M|\alpha}\, p(\alpha)\, d\alpha \tag{19}$$

Eq. (19) is identical to eq. (12) of [6] except that the arguments (i.e., SNR) of the complementary error function are different each other. Thus following the derivation procedure of [6], the solution of eq. (19) can be easily obtained as

$$P_{b,M} = \frac{1}{2}\exp(-R)\sqrt{\frac{d}{1+d}} \sum_{m=M}^{\infty}\binom{2m}{m} \times\left(\frac{1}{4+4d}\right)^m F_1\left(m+\frac{1}{2},\, m+1;\frac{R}{1+d}\right) \tag{20}$$

where $d = 2\sigma^2 H$, $R = \sum_{j=1}^{M}\frac{s_j^2}{2\sigma^2} = \sum_{j=1}^{M} R_j$, R_j denotes the Rician factor for *j*-th diversity branch, and $F_1(\cdots)$ denotes a confluent hypergeometric function. Also eq. (20) can be represented in a more computational efficient form [6]. For $M = 1$,

$$P_{b,1} = Q_1(u, w) - \frac{1}{2}\left(1+\sqrt{\frac{d}{1+d}}\right) \times \exp\left(-\frac{u^2+w^2}{2}\right) I_0(uw) \tag{21}$$

where

$$u = \sqrt{\frac{R\left[1+2d-2\sqrt{d(1+d)}\right]}{2(1+d)}} \tag{22}$$

$$w = \sqrt{\frac{R\left[1+2d+2\sqrt{d(1+d)}\right]}{2(1+d)}} \tag{23}$$

where $Q_1(u, w)$ is the first order Marcumm *Q*-function. For $M \geq 2$,

$$P_{b,M} = P_{b,1} - \frac{1}{2}\sqrt{\frac{d}{1+d}}\exp(-R)\sum_{m=1}^{M-1}\binom{2m}{m} \times\left(\frac{1}{4+4d}\right)^m F_1\left(m+\frac{1}{2},\, m+1;\frac{R}{1+d}\right) \tag{24}$$

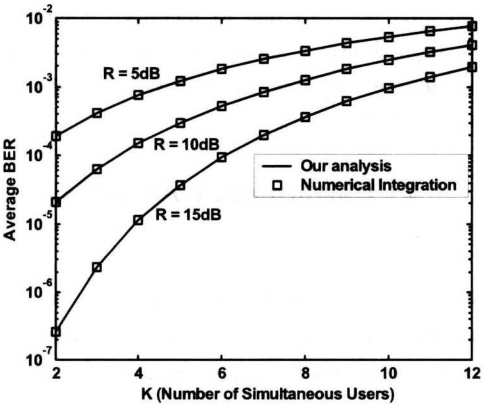

Fig. 1. Average BER for different Rician factors; E_b/N_0=15dB, *N*=127, *L*=*M*=3

To confirm the accuracy of the analysis, the BER estimates using the analytical results were evaluated and compared with the results obtained by integrating eq. (19) numerically. Figure 1 shows that both results match well for different Rician factors.

2. Outage probability

Another performance measurement is to find the distribution of the error probability. If the probability that the average error probability exceeds the value X is, say 0.1, the outage probability is said to be 0.1.

$$P_{outage} = Pr(BER \geq X) \tag{25}$$

To find the value α to be $P_{b,M|\alpha} = X$, we vary α from zero to α_0, where $P_{b,M|\alpha_0} = X$ is the bit error rate parameter. Then outage probability may be written as

$$\begin{aligned} P_{outage} &= Pr(P_{b,M|\alpha} \geq X) \\ &= Pr(0 \leq \alpha \leq \alpha_0) \end{aligned} \tag{26}$$

The probability $\Pr(0 \leq \alpha \leq \alpha_0)$ is easily obtained by integrating the pdf for MRC diversity as given in eq. (17). Using the cumulative density function (CDF) of α [9, eqn. 2-1-124], the outage probability can be obtained as

$$\begin{aligned} P_{outage} &= \int_0^{\alpha_0} p(\alpha)d\alpha = 1 - \int_{\alpha_0}^{\infty} p(\alpha)d\alpha \\ &= 1 - Q_M\left(\frac{\sqrt{P}}{\sigma}, \frac{\sqrt{\alpha_0}}{\sigma}\right) \end{aligned} \tag{27}$$

where $Q_M(x, y)$ is the generalized Marcumm Q-function.

Ⅳ. Numerical Results

For the performance evaluations of the investigated system, a wideband DS-CDMA system in a typical WLL channel model is considered. Table 1 shows the channel parameters of a typical WLL channel models [10]. The urban and hilly terrain WLL channels consist of three and four uncorrelated paths with Rician distribution, respectively.

Table 1. Parameters of a typical WLL Channels

Parameters	Urban	Hilly terrain
# of rosolvable paths L	3	4
# of Rician paths	3	4
Total Rician factor R (dB)	10	9.5

Figure 2 and 3 show average BER curves for different diversity orders in an urban and hilly terrain channel for K=10, with N=127 and N=255, respectively. From figures, we can see that urban channel provides much better BER performance than that of hilly terrain. To achieve the error probability of around 10^{-3}, the diversity order of M=2 is necessary for the case of N=255 in both channels. For the case of N=127, the diversity order of M=2 and 3 is necessary for urban channel and hilly terrain channel, respectively.

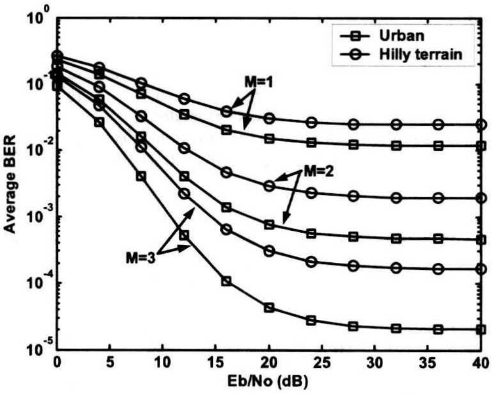

Fig. 2. Average BER for different diversity orders; N=127, K=10

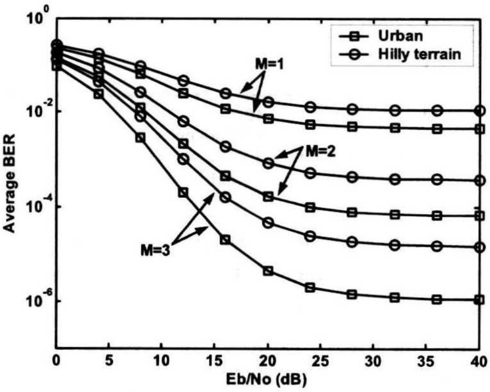

Fig. 3. Average BER for different diversity orders N=255, K=10

Figure 4 and 5 depict the outage probability for different diversity orders in hilly terrain channel and for different processing gains in urban channel, respectively. A typical system objective is that the outage threshold should be exceeded not more than 10 percent of the time. Accordingly, attention should focus on that portion of each graph where the ordinate is near 0.1 and the abscissa is, say 10^{-6}, the quality required by users.

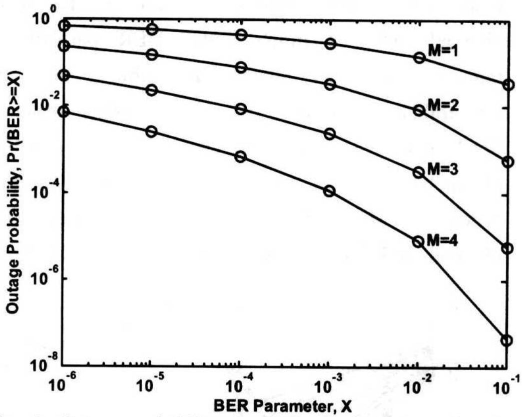

Fig. 4. Outage probability for different diversity orders in hilly terrain channel; E_b/N_0=30dB, N=255, K=10

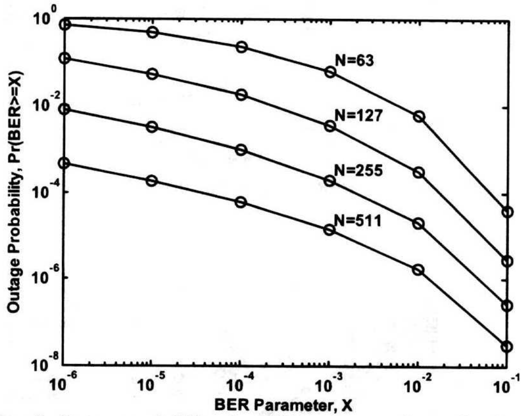

Fig. 5. Outage probability for different processing gains in urban channel; E_b/N_0=30dB, M=3, K=10

The number of users supported by the system, for a specified average BER, is also an important design criterion and performance measure. Table 2 shows the user capacity of system used in analysis, with a specified average BER of 10^{-3} and $E_b/N_0 = \infty$.

Table 2. User capacity of DS-CDMA system with different diversity orders and processing gains

Parameters		Urban			Hilly terrain			
Diversity order		1	2	3	1	2	3	4
Processing gain	63	1	6	13	0	3	8	13
	127	1	13	26	0	7	17	26
	255	3	26	52	1	15	33	53
	511	5	51	104	2	29	67	107

V. Conclusion

The closed-form expression of the average bit error probability was presented for a wideband DS-CDMA system with MRC diversity over a Rician fading channel. In addition to the average BER, outage probability and user capacity of system were investigated as performance measures. The accuracy of the bit error rate (BER) estimated by this expression was verified through comparison with the results obtained by numerical integration. It is shown that the processing gain, diversity order, and channel characteristics affect the system performance such as the average bit error probability, outage probability, and user capacity. The results are general enough so that it includes Rayleigh fading and nonfading additive white Gaussian noise (AWGN) channel as special cases, and our approach can be extended to the case of any other fading channels.

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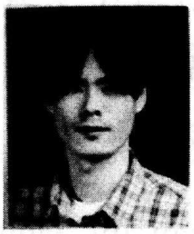
Seok Seo



Feb. 1997: B.S. degree in electronics engineering from Hanyang University.

Feb. 1999: M.S. degree in electronics engineering from Hanyang University.

Mar. 1999 ~ : Ph. D. candidate in electronics and computer engineering from Hanyang University.

Research interests: digital communiction, SDR system, fading channel modeling, multicarrier and spread-spectrum system.

Chankil Lee

Feb. 1981: B.S. degree in electronics engineering from Hanyang University.

Feb. 1983: M.S. degree in electronics engineering from Seoul National University.

June 1992: Ph. D. degree in electrical engineering from Georgia Inst. Tech., USA.

Mar. 1983 ~ Sep. 1985: ETRI, TDX researcher.

Mar. 1992 ~ Feb. 1994: ETRI, Moble telecommunication fileld researcher.

Mar. 1994 ~ Feb. 1995: Electrical engineering, Kyungpook University.

Mar. 1995 ~ : Professor, Dept. of Electrical and computer engineering, Hanyang University.

Research interests: digital communication, SDR system, CDMA modem, fading channel modeling, wireless sensor network.