



# STBC-OFDM Decoding Method for Fast-Fading Channels

Kyu In Lee\*, Jae Kwon Kim\*\* Regular Members Yong Soo Cho\* Lifelong Member

## ABSTRACT

In this paper, we propose a novel signal detection method that achieves the maximum likelihood (ML) performance but requires much less computational complexity than the ML detection. When the well-known linear decoding method is used for space-time block coded (STBC) OFDM systems in fast-fading channels, co-channel interference (CCI) as well as inter-carrier interference (ICI) occurs. A maximum likelihood (ML) method can be employed to deal with the CCI; however, its computational complexity is very high. In this paper, we propose a signal detection method for orthogonal space-time coded OFDM systems that achieves the similar error performance as the ML method, but requires much less computational complexity.

Key Words : OFDM, STBC, Fast fading, ML, MML

## I. Introduction

Recently, orthogonal frequency-division multiplexing (OFDM) has received a great deal of attention as a radio transmission technology for next generation cellular systems, due to advantages such as high spectral efficiency in multipath fading channels and simple equalization. Transmit diversity techniques using multiple antennas, especially Alamouti coding, have been studied extensively as a method of combating detrimental effects in wireless fading channels because of their relatively simple ML detection at the receiver and the feasibility of having multiple antennas at the base station [1-3].

The OFDM technique can be combined with the transmit diversity technique, dividing a broadband frequency selective channel into a number of flat fading channels, and applying the Alamouti space-time code to each subchannel. The successful combination of OFDM and Alamouti coding relies on a quasi-static channel condition. When the mobile unit is moving at a high speed, however, the simple Alamouti decoding incurs CCI as well as ICI, because the quasi-static assumption is no longer valid.

In order to cancel or reduce the ICI, various methods have been proposed [4-6]. As will be demonstrated in the simulation section, the CCI problem is more important than the ICI problem. The CCI problem was addressed in [4] and [6]. The method in [4] is basically based on the simple Alamouti decoding, and the decisions are used to cancel both the ICI and the CCI terms. Since the Alamouti decoding result at the first stage is not quite reliable, the performance improvement was quite limited. In [6] the ML decoding was used to deal with the CCI problem. Although the performance improvement is significant, as will be shown in Table 1, the computational complexity is very high especially when a large constellation is used.

In this paper, we propose a novel signal detection method that deals with the ICI problem

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<sup>\*</sup> School of Electrical and Electronic Engineering, Chung-Ang Univ. (yscho@cau.ac.kr)

<sup>\*\*</sup> Computer and Telecom. Engineering, Yonsei Univ. 논문번호: KICS2006-10-423, 첫 논문 접수일자: 2006년 10월 11일, 최종논문접수일자: 2007년 1월 22일

that achieves the ML performance but requires much less computational complexity than the conventional ML detection. It is demonstrated by computer simulations that the proposed method can achieve the same performance as that of ML method. We also evaluate the complexities of the conventional ML detection, the Alamouti decoding method, and the proposed detection method, and it is shown that the complexity of the proposed methods much less than the conventional ML detection.

## II. STBC OFDM in fast fading channels

In this section, we describe the STBC OFDM systems in fast fading channels, and we point out the problem that is addressed in this paper. When the channels are fast-fading, the received signal is expressed as follows: [3]

$$Y_t(k) = H_{1,t}(k)C_1(k) + H_{2,t}(k)C_2(k) + I_t(k) + N_t(k)$$
(1)

where  $Y_t(k)$  stands for the received signal at the k-th subchannel during the t-th symbol period k denotes subchannel index and the range of subchannel is 0 to N-1,  $C_i(k), i = 1, 2$  is the transmitted signal from the i-th transmit antenna at the k-th subchannel;  $H_{i,t}(k), i = 1, 2$  is the channel gain between the i-th transmit antenna and the receive antenna at the k-th subchannel;  $I_i(k)$  comprises the ICI components, and  $N_t(k)$  is an additive white Gaussian noise with mean of 0 and variance of  $\sigma_n^2$ . Assuming that the various existing schemes can be used to cancel the ICI term, we disregard the ICI term, then the received signal at the k-th subchannel can be expressed as

$$\begin{bmatrix} Y_{t}(k) \\ Y_{t+1}(k) \end{bmatrix} = \begin{bmatrix} H_{1,t}(k) & H_{2,t}(k) \\ H_{2,t+1}(k)^{*} & -H_{1,t+1}(k)^{*} \end{bmatrix} \begin{bmatrix} C_{1}(k) \\ C_{2}(k) \end{bmatrix} + \begin{bmatrix} N_{t}(k) \\ N_{t+1}(k) \end{bmatrix}$$

$$\mathbf{Y}(k) = \mathbf{H}(k)\mathbf{C}(k) + \mathbf{N}(k)$$
(2)

where, "\*" stands for complex conjugation.

When the channels are quasi-static, i.e.  $H_{i,t}(k)$  and  $H_{i,t+1}(k), i = 1, 2$  are the same, the simple Alamouti decoding offers the ML performance.

When the channels are fast-fading, however, the simple decoding experiences a severe performance degradation. The simple Alamouti decoding in a fast fading environment can be expressed as

$$\overline{\boldsymbol{H}}^{H}(k) \boldsymbol{Y}(k) = \overline{\boldsymbol{H}}^{H}(\boldsymbol{k}) \overline{\boldsymbol{H}}(k) \boldsymbol{\mathcal{C}}(k) + \overline{\boldsymbol{H}}^{H}(\boldsymbol{k}) \overline{\boldsymbol{\mathcal{H}}}(\boldsymbol{k}) \boldsymbol{\mathcal{C}}(\boldsymbol{k}) + \overline{\boldsymbol{H}}^{H}(\boldsymbol{k}) \boldsymbol{\mathcal{N}}(k)$$

$$= \rho(k) \boldsymbol{\mathcal{C}}(k) + CCI(k) + \overline{\boldsymbol{H}}^{H}(k) \boldsymbol{\mathcal{N}}(k)$$
(3)

where, superscript "H" stands for the Hermitian transpose, and  $\overline{H}(k)$ ,  $\widetilde{H}(k)$  are defined as

$$\boldsymbol{H}(k) = \boldsymbol{\overline{H}}(k) + \boldsymbol{\widetilde{H}}(k) \\ = \begin{bmatrix} \overline{H_1}(k) & \overline{H_2}(k) \\ \overline{H_2}(k)^* & -\overline{H_1}(k)^* \end{bmatrix} + \begin{bmatrix} \alpha_1(k) & \beta_2(k) \\ -\beta_2(k)^* & \alpha_1(k)^* \end{bmatrix}$$
(4)

and  $\rho(k) = |\overline{H_1}(k)|^2 + |\overline{H_2}(k)|^2$ ,  $CCI(k) (= \overline{H}^{-\mu}(k)\overline{H}(k)C(k))$ denotes the CCI component. We note that when the channels are quasi-static, CCI(k) becomes zero. Due to the nonzero CCI term in fast fading channels, performance degradation occurs if (3) is used as a decoding scheme.

## II. Signal Detection Methods

In this section, we describe the conventional ML method, then we propose a novel detection method to deal with the CCI term.

## 3.1 Conventional ML Detection Method

ML performance can no longer be obtained by the simple linear operation; instead following problem needs to be solved to achieve a better performance.

$$\widehat{\boldsymbol{C}}(k) = \arg \min_{C_1(k), C_2(k) \in C_M} \boldsymbol{Y}(k) - \boldsymbol{H}(k) \begin{bmatrix} C_1(k) \\ C_2(k) \end{bmatrix} (5)$$

Here  $C_M$  denotes the constellation points, and the constellation size is denoted as  $|C_M|$ . ML signal detection necessitates  $|C_M|^2$  times calculation of the ML metric to solve the equation (5), which makes it not attractive from a hardware implementation perspective.

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#### 3.2 Proposed Detection Method

We propose a signal detection method that achieves the ML performance at a much lower computational cost than the conventional ML detection method. The proposed method is based on the following equation:

$$C_{2,ML}(k) = \arg \min_{C_2(k) \in C_M} \| \boldsymbol{Y}(k) - \boldsymbol{H}_1(k) C_{1,ML}(k) - \boldsymbol{H}_2(k) C_2(k) \|$$

$$= \arg \min_{C_2(k) \in C_M} \left\| \frac{\boldsymbol{H}_2(k)^H}{\| \boldsymbol{H}_2(k) \|^2} (\boldsymbol{Y}(k) - \boldsymbol{H}_1(k) C_{1,ML}(k)) - C_2(k) \| \right\|$$

$$= Q \left( \frac{\boldsymbol{H}_2(k)^H}{\| \boldsymbol{H}_2(k) \|^2} (\boldsymbol{Y}(k) - \boldsymbol{H}_1(k) \boldsymbol{C}_{1,ML}(k)) \right)$$

$$= Q \left( \frac{\boldsymbol{H}_2(k)^H}{\| \boldsymbol{H}_2(k) \|^2} (\boldsymbol{Y}(k) - \boldsymbol{H}_1(k) \boldsymbol{C}_{1,ML}(k)) \right)$$
(6)

where  $C_{i,ML}(k), i = 1, 2$  stands for the ML solution of  $C_i(k), i = 1, 2$ ;  $H_j(k), j = 1, 2$  is the j-th column of the matrix H, and  $Q(\cdot)$  is the slicing function that finds the constellation point that is the nearest to the argument value. Equation (6) indicates that the ML solution  $C_{2,ML}(k)$  is expressed as a function of the ML solution  $C_{1,ML}(k)$ . Based on equation (6), we propose the following signal detection method:

#### *STEP*1 :

$$\hat{C}_{1,ML}(k) = \arg \prod_{C_1(k) \in C_M} | \boldsymbol{Y}(k) - \boldsymbol{H}_1(k)C_1(k)$$

$$- \boldsymbol{H}_2(k) Q \left( \frac{\boldsymbol{H}_2(k)^{\boldsymbol{H}}}{|\boldsymbol{H}_2(k)|^2} (\boldsymbol{Y}(k) - \boldsymbol{H}_1(k)C_1(k)) \right)$$

$$(7)$$

**STEP2**:  $C_{2,ML}(k)$  is obtained using equation (6).

Using the proposed method, the number of metric calculations is reduced from  $|C_M|^2$  to  $|C_M|$ . We note that the relation (6) can be modified into the following equation:

$$C_{1,ML}(k) = Q\left(\frac{\boldsymbol{H}_1(\boldsymbol{k})^{\boldsymbol{H}}}{\|\boldsymbol{H}_1(\boldsymbol{k})\|^2}(\boldsymbol{Y}(\boldsymbol{k}) - \boldsymbol{H}_2(\boldsymbol{k})\boldsymbol{C}_{2,ML}(\boldsymbol{k}))\right)$$
(8)

Therefore, the signal detection for  $C_{2,ML}(k)$  can be performed in Step 1 modifying equation (7), and  $C_{1,ML}(k)$  can be obtained in Step 2 using equation (8). In the case that the signals,  $C_1(k)$ and  $C_2(k)$  are drawn from signal constellations of different size, detection of the signal drawn from the smaller constellation can be performed in Step 1, and the remaining symbol can be detected in Step 2, in order to further reduce the complexity.

## **IV.** Simulation

In this section, we compare the ML decoding, Alamouti decoding, and the proposed decoding methods in terms of the error performance and the computational complexity.

**Error Performance:** The ITU-PA (Pedestrian A) channel is used; FFT size is 1024; CP length is 256 samples; carrier frequency is 2.3GHz, and the spectral bandwidth is 10MHz.

Fig. 1 shows the error performance of various decoding techniques in STBC-OFDM systems. The performance of the simple Alamouti decoding is denoted as "Alamouti" the ML performance is denoted as "ML", the performance of the proposed method is indicated as "Proposed". "SDFSE" indicates the sequential decision feedback sequence estimation (SDFSE) method with q = 1 to deal with the ICI problem [6]. "Alamouti-SDFSE" denotes the method in which the Alamouti decoding (ignoring the CCI term) is used to decode first, then the decoded symbols are used to cancel the ICI using the SDFSE. In a similar way, "Proposed-SDFSE" is a method in which the proposed method is used to deal with the CCI problem and decode first, then the



Fig. 1. Performances of STBC decoding methods

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decoded symbols are used to cancel the ICI terms using the SDFSE. "ML-SDFSE" method was not simulated, because the ML detection and the proposed method shows the similar performance, thus the "ML-SDFSE" obviously shows the similar performance as the "Proposed-SDFSE".

As can be seen in Fig. 1, when the mobile speed is 3Km/h (fdTs=0.00064), all detection methods (Alamouti, ML, and proposed method with or without SDFSE) achieve the similar performance. When the mobile speed is 120Km/h (fdTs=0.0256), however, the Alamouti decoding suffers a severe performance degradation due to the CCI as well as ICI. We can also see that the Alamouti decoding combined with SDFSE shows a negligible performance gain when compared with the Alamouti only decoding method. It is because that the unreliable Alamouti decoding results. The ML detection (without SDFSE) and the proposed method (without SDFSE) show the similar performance that is much better than that of Alamouti decoding. Furthermore, it can be seen that the proposed method is successfully combined with the SDFSE, showing significant additional error performance improvement. It seems that this is because that the reliable decoding results of the proposed method are used in the SDFSE.

**Computational Complexity**: Following are our assumptions in estimating the complexities of the various decoding methods.

- We assume that the constellation points  $C_1(k)$ ,  $C_2(k)$  have integer valued real parts and imaginary parts. (This condition can be made valid by applying an appropriate scaling factor in the automatic gain control (AGC) part or in the channel estimation part.)
- From the above assumption, the multiplication *H*(*k*)*C*(*k*) can be implemented as shift operations instead of multiplications.
- In estimating the complexities, only the number of complex multiplications and divisions are counted, excluding simple comparisons or shifting operations. In general shifting operation

is very simple from a hardware implementation perspective.

 The slicing operation using Q(·) was not included in estimating complexity, because the comparator is also very simple from a hardware implementation perspective.

Table 1 compares the Alamouti decoding, ML method, and the proposed method in terms of complex multiplication. In the Alamouti decoding, 4 complex multiplications are required for  $\hat{\boldsymbol{S}}(k) = \overline{\boldsymbol{H}}(k)^{H} \boldsymbol{Y}(k), \quad 2 \text{ complex multiplications for}$  $\rho(k) = \left| \overline{H_1}(k) \right|^2 + \left| \overline{H_2}(k) \right|^2$ . Finally, decision is made by  $Q\left(\frac{\hat{\boldsymbol{S}}_{i}(k)}{\rho(k)}\right)$ , i = 1, 2, where  $\hat{\boldsymbol{S}}_{i}(k)$ , i = 1, 2 is the i-th element of the vector  $\hat{\boldsymbol{S}}(k)$ . We note that the explicit divisions  $\frac{\hat{\boldsymbol{S}}_i(k)}{\rho(k)}, i = 1, 2$  can be avoided. Since the constellation points are integer values, the decision boundaries of used constellations are also integer values, therefore the multiplication of  $\rho(k)$  and the decision boundaries can be implemented as shifting operation instead of multiplications. We can compare  $\hat{\boldsymbol{S}}_{i}(k), i = 1, 2$  with the shifted decision boundaries, thus avoiding the Therefore, the complexity of the divisions. Alamouti decoding is 6 complex multiplications in total.

In the conventional ML method, the multiplication  $\boldsymbol{H}(k)\boldsymbol{C}(k)$  can be implemented as shift operations, thus the main complexity is caused in the norm value calculation  $\||\boldsymbol{Y}(k) - \boldsymbol{H}(k)\boldsymbol{C}(k)||^2$  that requires 2 complex multiplications. The number of possible  $\boldsymbol{C}(k)$  is  $|C_M|^2$ , thus the total number of required multiplications is  $2 \times |C_M|^2$ .

In order to estimate the complexity of the proposed method, we change the equation (6) into the following equation.

$$C_{2,ML}(k) = Q\left(\frac{H_2(k)^{H}}{\|H_2(k)\|^{\beta}} (Y(k) - H_1(k)C_1(k))\right)$$
$$= Q\left(\frac{1}{\|H_2(k)\|^{\beta}} (H_2^{H}(k)Y(k) - H_2^{H}(k)H_1(k)C_1(k))\right)$$
$$= Q\left(\frac{1}{\|H_2(k)\|^{\beta}} (a - bC_1(k))\right)$$
(9)

where  $a = \boldsymbol{H}_2^H(k) \boldsymbol{Y}(k)$ , and  $b = \boldsymbol{H}_2^H(k) \boldsymbol{H}_1(k)$ .

In the proposed detection method we need 4 multiplications for the calculation of a and b. The multiplication  $bC_1(k)$  can be implemented as shift operation, and the division by  $\|\boldsymbol{H}_2(k)\|^2$  can be avoided by adjusting the decision boundaries as in the Alamouti decoding method. In adjusting the decision boundary we have only to calculate  $\|\boldsymbol{H}_2(k)\|^2$ that requires 2 multiplications. The main complexity is caused by the  $\|\mathbf{Y}(k) - \mathbf{H}(k)\mathbf{C}(k)\|^2$  calculation that requires 2 multiplications. In the proposed method, the number of considered vector  $\boldsymbol{C}(k)$  is  $|C_M|$ . Therefore the complexity of the proposed method is  $6+2 \times |C_M|$  complex multiplications. The complexities of the decoding methods are compared in Table I. As can be seen in Table I, the complexity of the proposed method is dramatically reduced when compared to the ML method.

Table 1. The complexities of the Alamouti, ML, and the proposed decoding methods

Constellation	Alamouti	ML method	Proposed method
Complexity formula	6	$2 \times  C_M ^2$	$6+2 \times  C_M $
QPSK	6	32	14
16-QAM	6	512	38
64-QAM	6	8,192	134

## V. Conclusion

Given that the proposed method requires much less computational complexity than the conventional ML signal detection method and that an similar performance is achieved by the two methods, it can be concluded that the proposed method offers the best trade-off between performance and complexity.

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이 규 인 (Kyu In Lee) 정회원 2001년 2월: 중앙대학교 전자 전기 공학부 졸업 2003년 2월 : 중앙대학교 대학원 자전기공학부 졸업 2003년 3월~현재: 중앙대학교 대 학원 자전기공학부 박사과정 <관심분야> 디지털통신, 무선모뎀,

OFDM, MIMO

### 김 재 권(Jae Kwon Kim)



Kim) 정회원 1995년 8월 중앙대학교 전기공 학과 졸업 2000년 2월 중앙대학교 전기공

학과석사

2004년 5월 The University of Texas at Austin 공학박사 2004년 8월~2005년 8월 삼성종

합기술연구원 4G 시스템 Lab

2005년 9월~현재 연세대학교 원주캠퍼스 컴퓨터정보 통신공학부 교수

<관심분야> 디지털 통신, OFDM, MIMO, 실용적인 신 호검출 기법 조용수(Yong Soo Cho)



1984년 2월 : 중앙대학교 전자 공학과 졸업(학사) 1987년 2월 : 연세대학교 전자 공학과 졸업(석사) 1991년 2월 : The University of Texas at Austin 공학박사 1992년 3월~현재: 중앙대학교 전자

종신회원

전기공학과교수

2003년 8월~현재: TTA 휴대인터넷 프로젝트 그룹 (PG302) 무선접속 실무반 의장

<관심분야> 디지털통신, OFDM 모뎀 설계, MIMO-OFDM 모뎀 설계