Complex Infinite Impulse Response Filter Equalization for Digital Vestigial Side Band Signals

Wonzoo Chung* *Regular Member*

ABSTRACT

In this paper, we propose a complex-valued IIR filter for digital VSB signals based on CMA in order to efficiently mitigate multipath distortions, especially the leakage from the quadrature component. The proposed equalizer overcomes the drawback of the conventional real-valued IIR equalizers that it attempts to equalize Hilbert transform of quadrature component. We demonstrate via simulation that the proposed complex IIR filter successfully mitigates the leakages from the quadrature component, while the conventional real IIR filter requires a longer IIR filter to achieve the same performance. We present cost function analysis for a simple two-tap case showing that the proposed IIR equalizer with CMA for VSB signals has a global minimum at the desired location.

Key Words : Adaptive Blind Equalizer, 8-VSB, IIR-Equalization, Constant Modulus Algorithm, Minimum Output Energy Algorithm

Ⅰ. Introduction

In wireless data broadcasting systems, due to the increasing demand for high data rate blind adaptive equalization is preferred. For past decades, linear blind (adaptive) equalization has been intensively studied and widely used in industry[1-3]. Recently, recursive equalization scheme, or infinite impulse response (IIR) equalization scheme, is introduced to deal with much severe multipath channels. As a blind linear equalizer is switched to a decision-directed least mean squared (DD- LMS) equalizer after attaining reasonable symbol rate, IIR-equalizers are used in conjunction with DD-feedback equalizer (DFE)[4, 5]. Mostly used adaptation algorithms for IIR equalizers are Constant Modulus Algorithm (CMA)[2, 3], a popular blind adaptive algorithm for linear equalizers, and Minimum Output Energy (MOE) algorithm[6]. Although, their stability issue and global convergency issue are not thoroughly resolved[7, 8], blind IIR equalizers are currently the most efficient method to acquire open-eyed constellation from the cold start-up when used with DFE.

Especially in ATSC DTV receiver System, an IIR equalizer followed by the DFE plays a critical role to recover received 8-VSB signals distorted from long and severe multipath channel[9]. As illustrated in Figure 1, in a conventional IIR structure the received 8-VSB signal is equalized with a complex-valued linear feedforward filter, and then the real components are processed by a recursive filter to restore 8-PAM[9].

Although it is natural to use real-valued recursive filter for the real-valued PAM signals, it has a critical drawback in the presence of the

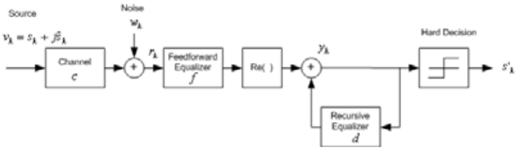

Fig. 1. Conventional IIR Equalization System for 8-VSB signals

※ 본 연구는 명지대학교 IT 교책 연구소의 지원으로 수행되었음.
* 명지대학교 통신공학과 통신신호연구실 (wchung@mju.ac.kr)
논문번호 : KICS2006-05-198, 접수일자 : 2006년 5월 8일, 최종논문접수일자 : 2006년 9월 14일

leakage from the quadrature component. Since the quadrature component of a VSB signal is Hilbert transformation of the in-phase component[10], the effective channel for in-phase component in the presence of quadrature leakage becomes an infinite length channel due to the Hilbert transform filter.

In this paper, we propose a blind adaptive complex-valued recursive IIR equalizer for digital VSB signals using Constant Modulus Algorithm to overcome the drawback of the real-valued IIR equalizer. Theoretical analysis on the convergence of the proposed complex algorithm is prohibitively difficult task due to recursive nature of IIR CMA, although simulation results suggest successful convergence. Hence, in this paper we present limited analysis on a feasible case, a simple one-tap IIR recursive-filter-only equalizer. For this simple case, we show that CMA complex IIR equalizer for VSB has a global local minima at the desired location, while another adaptive complex IIR equaizer, MOE complex IIR equalizer, does not.

In Section II we define the complex CM-IIR equalizer for VSB more rigorously with adaptation rule. In Section Ⅲ we present cost function analysis on the complex CMA IIR and complex MOE IIR equalizers for VSB. Section Ⅳ presents simulation results and Section Ⅴ conclude.

Ⅱ. Complex IIR Equalizers for VSB

Let $\{v_k\}$ denote a VSB signal sequence generated from an independent and identically distributed (i.i.d.) M-level PAM sequence $\{s_k\}$, i.e. $v_k = s_k + j\hat{s}_k$ with the second moment $m_2 = E\{s_k^2\}$ and the fourth moment $m_4 = E\{s_k^4\}$. The quadrature component $\hat{s}_k$ is given by Hilbert transformation filter h, $\hat{s}_k = \sum_n h[n]s_{k-n}$ where $h[n] = 2/(\pi n)$ for odd n and $h[n] = 0$ for even n [12]. Notice that there is a correlation between the real component and the imaginary component of a VSB sequence, $E\{s_m\hat{s}_l\} = h[l-m]$, while the real components and imaginary components are white, $E\{\hat{s}_m\hat{s}_l\} = \delta(m-l)$,

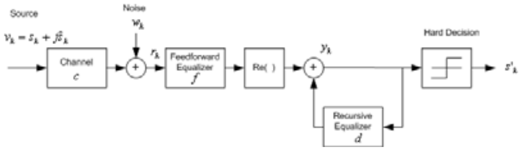

Fig. 2. IIR equalizer for 8-VSB with a complex recursive filter

from the fact that the magnitude response of Hilbert transformation in frequency domain is flat [12] and $E\{s_m s_l\} = \delta(m-l)$. Consequently, a VSB signal sequence is not white, although the original PAM sequence is white, $E\{v_m v_l^*\} = 2m_2 + 2m_2 h[l-m]$.

Consider an IIR equalizer system illustrated in Figure 2. Let r_k denote a received signal through a finite discrete time multipath channel model $\mathbf{c} = [c_0, c_1, \cdots c_{N_c-1}]$, i.e. $r_k = \sum_{n=0}^{N_c-1} c_n v_{k-n}$.

Let $\mathbf{f} = [f_0, \cdots, f_{N_f-1}]$ and $\mathbf{d} = [d_0, \cdots d_{Nd-1}]$ denote a complex-valued transversal filter and a complex-valued recursive filter, respectively. The equalizer output, denoted by y_k, is given by $y_k = \sum_{n=0}^{N_f-1} f_n r_{k-n} - \sum_{n=1}^{N_d-1} d_n y_{k-n}$. To find the transversal filter and recursive filter of the IIR equalizer minimizing MSE without help of training sequence, one uses a sub-optimal cost-function. One of such cost functions is minimum output energy (MOE) [6], which minimizes the output energy of the equalizer with a norm constraint on the feedforward equalizer $\|\mathbf{f}\|^2 = 1$,

$$J_{MOE}(\mathbf{f}, \mathbf{d}) = E\left\{\mathrm{Re}(y_k)^2\right\}, \tag{1}$$

However, the MOE approach does not work for the complex IIR with VSB signals, since VSB signal is not white. As an illustrative example, consider a two-tap FIR channel $[1, d]$. The power of real component of the filter output, $y_k = v_k + dv_{k-1}$, is given as $E\left\{\mathrm{Re}(y_k)^2\right\} = m_2 + m_2|d|^2 - 2m_2 h[1]\mathrm{Im}(d)$ and minmized to $2m_2 - 2m_2 h[1]$ when $d = j$. Hence, we cannot use MOE method, for VSB signals. Another approach is to minimize dispersion of the real component

$$J_{CM}(\mathbf{f},\mathbf{d}) = E\left\{\left(\mathrm{Re}(y_k)^2 - \gamma\right)^2\right\}, \qquad (2)$$

where $\gamma = \frac{m_4}{m_2}$ called dispersion constant [12]. This cost function is known as Constant Modulus (CM) cost function, perhaps the most popular blind algorithm for linear equalizers [12]. The cost function has the global minimum $\frac{m_4^2}{m_2^2} - m_4$ if and only if the received signal is perfectly recovered to original M-PAM signal (with delay ambiguity) in the absence of noise [12]. The stochastic update equation [13] for $\mathbf{f}$ and $\mathbf{d}$ minimizing (13) are given by

$$\begin{aligned} f_n(k+1) &= f_n(k) + \mu r_k^* e(k), \\ d_n(k+1) &= d_n(k) - \mu y_k^* e(k), \end{aligned} \qquad (3)$$

where μ is a step-size, $f_n(k)$ and $d_n(k)$ denote the n-th element of $\mathbf{f}$ and $\mathbf{d}$, respectively, at the k-th iteration, $e(k) = \left(\mathrm{Re}(y_k)^2 - \gamma\right)\mathrm{Re}(y_k)$. denotes the instaneous error at k and $(\cdot)^*$ denotes conjugator. Notice that in the conventional IIR filter for VSB, $\mathbf{d}$ is real-valued and $\mathbf{d}$ uses $\mathrm{Re}(y_k)$ instead of y_k as the input. In the next section, we attempt to provide limited theoretical analysis for a simple feasible case.

Ⅲ. Complex CM IIR equalizer solution for One-Tap Recursive Filter Case

In this section we show that complex CM-IIR equalizer for VSB (13) converges to cancel post cursor term, i.e a global minimum at a desired location, for a one-tap recurse filter in the absence of noise. We assume that carrier phase is perfectly recovered by a phase recover scheme such as [14], and the channel is given as $\mathbf{c} = [1, c]$ for a complex number c with $|c| < 1$ (or assume that the feedforward filter removes post-cursor channel perfectly). Hence, we focus on the recursive filter only assuming $\mathbf{f} = 1$.

Let d denote the coefficient of a one-tap complex recursive filter, and θ the parameter error, $\theta = c - d$. We further assume that $|d| < 1$. Then the equalizer output is $y_k = v_k + \sum_{n=0}^{\infty} (-d)^n \theta v_{k-1-n}$,

In the absence of noise, the zero forcing equalizer is given by $\mathbf{f} = 1$ and $\mathbf{d} = c$, i.e. $\theta = 0$. In the following, we will show that the complex CM-IIR equalizer converges to the desired location $\theta = 0$, equivalently, the CM cost function (2), has a global minimum at $\theta = 0$.

Writing θ and d in the polar coordinate,

$$\theta = |\theta| e^{j\phi_\theta}, \qquad d = re^{j\phi_d}, \text{ we have}$$

$$\mathrm{Re}(y_k) = s_k + |\theta| \sum_{n=0}^{\infty} (-r)^n (a_n s_{k-1-n} - b_n \hat{s}_{k-1-n}), \text{ where}$$

we defined $a_n = \cos(n\phi_d + \phi_\theta)$, $b_n = \sin(n\phi_d + \phi_\theta)$.

For the simplicity of notation, let further denote

$$f(d) = \sum_{n=0}^{\infty} (-r)^n (a_n s_{k-1-n} - b_n \hat{s}_{k-1-n}). \qquad (4)$$

Then

$$\begin{aligned} E\left\{\mathrm{Re}(y_k)^2\right\} &= E\left\{s_k^2\right\} + 2|\theta| E\left\{s_k f(d)\right\} + |\theta|^2 E\left\{f(d)^2\right\}, \\ E\left\{\mathrm{Re}(y_k)^4\right\} &= E\left\{s_k^4\right\} + 4|\theta|^2 E\left\{s_k^3 f(d)\right\} + 6|\theta|^2 E\left\{s_k^2 f(d)^2\right\} \\ &\quad + 4|\theta|^3 E\left\{s_k f(d)^3\right\} + |\theta|^4 E\left\{f(d)^4\right\}. \end{aligned}$$

Finding a closed form expression of above expectation for high order $f(d)^n$ terms is a tedious task. $E\left\{\mathrm{Re}(y_k)^2\right\}$ can be given relatively easily

$$\begin{aligned} E\left\{\mathrm{Re}(y_k)^2\right\} &= m_2 + m_2|\theta| \sum_{n=0}^{\infty} (-r)^n h[n+1] \sin(n\phi_d + \phi_\theta) \\ &\quad + m_2|\theta|^2 \sum_{n=0}^{\infty} r^{2n} + m_2|\theta|^2 \sum_{n \neq m}^{\infty} (-r)^{n+m} h[m-n] \sin((m-n)\phi_d) \end{aligned}$$

Notice that this is a cost function of MOE-IIR equalizer. Due to high order terms on $|d|$, it is difficult to analyze the minima of this cost function. Since the sin terms are not zero, the cost function does not have a unique global minimum at $\theta = 0$. We have

$$
\begin{aligned}
J_{CM}(d) &= m_4 + 4m_4|\theta|\sum_{n=0}^{\infty}(-r)^n h[n+1]\sin(n\phi_d + \phi_\theta) \\
&+ |\theta|^2\left(6E\left\{s_k^2 f(d)^2\right\} + 4|\theta| E\left\{s_k f(d)^3\right\} + |\theta|^2 E\left\{f(d)^4\right\}\right) \\
&- 2\frac{m_4}{m_2}m_2 - 4\frac{m_4}{m_2}m_2|\theta|\sum_{n=0}^{\infty}(-r)^n h[n+1]\sin(n\phi_d + \phi_\theta) \\
&- 2|\theta|^2 E\left\{f(d)^2\right\} + \left(\frac{m_4}{m_2}\right)^2
\end{aligned}
$$

Shortly,

$$
\begin{aligned}
J_{CM}(d) &= \frac{m_4^2}{m_2^2} - m_4 + |\theta|^2(6E\left\{s_k^2 f(d)^2\right\} - 2E\left\{f(d)^2\right\} \\
&+ 4|\theta| E\left\{s_k f(d)^3\right\} + |\theta|^2 E\left\{f(d)^4\right\}) \qquad (5)
\end{aligned}
$$

From a fact that a continuous real-valued function $|z|^2 g(z)$ for complex number z has a stationary point at $|z| = 0$ (from Leibniz Identity), we know that $\theta = 0$ is a stationary point of J_{CM}. Furthermore, J_{CM} attains the minimum achievable value, $m_4^2/m_2 - m_4$, when $\theta = 0$. Hence, $\theta = 0$ is a global minimum of J_{CM} (Figure 3).

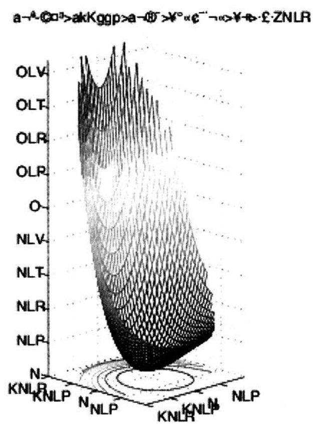

Fig 3. Cost function of the complex CM-IIR equalizer for a one-tap recursive filter for c=0 and |d| 〈 0.4

We have shown that the proposed algorithm globally converges to the desired location where the recursive filter cancels post cursor channel (d=c) for the two-tap channels. Mathematical analysis beyond the two-tap channel case is infeasible due to the complexity of the recursive structure. Hence in the next section we present simulation results showing the desired convergence property of proposed algorithm for general multi-tap channels.

Ⅳ. Simulation Results

In this section we present simulation results confirming the analysis in Section Ⅲ and showing the desired convergence of the proposed algorithm for multi-tap channel cases. In the simulations we compare the performance of the proposed complex- valued CM-IIR equalizer with the conventional real-valued IIR equalizer. The first channel is a following two-tap multipath under SNR= ∞.

$$
c = \delta(k) + 0.5j\delta(k-7) = [1,0,0,0,0,0,0,0.5j]. \qquad (6)
$$

The recursive filter is initialized to 0 and adapted the update rule (3) with the step-size $\mu = 0.0001$ without help of training signals. In Figure 4 we compare the cluster variance trajectories of the proposed complex CM-IIR output with the recursive filter length $N_d = 7$, convention real CM IIR equalizer output with $N_d = 7$ and $N_d = 14$, and real MOE-IIR equalizer of length $N_d = 14$. The conventional real-valued CM-IIR equalizer with $N_d = 7$

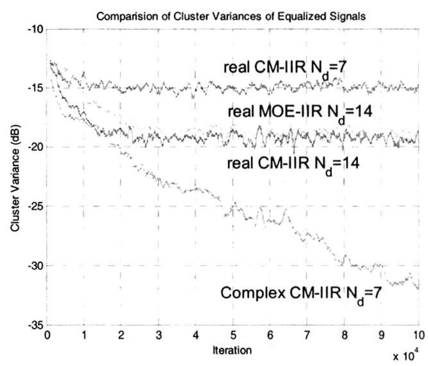

Fig. 4. Cluster Variance Performance Comparison $\mathbf{c} = \delta(k) + 0.5j\delta(k-7)$

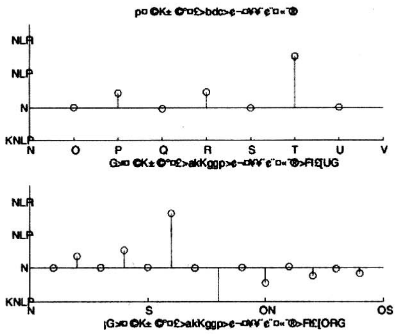

Fig. 5. Converged Recursive Filter Coefficients of the real CM-IIR for $\mathbf{c}=\delta(k)+0.5j\delta(k-7)$

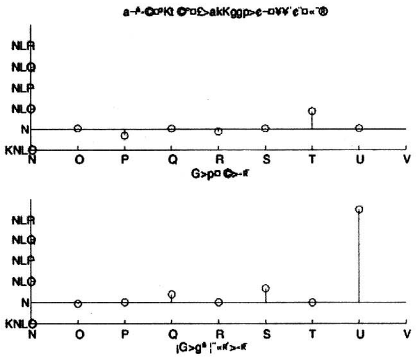

Fig. 6. Converged Recursive Filter Coefficients of the complex CM-IIR $\mathbf{c}=\delta(k)+0.5j\delta(k-7)$

does not open the eye, while an increased filter length $N_d=14$ barely opens eye (the constellation eye opens around -20 dB).

As one can observe in Figure 5, the real-valued recursive filter converged to the truncated Hilbert transform filter corresponding to the $0.5j$ path. On the contrary, the complex recursive filter directly converged to $0.5j\delta(k-7)$ to cancel the multipath (Figure 6). Hence, the complex CM-IIR performs better as the cluster variance trajectory shows in Figure 4.

In the second simulation, we used a more realistic channel, a multi-tap multipath channel,

$$c=[1,0,0.1+0.2*j,-0.4,0,0.1,0,0.1-0.2*j], \quad (7)$$

under 40dB SNR. Notice that the dominating taps (1, 0.4) are real valued and insignificant taps are complex valued. Even for this channel the proposed algorithm presents the same performance trend in the previous complex two tap channel. The complex CM-IIR equalizer with $N_d=7$ achieves superior cluster variance performance than the real CM-IIR with $N_d=14$. The fluctuation of the cluster variance is due to the self-stabilization of the IIR algorithm in the presence of noise as mentioned in [7].

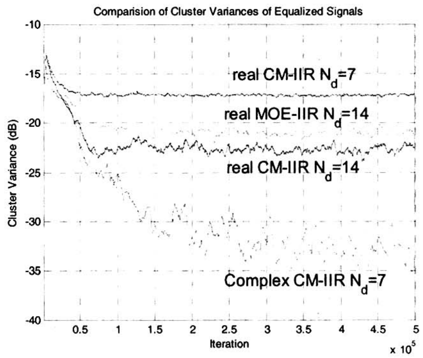

Fig. 7. Cluster Variance Performance Comparison for multi-tap multipath.

V. Conclusion

In this paper, we have presented a complex-complex CM-IIR equalizer for digital VSB signals to overcome a drawback of the conventional complex-real CM-IIR equalizer. Simulation results show that the complex CM-IIR equalizer successfully cancels complex-valued channels, while the conventional real CM-IIR equalizer needs an infinite length feedback filter to cancel them. Cost function analysis showing that the proposed complex CM-IIR equalizer has the global minimum at the desired location is presented for a feasible simple case.

Reference

〔1〕 L. Tong, G. Xu, and T. Kaliath, “Blind channel identification and equalization using second-order statistics: A time-domain approach,” IEEE Trans. Information Theory, vol.~40, pp. 340-349, Mar. 1994.

〔2〕 D.N. Godard, “Self-recovering equalization and carrier tracking in a two-dimensional data communication system," IEEE Trans. Communication, pp. 1867-1875, 1980.

〔3〕 J.R. Treichler and B.Agee, "A new approach to multipath correction of constant modulus signals," *IEEE Trans. Signal Processing*, Apr. 1983.

〔4〕 T. Endres et. al., "Carrier independent blind initialization of a DFE," IEEE SPAWC, Annapolis, MD, pp. 239-242, May 1999.

〔5〕 J. Labat, O. Macchi, and C. Laot, "Adaptive decision feedback eqaulization: Can you skip the training period?" *IEEE Trans. Communication*, pp.921-930, July 1998.

〔6〕 S. Verdu, B. Anderson, and K. R., "Blind equalization without gain identification," *IEEE Trans. Information Theory*, pp. 292-297, 1993.

〔7〕 O. Macchi, *Adaptive Processing,* John Wiley & Sons Inc., 1998.

〔8〕 P. Regalia, *Adaptive IIR Filtering in Signal Processing and Control*} CRC, 1994.

〔9〕 Henderson et al., "ATSC DTV receiver implementation," *Proceedings of the IEEE*, pp. 119-147, Jan. 2006.

〔10〕 J. Proakis, *Digital Communications,* McGraw-Hill, 1995.

〔11〕 A.V. Oppenheim and R.W. Shafer, *Discrete-Time Signal Processing,* Prentice Hall, 1998.

〔12〕 C. R. Johnson, et. al., "Blind equalization using the constant modulus criterion: A Review," *Proc. IEEE*, vol. 86, pp. 1927-1950, Oct. 1998.

〔13〕 J.R. Treichler, C.R. Johnson, Jr., and M.G. Larimore, *Theory and Design of Adaptive Filters,* Prentice Hall, 2001.

〔14〕 W. Chung, W. A. Sethares, and C. Johnson, Jr., "Performance analysis of blind adaptive phase offset correction based on dispersion minimization," *IEEE Trans. Signal Processing*, pp. 1750-1759, June 2004.

정 원 주 (Wonzoo Chung) 정회원

He received the B.A. degree in mathematics from Korea University, Seoul, Korea and the M.S. and Ph.D. degrees in Electrical Engineering from Cornell University, Ithaca, NY. He has worked as the Senior System Architect at Dotcast, Inc, Seattle, WA. He is currently Assoicate Professor at Myong Ji University. His main research area includes digital signal processing for digital communication systems.