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MIMO 시스템을 위한 Out-of-Constellation Point 보정 Lattice Reduction-aided 검출기법

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Lattice Reduction-aided Detection with Out-of-Constellation Point Correction for MIMO Systems

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요 약

MIMO 시스템을 위한 Lattice Reduction (LR) aided 검출 방법의 중요한 단점의 해결책으로 심벌 판별의 신뢰도가 의심되는 경우 추가적인 보정을 해주면서, 계산양은 매우 작은 개선된 LR-aided 검출방법을 제안한다. LR 검출방법의 오류들은 주로, 마지막 lattice 역 변환 단계에서, lattice point들이 constellation point의 바깥으로 벗어날 때 발생한다는 것을 발견하였다. 이를 기반으로, 제안된 기법에서는 LR-aided 검출방법을 통해 얻어진 lattice point들이 constellation의 바깥쪽에 있는지를 확인하고, 바깥쪽에 있는 경우, 모든 constellation point들이 아닌, 얻어진 lattice point의 이웃하는 point들에 대해서만 ML(Maximum Likelihood) 검출을 수행한다. 제안된 기법을 사용함으로, 검출 성능은 ML에 근접하면서, 계산양은 아주 작은 효과를 기대할 수 있다.

Key Words : MIMO, Lattice, Out-of-Constellation, ML

ABSTRACT

An important drawback in Lattice Reduction (LR) aided detectors has been investigated. For the solution, an improved LR aided detection with ignorable complexity overhead is proposed for MIMO system, where the additional correction operation is performed for the case of unreliable symbol decision. We found that LR aided detection errors mainly occur when the lattice points after the inverse lattice transform in the final step fall outside the constellation point set. In the proposed scheme, we check whether or not the lattice point obtained through LR detection is out of constellation. Only for the case of out of constellation, we additionally perform ML search with reduced search region restricted to the neighboring points near to the obtained lattice points. Using this approach, we can effectively and significantly improve the detection performance with just a slight complexity overhead which is negligible compared to full searched ML scheme. Simulation results show that the proposed scheme achieves the detection performance near to that of the ML detection with a lower computational complexity.

Ⅰ. INTRODUCTION

The concept of multiple input - multiple output (MIMO) based communications has received widespread attention from the communication society[1]. The use of MIMO techniques in wireless transmission promises a tremendous increase in spectral efficiency[2]. As the number of dimensions increases, the detection problem

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quickly becomes very complex, since with *M*-ary signaling in each dimension, a *K*-dimensional signaling vector allows for *MK* different transmit signals at each time step. This exponential growth of the signaling set calls for low complexity suboptimum detector structures like Zero Forcing (ZF), Minimum-Mean Square Error (MMSE), VBLAST, several of which have been explained in, e.g.,[3, 4, 5]. Unfortunately for ill-conditioned channel matrices, all these schemes are clearly inferior to Maximum-Likelihood (ML) detection. A very general approach to obtain approximate solutions to the so-called lattice closest vector problem is found in the reference[6]. The application of this method to MIMO transmission schemes has been called Lattice Reduction (LR) aided detection[7, 8]. The error rate curve for this detector show a very promising performance. Its BER curve slope to the SNR increase is the same to that of ML detection, i.e., the LR aided detector achieves the full diversity offered by the channel, even when simple linear ZF[7] and MMSE[8] equalization is used.

In this paper, we investigated important drawback in LR aided detectors and developed an improved LR-aided detection scheme. We found that LR aided detection errors mainly occur when the lattice points after the inverse lattice transform in the final detection step falls outside of the signaling constellation. One solution of this problem is discussed in[9], where the reduced ML search algorithm with Lenstra-Lenstra-Lovasz(LLL) LR algorithm is implemented. That scheme turns out to be rather complex, since it uses parallel detectors and LR techniques.

Based on our proposed scheme, we showed that this problem can be avoided even with a slight modification of conventional LR-aided detectors. In the proposed scheme, the additional reduced ML search is applied only for the case of unreliable symbol decision. Using this approach, we can effectively and significantly improve the detection performance with just a slight complexity overhead. Simulation results show that the proposed scheme achieves the detection performance near to that of the ML detection with a lower computational complexity.

The paper is structured as follows; The Section II is devoted to description of suboptimal linear detectors (ZF, MMSE) and their improvement by combining LR. In the Section III the weakness of LR detectors and as a solution, the proposed algorithm is described. The Section IV contains simulation results. Finally, some concluding remarks are offered in the Section V.

Ⅱ. INTRODUCTION TO LR AIDED DETECTORS

Consider a MIMO system with N_T transmit and $N_R(\geq N_T)$ receive antennas. First, the data is demultiplexed into N_T data sub streams. These sub streams are mapped onto M-QAM symbols and transmitted over the N_T antennas simultaneously (Fig.1). In the traditional MIMO system the received signal can be expressed as:

$$\boldsymbol{y} = \boldsymbol{Hs} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{H}$ is the channel matrix whose elements are the uncorrelated complex Gaussian random variables with unit variance, $\boldsymbol{s} \in A^{N_T}$ is transmitted signal vector where A denotes the lattice of the symbol alphabet and the entries of vector $\boldsymbol{n}$ are independent Gaussian noise component with variance $\sigma^2 = N_0/2$, where $N_0/2$ is a background noise power spectral density. The optimal ML detector searches the whole set of possible cases and decides which minimizes the Euclidean distance:

$$\hat{s} = \underset{s \in A^{N_T}}{\arg\min} \left\| y - \boldsymbol{H}\boldsymbol{s} \right\|^2 \tag{2}$$

For the detection process, although the performance of ML detector is optimal, its complexity is very

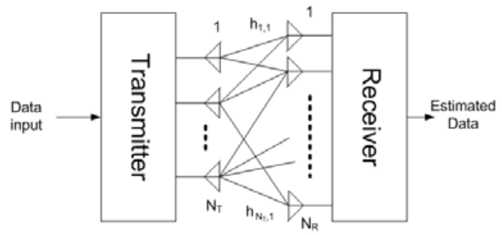

Fig. 1 Proposed MIMO system

high. Whereas the LR aided detectors offer substantially lower complexity and very close performance to ML. As the linear detection methods are used in LR aided detectors, we made brief introduction to them.

2.1 Linear Detectors

More simple detection methods are so called linear detectors. A straightforward approach to recover s from y is to use an $(N_T \times N_R)$ weight matrix W to linearly combine the elements of y to estimate $\boldsymbol{s}$, i.e. $\hat{\boldsymbol{s}} = Wy$

2.1.1 Zero-Forcing (ZF)

The ZF is a linear equalization with respect to the Zero-Forcing criterion. The ZF algorithm attempts to null out the interference introduced from the matrix channel by directly inverting the channel with the weight matrix:

$$\boldsymbol{W}^{ZF} = \boldsymbol{H}^{\perp} = (\boldsymbol{H}\boldsymbol{H}^{*})^{-1}\boldsymbol{H}^{*} \qquad (2)$$

where $\boldsymbol{H}^{*}$- refers to conjugate and $\boldsymbol{H}^{\perp}$ is the pseudo inverse of channel matrix $\boldsymbol{H}$.

2.1.2 Minimum Mean-Squared Error (MMSE):

A drawback of the ZF is that nulling out the interference without considering the chance that noise could boost up the noise power significantly, which in turn results in performance degradation. To solve this, MMSE minimizes the mean squared-error:

$$\boldsymbol{W}^{MMSE} = (\boldsymbol{H}^{*}\boldsymbol{H} + \sigma_n^2 / \sigma_s^2 \boldsymbol{I}_{N_T})^{-1}\boldsymbol{H}^{*} \qquad (2)$$

where $\boldsymbol{H}^{*}$refers to conjugate transpose of channel matrix and σ_s^2, σ_n^2 are the signal and noise variances respectively.

2.2 Conventional Lattice basis reduction

In the case of bad channel decision boundaries for suboptimal detectors are undesirably elongated, therefore small amount of noise would lead them to detection error[7]. The LR technique is a good solution of this problem. By using LR technique, the performance of linear detectors can be improved by using more orthogonal basis vectors for lattice because the decision boundaries of linear detection methods are determined by the basis vectors.

A lattice in N_T complex dimensions can be represented by $\boldsymbol{L} = \{\boldsymbol{x} | \boldsymbol{x} = \boldsymbol{H}\Lambda\}$ where the channel matrix $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \dots, \boldsymbol{h}_{N_T}]$ is a matrix of basis vectors for lattice. The vector $\boldsymbol{h}_i$ is the i-th column of the real-valued channel matrix $\boldsymbol{H}$. $\Lambda = [\lambda_1, \lambda_2, \dots, \lambda_{N_T}]^T$ is a vector of complex integer weights. For any lattice L, there are many possible bases. Therefore, we may consider more orthogonal channel matrix $\boldsymbol{H}'$ for lattice basis. The lattice $\boldsymbol{L}$ can be represented by using this $\boldsymbol{H}'$ channel matrix, which makes the received signal more detectable for linear detectors. If $\boldsymbol{H}$ and $\boldsymbol{H}'$ are bases for lattice L, then following represents the connection between them:

$$\boldsymbol{H}' = \boldsymbol{HP} \qquad (5)$$

where $\boldsymbol{P}$ is chosen such that both $\boldsymbol{P}$ and $\boldsymbol{P}^{-1}$ have integer entries, where the determinant for each matrix is equal to +/- 1.

The main idea of LR technique is to operate in chosen nearly orthogonal lattice basis, since linear detectors can provide optimal estimates of the transmitted symbols if the channel matrix is orthogonal. Lattice reduction methods generate the equivalent channel matrix, which is nearly orthogonal.

In the traditional linear detection systems, the detector compensates the effect of original channel $\boldsymbol{H}$ to produce the estimate of $\hat{\boldsymbol{s}}$. On the other hand, in the LR technique, we use a new nearly orthogonalized channel $\boldsymbol{H}'(=\boldsymbol{HP})$ in stead of $\boldsymbol{H}$ in order to produce the estimate of $\hat{\boldsymbol{z}}$ which is one-to-one mapped into a point in the channel $\boldsymbol{H}$. For example, if ZF is used, then $(\boldsymbol{H}')^{-1}\boldsymbol{y}$ is quantized to produce $\hat{\boldsymbol{z}}$ and then, $\hat{\boldsymbol{s}}$ is produced by $\hat{\boldsymbol{s}} = P\hat{\boldsymbol{z}}$ as shown in Fig. 2.

Nowadays, many kinds of LR algorithms have been developed. Lattice reduction in high dimensions is a known Nondeterministic Polynomial-time(NP)-hard

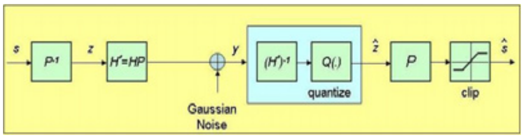

Fig. 2 Lattice reduction technique

problem. In this paper, we used optimal iterative complex LR algorithm investigated in [7]. If we assume that vectors $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are first and second columns of 2×2 channel matrix $\boldsymbol{H}$, the general procedure of the lattice basis reduction algorithm can be represented as follows:

2.2.1 Correlation check.

If $|Re(\langle \boldsymbol{h}_1, \boldsymbol{h}_2' \rangle)| \le 0.5 \| \boldsymbol{h}_1 \|^2$ and $|Im(\langle \boldsymbol{h}_1, \boldsymbol{h}_2' \rangle)| \le 0.5 \| \boldsymbol{h}_1 \|^2$ then stop. Otherwise replace $\boldsymbol{h}_2$ with $\boldsymbol{h}_2' = \boldsymbol{h}_2 - \left\lfloor \frac{< \boldsymbol{h}_1, \boldsymbol{h}_2 >}{\| \boldsymbol{h}_1 \|^2} \right\rceil$, where $\lfloor \cdot \rceil$ is the rounding operation, then go to the second step.

2.2.2 Length check.

Check whether $\| \boldsymbol{h}_1 \| < \| \boldsymbol{h}_2 \|$. Stop, if vectors satisfy this condition. Otherwise, swap $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ and go to the first step.

After such operations we will have the reduced basis $\boldsymbol{H}'$ with properties:

$$\| \boldsymbol{h}_1 \| < \| \boldsymbol{h}_2 \|, \ |Re(\langle \boldsymbol{h}_1, \boldsymbol{h}_2' \rangle)| \le 0.5 \| \boldsymbol{h}_1 \|^2$$
$$\text{and } |Im(\langle \boldsymbol{h}_1, \boldsymbol{h}_2' \rangle)| \le 0.5 \| \boldsymbol{h}_1 \|^2 .$$

Although, for simplicity, we focused on the 2×2 case ($N_R = N_T = 2$) that arises frequently in practice, the proposed algorithm can be successfully extended to higher number of transmit and receive antennas, while maintaining the same performance improvement.

Ⅲ. PROPOSED DETECTION SCHEME

Lattice reduction realizes decision regions much closer to that ML, and significantly increases the performance of suboptimal detectors and gives us the same diversity as ML. It has been shown that LR technique achieves the same slop to ML within 3dB [7, 8]. We notice that this difference in performance is mostly caused by incorrect detection of LR aided detectors, when the lattice point ($\boldsymbol{P}\hat{\boldsymbol{z}}$) at the final step in LR detection procedure, falls outside the constellation point set (see Fig. 3).

For simple understanding of our purpose, we used one dimensional signaling constellation to demonstrate the noticed problem in LR aided detection methods (see Fig. 3, Fig. 4). In traditional detectors, where the constellation has the rectangular shape it is guaranteed that after slicing and clipping operations, the received signal point falls inside of the signaling constellation. In LR aided detection technique, we first slice the received signal value to the nearest integer point in the constellation of $z \in P^{-1}A^2$, however, this can not always guarantee that the sliced point belongs to the constellation of z whose lattice shape is skewed. Also, the clipping operation is not applicable due to skewed constellation shape unlike the original constellation of $s \in A^2$.

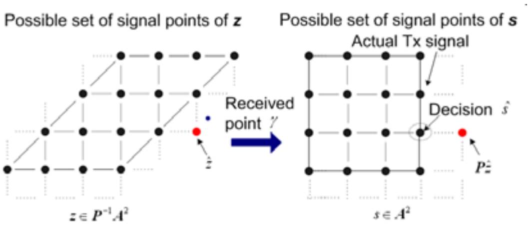

Fig. 3 Incorrect decision in the case of out-of constellation event

As shown in Fig.3, even though the received point is closest to the actual transmitted signal in the constellation set of z, it is sliced into the closest but out-of-constellation point, $\hat{\boldsymbol{z}}$, which is mapped into the incorrect point $\hat{\boldsymbol{s}}$ after the lattice-inverting and clipping operation.

Let us denote the probability of detection error conditioned on out-of-constellation event as P(A|B), where A is a symbol error event and B is a out-of-constellation event. We simulate and tabulate (see Table 1) the probability P(A|B) for different kinds of QAM modulation in order to show the weakness of the LR in the case of out-of-constellation event.

We can see that in the case of out-of-constellation event, the probability of detection error is very high. This implies that most of the errors are due to this event. Therefore, additional compensation should be selectively applied for the out-of-constellation signal points. The best way is to conditionally use

Table 1. Detection Error Probability Conditioned on Out-of-Constellation Event

P(A|B)			
Es/No[dB]	15	20	25
16-QAM	0.7112	0.8005	0.8452
64-QAM	0.7051	0.7691	0.8301

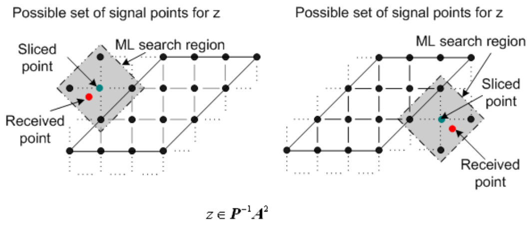

Fig. 4 Reduced ML search for out-of-constellation point in 16 QAM modulation, 2×2 scheme, case when both received signals are out of constellation

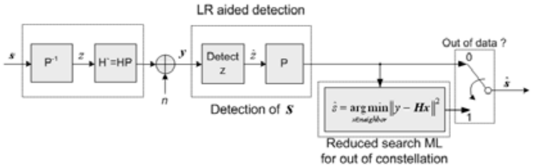

Fig. 5 Block diagram for proposed scheme

the ML search only for the case of out of constellation event. Let us denote this scheme as “LR+Conditioned ML”. However, the “LR+Conditioned ML” is still practically prohibitive due to high complexity since the probability that the signal will be out of constellation is rather high. Therefore, we developed reduced ML search algorithm to mitigate the complexity problem.

Let us assume that at the final step in LR detection procedure, lattice point fell outside the constellation. In that case there is a high probability that the original point lies somewhere near that point. Therefore, we can apply the ML search only for the 4 neighboring points around the sliced point (see Fig. 4). In the Fig. 4, “received point” is the corresponding point to the received signal y in $\boldsymbol{H}'$ domain, and “received point” quantized to the nearest integer value noted as “sliced point”.

The block diagram corresponding to new detector is illustrated below (see Fig. 5). In proposed scheme, we determine whether the point, Pz is out of constellation before the final clipping operation. We apply reduced search ML algorithm only for the out-of-constellation points after traditional LR aided detection in $\boldsymbol{H}'$ basis. Using this approach, we propose a computationally efficient algorithm to mitigate the detection problem for LR aided detectors.

Ⅳ. SIMULATION RESULTS

In this section, we compare the average complexity

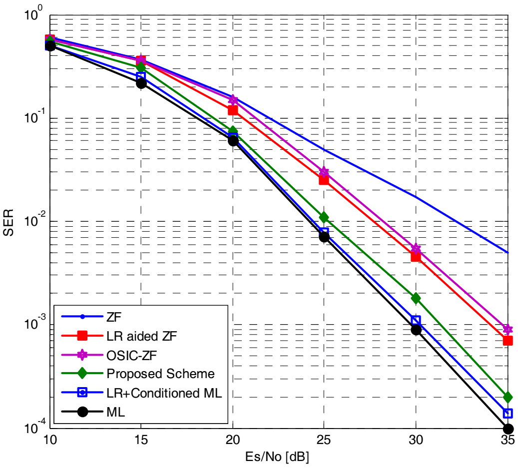

Fig. 6 SER performance (16QAM, 2×2 MIMO systems)

and the error rate performance of the proposed scheme with optimal ML, OSIC-ZF (Ordered Successive Interferrence Cancellation with ZF) in [10] and traditional LR aided detections for 2×2 MIMO system. Even the sphere decoding is widely being used for comparison with newly developed schemes, we did not include the simulation results for that method. However, we note that the complexity of the tree search sphere decoding is a function of transmit / receive antennas and modulation level, whereas the complexity of our proposed scheme only depends on the number of transmit/receive antennas. Consequently, as the modulation level increases, the sphere decoding has more higher complexity than our proposed method. This is an outstanding advantage of the LR-based detection over the sphere decoding.

These simulated algorithms in our paper are chosen for better explanation and illustration of noticed disadvantage in basic LR methods and the performance improvement by the proposed scheme. First, as a comparison with a well-known low compexity detection scheme other than LR-aided detection, it is shown in Fig. 6 and Fig. 7 that the LR-based schemes (conventional and proposed) achieve improved SER performance compared to OSIC scheme.

In this paper, the traditional Zero-Forcing LR (ZF/LR) aided detection method has been used for the performance comparison reference. We noticed that proposed out-of-constellation checking scheme

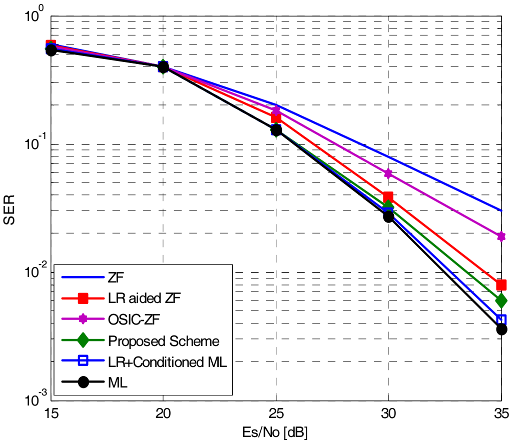

Fig. 7 SER performance (16QAM, 2×2 MIMO systems)

can significantly improve the performance of the traditional LR aided detectors. To see the improvement by our proposed scheme, average symbol-error rate (SER) curves plotted in Fig. 6 for 16-QAM modulation. It is observed that the proposed adaptive scheme is almost same to ML, while the gap between traditional LR technique and the proposed scheme is about 3 dB at Es/No=30dB for 16-QAM modulation.

The SER curves for 64-QAM modulation are also plotted (see Fig. 7). We can see that even for high level modulation the proposed scheme shows the performance almost close to ML case. It is also meaningful to examine the complexity of the proposed detection method. To compare the complexity of the proposed scheme with ML and LR+Conditioned ML methods, we used the average number of searched points per one transmission, which are tabulated in Table II. In every transmission, we transmit two 16-QAM symbols (2×2 MIMO system). The number of flops (operations) per each searched point are the same for all simulated methods. Therefore, we used average number of searched points for complexity comparison. Simulation results showed that in general, the average searched points per one transmission for proposed scheme is about 4, which is significantly smaller than other schemes.

Let us consider the worst case, where both of transmitted signals from two transmitter antennas appear out-of-constellation. In this case the proposed algorithm searches the points whose number is 6 times smaller than LR combined with conditional ML detection techniques. Further more, we note that, the occurrence probability of this worst case is very small (Table 3).

Table 2. Average Number of Search Points per One Transmission 16-QAM Modulation, 2×2 MIMO System

Es/No [dB]	15	20	25
Proposed Scheme	4.05	3.52	3.08
LR with conditional ML search	25.29	22.72	18.83
ML detection	256	256	256

Table 3. The Worst Case Number of Search Points per One Transmission, 16-QAM Modulation, 2×2 MIMO System, Es/No=15dB

Scheme	Search Points	Occurence Probability
Proposed Scheme	16	0.078
LR with conditional ML search	256	0.078
ML detection	256	1.0

Notice that the number of searched point decreases by increasing SNR, since the probability of out-of-constellation event becomes smaller for higher SNR.

Ⅴ. CONCLUSION

In this paper, we proposed the improved LR aided MIMO signal detection technique with low complexity, where the additional correction operation after traditional LR aided detection is performed for the case of unreliable symbol decision. We showed that with the new approach, the large part of the remaining gap between LR aided detectors and ML detection can be reduced. We showed that the complexity of the proposed scheme is significantly low than ML case, while keeping very close performance to it. The significantly low complexity compared with ML detection makes the algorithm applicable for higher level modulation systems.

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