On the Signal Power Normalization Approach to the Escalator Adaptive Filter Algorithms

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ABSTRACT

A normalization approach to coefficient adaptation in the escalator(ESC) filter structure that conventionally employs least mean square(LMS) algorithm is introduced. Using Taylor's expansion of the local error signal, a normalized form of the ESC-LMS algorithm is derived. Compared with the computational complexity of the conventional ESC-LMS algorithm employs input power estimation for time-varying convergence coefficient using a single-pole low-pass filter, the computational complexity of the proposed method can be reduced by 50% without performance degradation.

key Words : Normalization, escalator, LMS, complexity, equalization

Ⅰ. Introduction

Many researchers have studied various adaptive equalizer structures and coefficient-adjustment algorithms. The tapped delay line(TDL) equalizer structure using the least mean square(LMS) algorithm(TDL-LMS) of Widrow and Hoff[1] has been being widely used dueto its simplicity in realization. One drawback of the LMS algorithm is that its convergence speed decreases as the ratio of the maximum to the minimum eigenvalues of the input autocorrelation matrix increases. To cope with this problem, it has been proposed to orthogonalize the input signal using Gram-Schmidt orthogonalization procedure that can be implementedby the escalator (ESC) structure[2]. Due to the complete orthogonalization property of the ESC structure, its convergence speed is independent of the eigenvalue spread ratio(ESR) of the input signals. The learning speed of the algorithms employing stochastic-gradient descent method is dependent on convergence coefficients. The LMS algorithm for the ESC that employs input power estimation for time-varying convergence coefficient using a single-pole low-pass filteris currently used in the ESC filtering problems[3,4]. Using Taylor's expansion of error signal in the TDL structure, a method of obtaining optimum convergence coefficients has been studiedin[5], and it is quite similar to that of the normalized least mean square(NLMS) where convergence coefficient is normalized with respect to the squared Euclidean norm of the input vector. If we apply Taylor's expansion of the local error signal to obtaining the optimum convergence coefficients of the ESC adaptive filter algorithm, we can acquire fast convergence speed and reduce its computational complexity. In this paper, we introduce a method of coefficient adaptation in the ESC-LMS algorithm by applying the Taylor's expansion approach of local error signal to the adaptation of coefficients.

Ⅱ. Escalator Equalizer Structure[2]

Given a symmetric matrix R, there exists a unit lower triangular(ULT) matrix W, such that WRW^T is a diagonal matrix. The ULT matrix W can be computed in the form of W = $W_N W_{N-1} \ldots W_1$. The ULT transformation $Y(k) = W \cdot X(k)$ means that system W generates the uncorrelated output vector $Y(k)$ for the input vector $X(k)$ where its symmetric

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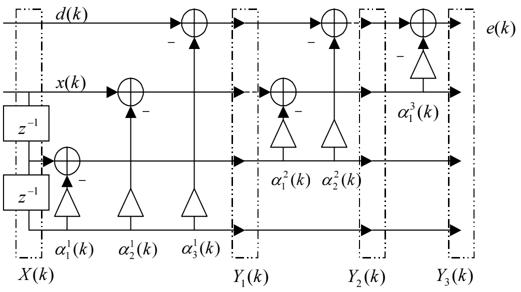

Figure 1. ESC filter realization for N=3

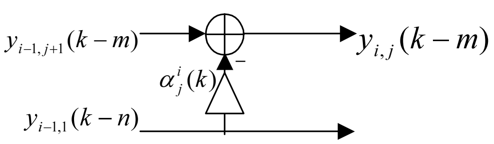

Figure 2. Part of Esacalator filter with $\alpha_j^i(k)$.

autocorrelation matrix $R = E[X(k)X^T(k)]$. If we define $X(k)$ as an input vector augmented with the desired sample $d(k)$, $X(k) = [x(k-N+1), x(k-N+2), ..., x(k), d(k)]^T$ and $Y(k) = [y(k-N+1), y(k-N+2), ..., y(k), e(k)]^T$ as an output vector augmented with the error sample $e(k)$, $Y(k) = W \cdot X(k)$ becomes filtering process of ESC structure.

We can realize the ULT transformation sequentially like $Y_1(k) = W_1 \cdot X(k)$, $Y_2(k) = W_2 \cdot Y_1(k)$ and $Y_3(k) = W_3 \cdot Y_2(k)$ etc. The final stage's output vector $Y_N(k)$ becomes $Y(k)$. The correspondingESC filter realization for N=3(N is the total number of stages) is shown in Fig. 1 and the general ESC filter equations are

$$y_{i,j}(k-m) = y_{i-1,j+1}(k-m) - \alpha_j^i(k) y_{i-1,1}(k-n),$$
$$y_{0,N+1}(k+1) = d(k), y_{0,j}(\cdot) = x(\cdot), y_{i,j}(k+1) = e_i(k), \tag{1}$$

for $1 \le i \le N$, $1 \le j \le N-i+1$, $m = N-i-j$ and $n = N-i$.

For the escalator structure the global minimization of the output energy can be accomplished as a sequence of local minimization problems, one at each stage of the escalator filters. Figure 2 shows the part of the escalator filter that corresponds to the weight $\alpha_j^i(k)$. We can see that part of escalator filter can be considered as one-tap coefficient TDL filter where $1 \le i \le N$, $1 \le j \le N-i+1$.

Ⅲ. Escaltor filter coefficient adaptation by the LMS algorithm

The escalator weight $\alpha_j^i(k)$, which can be considered as the coefficient of one-tap TDL, can be optimized according to a MSE criterion or by employing the method of least squares. Suppose we adopt the MSE criterion and select the parameter to minimize the sum of the mean-square errors where the error is $y_{i,j}(k-m)$. In other words, the MSE is given as

$$MSE = E[y^2_{i,j}(k-m)]$$
$$= E[(y_{i-1,j+1}(k-m) - \alpha_j^i(k) y_{i-1,1}(k-n))^2] \tag{2}$$

Differentiation of MSE with respect to $\alpha_j^i(k)$ yields the solution

$$\alpha_j^i(k) = \frac{E[y_{i-1,j+1}(k-m) y_{i-1,1}(k-n)]}{E[y_{i-1,1}^2(k-n)]} \tag{3}$$

It's instructive to note that each $\alpha_j^i(k)$ defined above is in the form of ration whose numerator and denominator are cross-correlation and autocorrelation(variance) terms, respectively.

Assuming that the variance of a random variable $f(k)$ changes slowly, one common method for estimating the variance, $E[f^2(k)]$, is to use a single-pole low-pass filter, i.e.,

$$\sigma_f^2(k) = \beta \sigma_f^2(k-1) + (1-\beta) f^2(k) \tag{4}$$

where $0 < \beta < 1$ is a smoothing parameter which controls the bandwidth and time constant of the system whose transfer function with it's input $f^2(k)$ is given by

$$S(z) = (1-\beta) \frac{z}{z-\beta} \tag{5}$$

Similarly the instantaneous cross-correlation function of two random variables $f(k)$ and $g(k)$ can be estimated using a simple low-pass filter; i.e.,

$$r_{fg}(m,k) = E[f(k)g(k-m)]$$
$$= \beta r_{fg}(m,k-1) + (1-\beta)f(k)g(k-m) \quad (6)$$

Thus the denominator and numerator terms in (3) can be updated via a single-pole low-pass system (4) and (6) to yield adaptive escalator filter weights.

$$\alpha_j^i(k) = \frac{c_{i,j}(k)}{v_y{}^i(k)} \quad (7)$$

$$c_{i,j}(k) = \beta c_{i,j}(k-1) + (1-\beta)y_{i-1,j+1}(k-m)y_{i-1,1}(k-n) \quad (8)$$

$$v_y{}^i(k) = \beta v_y{}^i(k-1) + (1-\beta)y_{i-1,1}^2(k-n) \quad (9)$$

where $0 < \beta < 1$, and the initial values of $c_{i,j}(k)$ and $v_i{}^i(k)$ are usually zero.

Instead of estimating the cross-correlation functions by using the simple low-pass filter, we can use the steepest descent method with time-varying convergence parameter $\mu_i(k)$ as the following[4]:

$$\alpha_j^i(k+1) = \alpha_j^i(k) + \mu_i(k)y_{i,j}(k-m)y_{i-1,1}(k-n) \quad (10)$$

where $\mu_i(k) = 2\mu / v_y{}^i(k)$ and $v_y{}^i(k)$ is estimated using a recurrence relation given by (9) for $0 < \beta < 1$. Here, the time constant of the LPF is determined by β, whereas the average time constant of the adaptation process of the LMS-type algorithm is dependent on μ. As β becomes smaller, the time constant of the LPF decreases, which yields faster estimation of the signal power but a larger estimation noise in the steady state. For convenience's sake, we call the algorithm (10) ESC-LMS in this paper.

Ⅳ. The proposed normalization approach to the ESC-LMS algorithm

From (10), the coefficient difference $\Delta\alpha_j^i(k) = \alpha_j^i(k+1) - \alpha_j^i(k)$ can be expressed as

$$\Delta\alpha_j^i(k) = \mu_i(k)y_{i,j}(k-m)y_{i-1,1}(k-n) \quad (11)$$

By carrying out an analysis similar to [5], we can write $y_{i,j}(k-m+1)$ as a Taylor's expansion of $y_{i,j}(k-m)$,

$$y_{i,j}(k-m+1) = y_{i,j}(k-m) + \frac{\partial y_{i,j}(k-m)}{\partial \alpha_j^i(k)} \cdot \Delta\alpha_j^i(k) + \frac{\partial^2 y_{i,j}(k-m)}{\partial \alpha_j^i(k)^2} \cdot [\Delta\alpha_j^i(k)]^2 + \cdots, \quad (12)$$

where

$$\frac{\partial y_{i,j}(k-m)}{\partial \alpha_j^i(k)} = \frac{\partial[y_{i-1,j+1}(k-m) - \alpha_j^i(k) \cdot y_{i-1,1}(k-n)]}{\partial \alpha_j^i(k)}$$
$$= -y_{i-1,1}(k-n) \quad (13)$$

The second-order term of (12) can be negligible in the steady state.

Substituting (11) and (13) into (12) yields

$$y_{i,j}(k-m+1) = y_{i,j}(k-m) - \mu_i(k)y_{i,j}(k-m)[y_{i-1,1}(k-n)]^2$$

Also,

$$[y_{i,j}(k-m+1)]^2 = [y_{i,j}(k-m)]^2\left[1 - \mu_i(k)[y_{i-1,1}(k-n)]^2\right]^2. \quad (14)$$

By differentiating (14) with respect to $\mu_i(k)$, we can acquire

$$\frac{\partial[y_{i,j}(k-m+1)]^2}{\partial \mu_i(k)} = -2[y_{i,j}(k-m)]^2 [y_{i-1,1}(k-n)]^2\left[1 - \mu_i(k)[y_{i-1,1}(k-n)]^2\right] \quad (15)$$

Letting (15) be equal to zero yields the optimum time-varying convergence coefficient, $\mu_i^*(k)$.

$$\mu_i^*(k) = \frac{1}{[y_{i-1,1}(k-n)]^2} \quad (16)$$

This convergence coefficient may be viewed as a similar form of the normalized LMS algorithm where the convergence coefficient is normalized with respect to the squared Euclidean norm of the input vector. When the input vector in the normalized LMS algorithm is small, numerical difficulties may arise because then we have to divide by a small value for the squared norm of the input vector[6]. To overcome this kind of problem (16) is slightly modified as follows:

$$\mu_i^*(k) = \frac{1}{a + [y_{i-1,1}(k-n)]^2}, \quad (17)$$

where constant $a > 0$.

This suggests the following adaptation algorithm for ESC structure.

$$\alpha_j^i(k+1) = \alpha_j^i(k) + \frac{y_{i,j}(k-m)y_{i-1,1}(k-n)}{a + [y_{i-1,1}(k-n)]^2} \quad (18)$$

It is noticeable to compare the computational complexity (multiplications and divisions) of the proposed algorithm and the ESC-LMS. The ESC-LMS algorithm of (9) and (10) require 5 multiplications and 1 division but (18) requires 2 multiplications and 1 division. This means the proposed method reduces the computational complexity by 50%.

V. Simulation results

In this section the performance of three adaptive equalizer algorithms for two channels that have different eigenvalue spread ratiosare compared. The three algorithms considered are: the TDL-LMS algorithm[6], ESC-LMS algorithm[4] and the proposed method (18) for the ESC structure. The impulse response h_i of the channel model[6] is

$$h_i = \frac{1}{2}\{1 + \cos[2\pi(i-2)/B]\}, \quad i = 1,2,3 \quad (19)$$

where i is discrete sample time and the parameter B determines the channel bandwidth and controls the eigenvalue spread ratio of the correlation matrix of the inputs in the equalizer[5]. The eigenvalue spread ratio increases with B. The experiment is carried out in two channels that are intended to evaluate the convergence performance of the three algorithms using TDL and ESC structure to changes in the eigenvalue spread.

Channel 1: B =3.1, ESR=11.12,
Channel 2: B =3.3, ESR=21.71

The TDL-LMS equalizer has 11 tap coefficients. The ESC-LMS and the proposed method consist of 11 stages. A zero mean white Gaussian noise sequence with variance 0.001 was added to yield the equalizer input. The convergence parameter for the TDL-LMS was $2\mu = 0.02$. The convergence parameter 2μ and smoothing parameter β for ESC-LMS are 0.02 and 0.99, respectively. The value of constant a for $\mu_i^*(k)$ in the proposed method is set to 60. These values of convergence parameters for those algorithms were determined experimentally so that the related steady-state mean squared errors were the same. We see from Fig. 3 and Fig. 4 that increasing the ESR has the effect of increasing the steady-state MSE value of the three algorithms and slowing down the rate of the convergence of the TDL-LMS equalizer algorithm. In the channel 1 approximately 900 samples are required for the TDL-LMS to converge and in the channel 2, TDL-LMS requires more than 1100 samples to converge. On the other hand, the algorithms having ESC structure show no decrease in their convergence speed. In both cases, the proposed has shown no degradation of learning performance compared with the ESC-LMS that converges in about 300 samples.

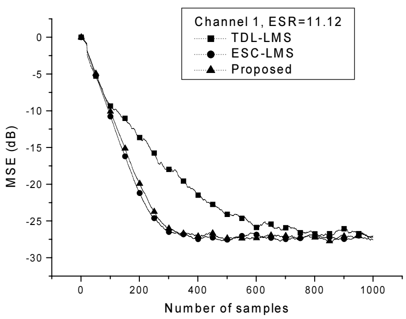

Figure 3. Convergence performance for eigenvalue spread ratio(ESR) = 11.12

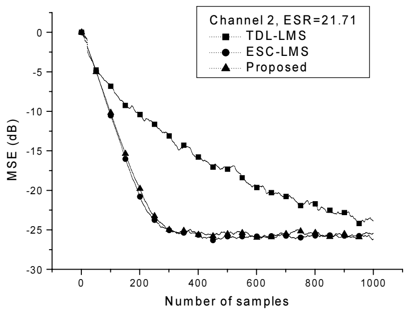

Figure 4. Convergence performance for eigenvalue spread ratio (ESR) = 21.71

Ⅵ. Conclusions

In this paper we proposed a new escalator-coefficient adaptation algorithm that is independent of eigenvalue spread ratio and has lower computational complexity than the conventional ESC LMS algorithm. Using Taylor's expansion of error signal, a method of obtaining optimum convergence coefficient value is derived. Compared with the computational complexity of the ESC-LMS algorithm, the computational complexity of the proposed method can be reduced by 50% without performance degradation. This result indicates that the proposed method does appear to be an attractive alternative to the conventional ESC algorithms for adaptive signal processing applications requiring orthogonalization of the input signal.

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