

신호부공간 추정 성능 향상을 위한 전후방 상관을 이용한 PAST(projection approximation subspace-tracking) 알고리즘 연구

정회원 임준석*, 편용국**

A Forward backward Correlation-based Projection Approximation for Subspace Tracking

Jun-Seok Lim*, Yong-Kug Pyeon** *Regular Members*

요약

본 논문에서는 상관계수를 바탕으로 신호부공간을 추정하는 COPAST(correlation-based projection approximation subspace tracking)의 성능을 향상시키기 위하여 상관계수를 구하는 부분을 순방향 신호 벡터로부터 상관계수를 구하고 동시에 역방향 신호 벡터에서도 상관계수를 구하여 신호 부공간을 추정하는 방법을 제안한다. 컴퓨터 모의 실험을 통해서 제안된 방법이 기존의 COPAST에 비해서 약 5dB의 신호 부공간 추정 정확도에 향상이 있었음을 확인하였다. 이로써 순방향과 역방향 모두를 사용하는 것이 성능향상에 도움이 됨을 보였다.

Key Words : Subspace Estimation, PAST, COPAST

ABSTRACT

In this paper, we propose a forward/backward correlation-based subspace estimation technique, which is called forward/backward correlation-based projection approximation subspace tracking (FB-COPAST). The FB-COPAST utilizes the forward and backward correlation matrix in projection approximation approach to develop the subspace tracking algorithm. With the projection approximation, the RLS-based FB-COPAST is presented. The RLS-based FB-COPAST algorithm has the better performance than the recently developed COPAST method. From the simulation results, we can confirm the proposed FB-COPAST outperforms the COPAST as well as the conventional PAST in the estimation accuracy by about 5dB in steady state.

I. Introduction

In recent years, subspace-tracking algorithms have been intensively studied and widely applied to reduce the computational complexity of subspace estimation. Instead of updating the whole eigen-structure, subspace-tracking algorithm only works with the signal

or noise subspace. This makes subspace-tracking algorithm more efficient than conventional methods using eigenvalue decomposition (ED) or singular value decomposition (SVD). One of the attractive subspace-tracking algorithms is the projection approximation subspace-tracking (PAST) algorithm [1]. The idea of the PAST is to make the expectation of the

※ This work was supported by the faculty research fund of Sejong University in 2007.

* 세종대학교 전자공학과 (jslim@sejong.ac.kr), ** 강원도립대학 정보통신과(ykpyeon@wmail.gw.ac.kr)

논문번호 : 08015-0311, 접수일자 : 2008년 3월 11일

squared difference between the input vector and the projected vector minimum. With proper projection approximation, the PAST derives a recursive least squares (RLS) algorithm for tracking the signal subspace. However, the PAST still has room for the improvement in the subspace estimation accuracy. To improve the PAST algorithm, many different algorithms have been proposed. For example, Jung-Lang Yu developed a correlation-based projection approximation subspace tracking (COPAST), to improve the convergence property of the sub-space tracking [2] and Gustafsson proposed an instrumental variable based PAST to cope with the colored noise case [3]. In this paper, we propose a new algorithm to improve the subspace estimation accuracy in the COPAST algorithm [2]. The proposed algorithm utilizes the normal forward ordered input vector and the reversal ordered input vector simultaneously. These forward and backward input vectors can build up the better sample covariance. Therefore, we expect an improved COPAST algorithm with the better subspace estimation accuracy.

The outline of the paper is as follows. In section 2, the COPAST is summarized. Section 3 discusses the property of covariance matrix and the estimated covariance matrix. Then the forward and backward COPAST (FB-COPAST) algorithm is derived. Section 4 shows simulation results to demonstrate that the FB-COPAST performs very well with respect to the subspace estimation accuracy. Finally, Section 5 contains the conclusions.

II. Correlation-based PAST

The PAST (Projection Approximation Subspace Tracking) algorithm is one of the low complex recursive subspace estimation method, which is useful for DOA estimation. The PAST algorithm is based on the minimum property of the unconstrained cost function.

$$J(\mathbf{W}(t)) = \sum_{i=1}^t \beta^{t-i} \|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{W}^H(t)\mathbf{x}(i)\|^2 \quad (1)$$

where $\mathbf{x}(i) = [x_1(i) \ \cdots \ x_N(i)]^T$ is the input vector and $\mathbf{W}(t)$ is a $N \times r$ matrix representing the signal

subspace. To derive recursive update of $\mathbf{W}(t)$ from $\mathbf{w}(t-1)$, Yang in [1] approximates $\mathbf{W}^H(t)\mathbf{x}(i)$ by the expression $y(i) = \mathbf{W}^H(t-1)\mathbf{x}(i)$ which can be calculated for $1 \leq i \leq t$ at the time instant t . Then (1) results in a modified cost function,

$$J'(\mathbf{W}(t)) = \sum_{i=1}^t \beta^{t-i} \|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{y}(i)\|^2 \quad (2)$$

This becomes the exponentially weighted least squares cost function which is well studied in adaptive filtering. $J'(\mathbf{W}(t))$ is minimized if

$$\begin{aligned} \mathbf{W}(t) &= \mathbf{C}_{yy}(t)\mathbf{C}_{yy}^{-1}(t), \\ \mathbf{C}_{yy}(t) &= \sum_{i=1}^t \beta^{t-i} \mathbf{y}(i)\mathbf{y}^H(i) = \beta\mathbf{C}_{yy}(t-1) + \mathbf{y}(t)\mathbf{y}^H(t), \\ \mathbf{C}_{yx}(t) &= \sum_{i=1}^t \beta^{t-i} \mathbf{y}(i)\mathbf{x}^H(i) = \beta\mathbf{C}_{yx}(t-1) + \mathbf{y}(t)\mathbf{x}^H(t). \end{aligned} \quad (3)$$

Yu developed the COPAST algorithm in [2]. The COPAST defines the cost function in terms of the correlation matrix as follows

$$\begin{aligned} J(\mathbf{W}) &= \|\mathbf{C}_{xx} - \mathbf{W}\mathbf{W}^H\mathbf{C}_{xx}\|_F^2 \\ &= \text{Tr}\{\mathbf{C}_{xx}\mathbf{C}_{xx}^H - 2\mathbf{W}^H\mathbf{C}_{xx}\mathbf{C}_{xx}^H\mathbf{W} + \mathbf{W}^H\mathbf{C}_{xx}\mathbf{C}_{xx}^H\mathbf{W}\mathbf{W}^H\mathbf{C}_{xx}\mathbf{C}_{xx}^H\mathbf{W}\} \quad (4) \end{aligned}$$

where $\text{Tr}\{\bullet\}$ denotes the trace of a matrix and F indicates the Frobenius norm. Since the correlation matrix is the second-order statistic of the input vector, the input SNR seems to be square of that in the PAST. Consequently, the COPAST using the correlation matrix improves the performance of the PAST. For convenience of finding the solution of (4), the ensemble correlation matrix is replaced with the exponentially weighted sum

$$\mathbf{C}_{xx}(t) = \sum_{i=1}^t \beta^{t-i} \mathbf{x}(i)\mathbf{x}^H(i) \quad (5)$$

Thus, the cost function becomes

$$J(\mathbf{W}(t)) = \|\mathbf{C}_{xx}(t) - \mathbf{W}(t)\mathbf{W}^H(t)\mathbf{C}_{xx}(t)\|_F^2 \quad (6)$$

Using the projection approximation concept and (5), $\mathbf{W}^H(t)\mathbf{C}_{xx}(t)$ in (6), which is termed $\mathbf{C}_{yx}(t)$, is approximated to

$$\mathbf{C}_{\mathbf{y}\mathbf{x}}(t) = \mathbf{W}^H(t)\mathbf{C}_{\mathbf{x}\mathbf{x}}(t) \cong \sum_{i=1}^t \beta^{t-i} \mathbf{W}^H(i-1)\mathbf{x}(i)\mathbf{x}^H(i) \quad (7)$$

Using (7), the criterion of (6) degenerates to a quadratic optimization problem, and is minimized by

$$\mathbf{W}(t) = \mathbf{C}_{\mathbf{x}\mathbf{x}}(t)\mathbf{C}_{\mathbf{y}\mathbf{x}}^+(t) \quad (8)$$

where $\mathbf{C}_{\mathbf{x}\mathbf{x}}(t) = \beta\mathbf{C}_{\mathbf{x}\mathbf{x}}(t-1) + \mathbf{x}(t)\mathbf{x}^H(t)$ and $^+$ denotes the Moore-Penrose pseudo-inverse [2]. The COPAST algorithm is summarized in Table 1.

Table 1. Summary of COPAST Algorithm

$\mathbf{W}(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \vdots \end{bmatrix}, \quad [L \times r]$
$\mathbf{P}(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \quad [r \times r]$
Do $t=1, \dots$ $\mathbf{y}(t) = \mathbf{W}^H(t-1)\mathbf{x}(t)$ $\mathbf{V}(t) = [\mathbf{C}_{\mathbf{x}\mathbf{x}}(t-1)\mathbf{x}(t) \quad \mathbf{x}(t)]$ $\boldsymbol{\omega}(t) = \mathbf{C}_{\mathbf{y}\mathbf{x}}(t-1)\mathbf{x}(t)$ $\beta^2 \boldsymbol{\Lambda} = \begin{bmatrix} -\mathbf{x}^H(t)\mathbf{x}(t) & \beta \\ \beta & 0 \end{bmatrix}$ $\boldsymbol{\Phi}(t) = [\boldsymbol{\omega}(t) \quad \mathbf{y}(t)]$ $\mathbf{K}(t) = [\beta^2 \boldsymbol{\Lambda} + \boldsymbol{\Phi}^H(t)\mathbf{P}(t-1)\boldsymbol{\Phi}(t)]^{-1} \boldsymbol{\Phi}^H(t)\mathbf{P}(t-1)$ $\mathbf{P}(t) = [\mathbf{P}(t-1) - \mathbf{P}(t-1)\boldsymbol{\Phi}(t)\mathbf{K}(t)] / \beta^2$ $\mathbf{C}_{\mathbf{y}\mathbf{x}}(t) = \beta\mathbf{C}_{\mathbf{y}\mathbf{x}}(t-1) + \mathbf{y}(t)\mathbf{x}^H(t)$ $\mathbf{C}_{\mathbf{x}\mathbf{x}}(t) = \beta\mathbf{C}_{\mathbf{x}\mathbf{x}}(t-1) + \mathbf{x}(t)\mathbf{x}^H(t)$ $\mathbf{W}(t) = \mathbf{W}(t-1) + [\mathbf{V}(t) - \mathbf{W}(t-1)\boldsymbol{\Phi}(t)]\mathbf{K}(t)$ END

III. Forward-Backward COPAST Algorithm

PAST style algorithms such as PAST and COPAST, use the sample covariance matrix $\mathbf{C}_{\mathbf{x}\mathbf{x}}(t)$ which is,

$$\mathbf{C}_{\mathbf{x}\mathbf{x}}(t) = \beta\mathbf{C}_{\mathbf{x}\mathbf{x}}(t-1) + \begin{bmatrix} x_1(t) \\ \vdots \\ x_L(t) \end{bmatrix} \begin{bmatrix} x_1^*(t) & \cdots & x_L^*(t) \end{bmatrix} \quad (9)$$

Generally, the theoretical covariance matrix $\mathbf{c}_{\mathbf{x}}$ is Toeplitz and persymmetric [4]. However, the estimated covariance matrix $\mathbf{C}_{\mathbf{x}\mathbf{x}}(t)$ in (9) does not guarantee all the properties, because it does not satisfy the persymmetric property such as $\mathbf{J}(\mathbf{C}_{\mathbf{x}\mathbf{x}}(t))^T \mathbf{J} \neq \mathbf{C}(t)$,

where $\mathbf{J} = \begin{bmatrix} 0 & & & 1 \\ & \ddots & & \\ & & \ddots & \\ 1 & & & 0 \end{bmatrix}$ is the so-called reversal matrix.

We can make the sample covariance matrix Toeplitz and persymmetric by some modification of (9). The modified sample covariance matrix $\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}$ is,

$$\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t) = \frac{1}{2}(\mathbf{C}_{\mathbf{x}\mathbf{x}}(t) + \mathbf{J}\mathbf{C}_{\mathbf{x}\mathbf{x}}^T(t)\mathbf{J}) \quad (10)$$

The second term of (10) has the following recursive form,

$$\mathbf{J}\mathbf{C}_{\mathbf{x}\mathbf{x}}^T(t)\mathbf{J} = \beta\mathbf{J}\mathbf{C}_{\mathbf{x}\mathbf{x}}^T(t-1)\mathbf{J} + \begin{bmatrix} x_L^*(t) \\ \vdots \\ x_1^*(t) \end{bmatrix} \begin{bmatrix} x_L(t) & \cdots & x_1(t) \end{bmatrix} \quad (11)$$

The modified sample covariance matrix $\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t)$ utilizes the sample covariance matrix of the reverse ordered vector as well as that of the normal forward ordered vector. Therefore, we can call the $\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t)$ as the forward-backward covariance matrix. The forward-backward covariance matrix, $\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t)$, is invariant to the transform, $\mathbf{J}(\bullet)^T \mathbf{J}$,

$$\mathbf{J}(\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t))^T \mathbf{J} = \tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t) \quad (12)$$

(12) shows that $\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t)$ is persymmetric [4]. Therefore, We may expect $\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t)$ to be a better estimate of $\mathbf{C}_{\mathbf{x}\mathbf{x}}$ than $\mathbf{C}_{\mathbf{x}\mathbf{x}}(t)$. In turn, this means that the estimated principal components derived from $\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t)$ are likely to be more accurate than those obtained from $\mathbf{C}_{\mathbf{x}\mathbf{x}}(t)$. To apply the forward-backward covariance matrix to the PAST style algorithm, we should modify (9) considering a forward-backward covariance matrix. The recursive forward-backward covariance matrix is as follows.

$$\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t) = \beta\tilde{\mathbf{C}}_{\mathbf{x}\mathbf{x}}(t-1) + \begin{bmatrix} x_1(t) & x_L^*(t) \\ \vdots & \vdots \\ x_L(t) & x_1^*(t) \end{bmatrix} \begin{bmatrix} x_1^*(t) & \cdots & x_L^*(t) \\ x_L(t) & \cdots & x_1(t) \end{bmatrix} \quad (13)$$

Comparing (13) with (10), (13) needs a scaling factor, 1/2. However, the scaling factor does not affect the subspace so that we dismiss the factor. Applying (13) to COPAST algorithm, we can derive a new COPAST algorithm with the forward-backward covariance matrix.

To handle the forward data vector and the backward data vector simultaneously, we can modify the data

vector $\mathbf{X}(t)$ in (8) to $\tilde{\mathbf{X}}(t) = \begin{bmatrix} x_1(t) & x_L^*(t) \\ \vdots & \vdots \\ x_L(t) & x_1^*(t) \end{bmatrix}$. Then

COPAST can be modified as follows.

$$\mathbf{W}(t) = \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t) \quad (14)$$

$$\mathbf{W}(t) = \mathbf{W}(t-1) + (\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) - \mathbf{W}(t-1) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t)) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \quad (15)$$

Table 2. Summary of FB-COPAST Algorithm

$\mathbf{W}(0) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}, \quad [L \times L]$
$\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(0) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}, \quad [r \times r]$
<p>Do $t=1, \dots$</p>
$\tilde{\mathbf{X}}(t) = \begin{bmatrix} x_1(t) & x_L^*(t) \\ \vdots & \vdots \\ x_L(t) & x_1^*(t) \end{bmatrix}$
$\tilde{\mathbf{Y}}(t) = \mathbf{W}^H(t-1) \tilde{\mathbf{X}}(t)$
$\tilde{\mathbf{V}}(t) = [\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1) \tilde{\mathbf{X}}(t) \quad \tilde{\mathbf{X}}(t)]$
$\mathbf{\Omega}(t) = \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \tilde{\mathbf{X}}(t)$
$\beta^2 \tilde{\mathbf{\Lambda}}(t) = \begin{bmatrix} -\tilde{\mathbf{X}}^H(t) \tilde{\mathbf{X}}(t) & \beta \mathbf{I}_2 \\ \beta \mathbf{I}_2 & \mathbf{0}_2 \end{bmatrix}$
$\tilde{\mathbf{\Phi}}(t) = [\mathbf{\Omega}(t) \quad \tilde{\mathbf{Y}}(t)]$
$\mathbf{K}(t) = [\beta^2 \tilde{\mathbf{\Lambda}}(t) + \tilde{\mathbf{\Phi}}^H(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1) \tilde{\mathbf{\Phi}}(t)]^{-1} \times \tilde{\mathbf{\Phi}}^H(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1)$
$\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) = [\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1) - \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1) \tilde{\mathbf{\Phi}}(t) \mathbf{K}(t)] / \beta^2$
$\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t) = \beta \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t-1) + \tilde{\mathbf{Y}}(t) \tilde{\mathbf{X}}^H(t)$
$\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) = \beta \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1) + \tilde{\mathbf{X}}(t) \tilde{\mathbf{X}}^H(t)$
$\mathbf{W}(t) = \mathbf{W}(t-1) + [\tilde{\mathbf{V}}(t) - \mathbf{W}(t-1) \tilde{\mathbf{\Phi}}(t)] \mathbf{K}(t)$
<p>END</p>

where $\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) = (\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t))^{-1}$. We should derive the recursive equations for $(\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) - \mathbf{W}(t-1) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t)) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t)$ and $\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) = (\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}^+(t))^{-1}$.

We can derive the recursive equations from the derivation procedure in [2] and [3] with a little modification. Then, (15) becomes the following recursive equation.

$$\mathbf{W}(t) = \mathbf{W}(t-1) + [\tilde{\mathbf{V}}(t) - \mathbf{W}(t-1) \tilde{\mathbf{\Phi}}(t)] \mathbf{K}(t) \quad (16)$$

where $\tilde{\mathbf{V}}(t) = [\mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1) \tilde{\mathbf{X}}(t) \quad \tilde{\mathbf{X}}(t)]$, $\mathbf{\Omega}(t) = \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t) \tilde{\mathbf{X}}(t)$,

$\tilde{\mathbf{\Phi}}(t) = [\mathbf{\Omega}(t) \quad \tilde{\mathbf{Y}}(t)]$, $\beta^2 \tilde{\mathbf{\Lambda}}(t) = \begin{bmatrix} -\tilde{\mathbf{X}}^H(t) \tilde{\mathbf{X}}(t) & \beta \mathbf{I}_2 \\ \beta \mathbf{I}_2 & \mathbf{0}_2 \end{bmatrix}$ and

$\mathbf{K}(t) = [\beta^2 \tilde{\mathbf{\Lambda}}(t) + \tilde{\mathbf{\Phi}}^H(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1) \tilde{\mathbf{\Phi}}(t)]^{-1} \times \tilde{\mathbf{\Phi}}^H(t) \mathbf{C}_{\tilde{\mathbf{X}}\tilde{\mathbf{X}}}(t-1)$.

The proposed forward backward COPAST (FB-COPAST) is summarized in Table 2.

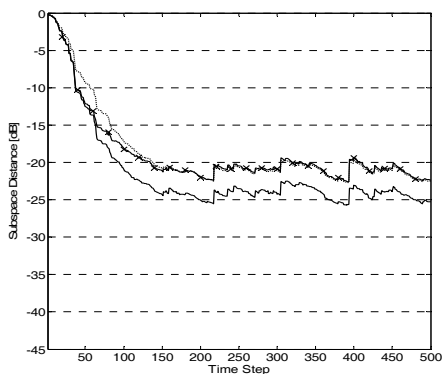
IV. Simulation Results

In this section, we demonstrate the applicability of the proposed algorithm to the subspace estimation. We assume the signal subspace comes from a narrow band far-field source by using a linear uniform array with 8 sensors. For the experimental purpose, we set the scenario that the angle of arrival of the signal comes from -30° .

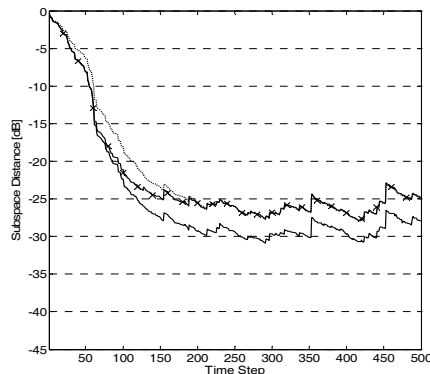
In Fig.1, we compare the estimation accuracy of the proposed algorithm with the conventional PAST algorithm and COPAST algorithm in the fixed forgetting factors of 0.98 under the four different SNR cases of 5dB, 10dB, 15dB and 20dB, respectively. To compare the quality of the estimated subspace, we show the distance between the estimated subspace and the true subspace, which is defined in [3].

$$\sin \theta(\mathbf{S}, \tilde{\mathbf{S}}) = \left\| (\mathbf{I} - \mathbf{P}) \tilde{\mathbf{P}} \right\|, \quad (29)$$

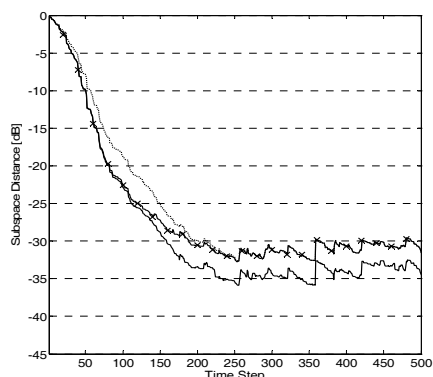
where \mathbf{s} is the true subspace, $\tilde{\mathbf{S}}$ is the estimated sub-space, \mathbf{P} is the projector onto \mathbf{S} and $\tilde{\mathbf{P}}$ is the projector onto $\tilde{\mathbf{S}}$. The results in Fig. 1 show that the proposed algorithm estimated the subspace more accurately than the conventional PAST algorithm and the COPAST algorithm in all SNR cases.



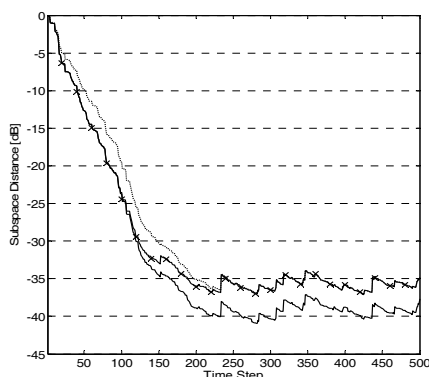
(a) Subspace estimation accuracy comparison in SNR 5dB



(b) Subspace estimation accuracy comparison in SNR 10dB



(c) Subspace estimation accuracy comparison in SNR 15dB



(d) Subspace estimation accuracy comparison in SNR 20dB

Fig.1 The subspace estimation accuracy comparisons (solid line: the proposed algorithm, -x-: COPAST in [2], dotted line: the conventional PAST).

V. Conclusion

In this paper, we have proposed the forward-backward COPAST (FB-COPAST) algorithm to estimate the signal subspace. The FB-COPAST applies the forward-backward covariance matrix to the COPAST. It improves the property of the estimated covariance matrix to get closer to the ideal covariance matrix. From the simulation results, we can confirm the proposed FB-COPAST outperforms the COPAST as well as the conventional PAST in the estimation accuracy by about 5dB in steady state. This result leads us to the conclusion that the forward-backward modification improves the estimation performance of the COPAST.

Reference

[1] Bin Yang, 'Projection Approximation Subspace

Tracking', IEEE Trans. Signal Proc., 1995, 43, (1), pp. 95-107

[2] Jung-Lang Yu, "A Novel Subspace Tracking Using Correlation-Based Projection Approximation," Signal Processing, 2000, 80, pp.2517-2525.

[3] Tony Gustafsson, "Instrumental Variable Subspace Tracking Using Projection Approximation," IEEE Trans. Signal Proc., 1998, 46, (3), pp.669-681.

[4] P. Stoica and R. L. Moses, Spectral Analysis of Signals, Prentice Hall, Upper Saddle River, N.J., 2005.

임 준 석 (Jun-Seok Lim)

정희원

제28권 12T호 참조

편 용 국 (Yong-Kug Pyeon)

정희원

제28권 12T호 참조