

# Nakagami- $m$ 페이딩 채널에서 GCIODs로 얻은 STBCs의 에르고딕 용량에 대한 연구

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## On the Ergodic Capacity of STBCs from GCIODs over Nakagami- $m$ Fading Channels

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### 요약

본 논문에서는 주파수 비선택성이고 준정적인 동일 분포 독립 Nakagami- $m$  페이딩 채널에서 GLCUDs(Generalized Linear Complex Orthogonal Designs)와 GCIODs(Generalized Coordinate Interleaved Orthogonal Designs)로 얻은 STBCs(Space-Time Block Codes)의 에르고딕 용량을 Meijer의 G함수로 표현되는 단항꼴로 유도한다. 유도된 해석적 결과는 Monte-Carlo 모의실험 결과와 매우 잘 일치함을 보였다. 그러므로 대규모 Monte-Carlo 모의실험 없이 다양한 안테나 구성과 상이한 채널환경에서 GLCUDs와 GCIODs로 얻은 STBCs의 에르고딕 용량 성능을 분석하고 예측하는 유용한 기법이 제안되었다고 할 수 있다. 마지막에는 유도된 식의 정확도를 입증하는 수치적인 결과들을 제시한다.

**Key Words** : STBCs, GLCUDs, GCIODs, Ergodic Capacity, Nakagami- $m$  Fading Channels

### ABSTRACT

In this paper, we derive exact closed-form formulas, in terms of Meijer's G-function, for the ergodic capacity of space-time block codes (STBCs) from generalized linear complex orthogonal designs (GLCUDs) and generalized coordinate interleaved orthogonal designs (GCIODs) in quasi-static frequency-nonselective i.i.d. Nakagami- $m$  fading channels. The derived analytical results show an excellent agreement with Monte-Carlo simulation results. Thus, a useful means for analyzing and predicting the ergodic capacity performance of STBCs from GLCUDs or GCIODs can be provided in various antenna configurations and different channel conditions without extensive Monte-Carlo simulations. We present some numerical results to verify the accuracy of the derived formulas.

### I. Introduction

In multiple-input multiple-output (MIMO) wireless communication systems, spatial diversity employing space-time coding is considered as a powerful tool for effectively combating fading [1]-[5].

Among several types of space-time codes, space-time block codes (STBCs) (especially, from complex orthogonal designs (CODs) or generalized linear complex orthogonal designs (GLCUDs)) have been of special interest, since they transform the MIMO channel into a number of independent

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scalar channels and achieve diversity gain with simple encoding and decoding [1]-[5]. More recently, Khan *et al.* and Yuen *et al.* have proposed STBCs from generalized coordinate interleaved orthogonal designs (GCIODs) [5] and STBCs from ABBA-type quasi-orthogonal designs with minimum decoding complexity (MDC-ABBA) [6], respectively, which can achieve full-diversity full-rate transmission and single-symbol decoding for the systems with more than two transmit antennas and complex signal constellations.

When the channel is ergodic, ergodic capacity is a typical measure of link performance [7]-[9]. Although some fundamental investigations were made in [5], most recent research for STBCs from GCIODs has focused on the evaluation of error rate (e.g., symbol-error rate (SER)), outage performance (e.g., outage capacity and information outage probability), and asymptotic/instantaneous diversity orders [10]-[13]. Therefore, in this paper, our aim is to derive an exact closed-form formula for the ergodic capacity of STBCs from GCIODs, especially in quasi-static frequency-nonselective i.i.d. Nakagami- $m$  fading channels. In particular, it is well-known that the Nakagami distribution is analytically tractable and provides a good fit for measured data in various fading environments [12]-[14]. Thus, starting from the idea in [15], where the authors presented closed-form expressions in terms of Meijer's G-function [16],[17] for the ergodic capacity of single-branch receivers, we first derive a closed-form formula for the ergodic capacity of STBCs from GLCODs. Then, from the fact that GCIODs consist of two component GLCODs, an exact closed-form formula for the ergodic capacity of STBCs from GCIODs is derived. Furthermore, since the MDC-ABBA is also constructed by two component GLCODs (but with a different construction rule from GCIODs), we present a closed-form expression for the ergodic capacity of MDC-ABBA [6],[18]. The resultant closed-form formulas allow us to accurately predict the ergodic capacity performance of STBCs from GLCODs or GCIODs in various fading environments, while avoiding the needs for extensive

Monte-Carlo simulations and computationally complex numerical integration methods.

*Notation:* The superscript  $(\cdot)^T$  denotes transpose operation.  $\mathbb{E}\{\cdot\}$  is reserved for expectation with respect to all the random variables within the braces.  $\mathcal{R}\{\alpha\}$  and  $\mathcal{I}\{\alpha\}$  denotes the real and imaginary part of complex number  $\alpha$ .  $\langle a \rangle_K$  denotes  $a \bmod K$  with the slight variation as  $\langle K \rangle_K = K$ . We use  $\|\mathbf{P}\|_F$  for the Frobenius norm.  $x \sim \text{CN}(m_x, \sigma_x^2)$  stands for a circular symmetric complex Gaussian variable  $x$  with mean  $m_x$  and variance  $\sigma_x^2$ .  $z \sim \text{G}(p, q)$  denotes the Gamma distribution, where  $p$  and  $q$  represent a scale parameter and a shape parameter, respectively.

## II. System and Channel Models

For a MIMO system with  $N_T$  transmit and  $N_R$  receive antennas, the  $T \times N_T$  transmission codeword matrix can be represented by  $\mathbf{S} = \{s_{tl}; 1 \leq t \leq T, 1 \leq l \leq N_T\}$ , where the entry  $s_{tl}$  is the symbol transmitted from the  $t^{\text{th}}$  antenna at time  $t$ . Then, the corresponding input-output relationship within a codeword duration  $T$  can be expressed as [11],[12]

$$\mathbf{R} = \mathbf{H}\mathbf{S}^T + \mathbf{V}, \quad (1)$$

where  $\mathbf{R} = \{r_{nt}; 1 \leq n \leq N_R, 1 \leq t \leq T\}$  is the  $N_R \times T$  received signal matrix,  $\mathbf{H} = \{h_{nl}; 1 \leq n \leq N_R, 1 \leq l \leq N_T\}$  is the  $N_R \times N_T$  MIMO channel matrix, where the element  $h_{nl}$  is the channel path gain between the  $l^{\text{th}}$  transmit antenna and the  $n^{\text{th}}$  receive antenna, and  $\mathbf{V} = \{v_{nt}; 1 \leq n \leq N_R, 1 \leq t \leq T\}$  is the receiver noise matrix of size  $N_R \times T$  with elements modeled as i.i.d. complex circular Gaussian random variables, each with a  $\text{CN}(0, \sigma_v^2)$  distribution. We assume that 1) the Nakagami- $m$  fading channel is quasi-static within a codeword interval and varies independently between codewords, 2) all the subchannels between

transmit and receive antenna pairs are i.i.d., and 3) the receiver knows the perfect channel state information while the transmitter has no knowledge of it. We further assume frequency-nonselective Nakagami- $m$  fading channels with a common fading parameter  $m$  and uniform average power gain  $\Omega=1$ . Let the channel coefficients  $h_{nl}=\eta_{nl}e^{j\theta_{nl}}$ , where  $\theta_{nl}$  denotes the phase uniformly distributed over  $[0, 2\pi)$  and  $\eta_{nl}=|h_{nl}|$  is a Nakagami distributed signal envelope, the probability density function (pdf) of which is given by<sup>[14]</sup>,

$$f_{\eta_{nl}}(\eta; \Omega, m) = \frac{2m^m \eta^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{m}{\Omega}\eta^2} U(\eta), \quad (2)$$

where  $m \geq 0.5$ ,  $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$  is the complete Gamma function<sup>[16]</sup>, and  $U(x)$  is the unit step function. Then,  $u_{nl}=\eta_{nl}^2$  follows a Gamma distribution denoted by  $u_{nl} \sim G(p, q)$  with the relation between the parameters (i.e.,  $p$  and  $q$ ) and Nakagami parameters as  $p=\Omega/m=1/m$  and  $q=m$ , and its pdf is<sup>[14]</sup>,

$$f_{u_{nl}}(u; p, q) = \frac{u^{q-1}}{\Gamma(q)p^q} e^{-\frac{u}{p}} U(u). \quad (3)$$

According to<sup>[5]</sup>, a GCIOD  $S(s_1, \dots, s_K)$  (where  $K$  is even) of size  $T \times N_T$  and code rate  $R_c = K/T$  in variables  $s_i$ ,  $i=1, \dots, K$  consists of two GLCQDs,  $\mathcal{O}_1 = (\tilde{s}_1, \dots, \tilde{s}_{K/2})$  and  $\mathcal{O}_2 = (\tilde{s}_{K/2+1}, \dots, \tilde{s}_K)$  of size  $T_1 \times N_{T_1}$ ,  $T_2 \times N_{T_2}$ , and code rates  $R_{c1} = K/2T_1$ ,  $R_{c2} = K/2T_2$ , respectively, where  $T_1 + T_2 = T$ ,  $N_{T_1} + N_{T_2} = N_T$ , and  $\tilde{s}_i = \mathcal{R}\{s_i\} + j\{s_{\langle i+K/2 \rangle_K}\}$ ,  $i=1, \dots, K$  are coordinate interleaved variables of complex variables  $s_i$ . In particular, when  $\mathcal{O}_1 = \mathcal{O}_2$ , it is referred to as a (symmetric) CIOD<sup>[5],[11]</sup>.

### III. Meijer's G-Function

The Meijer's G-function is a generalization of

the generalized hypergeometric function, which can be reduced to simpler special functions in many common cases. The Meijer's G-function can be defined by the following contour integral representation<sup>[16],[17]</sup>,

$$G_{p,q}^{m,n} \left[ z \middle| \begin{matrix} a_1, \dots, a_n, a_{n+1}, \dots, a_p \\ b_1, \dots, b_m, b_{m+1}, \dots, b_q \end{matrix} \right] = \frac{1}{j2\pi} \int \frac{\prod_{i=1}^m \Gamma(b_i - s) \prod_{i=1}^n \Gamma(1-a_i + s) x^s}{\prod_{i=m+1}^q \Gamma(1-b_i + s) \prod_{i=n+1}^p \Gamma(a_i - s)} ds, \quad (4)$$

where  $z$ ,  $\{a_1, \dots, a_n, a_{n+1}, \dots, a_p\}$ , and  $\{b_1, \dots, b_m, b_{m+1}, \dots, b_q\}$  are generally complex-valued.

In some special cases, the exponential and logarithmic functions can be expressed by the Meijer's G-functions as follows<sup>[19],[20]</sup>.

$$\begin{aligned} G_{0,1}^{1,0} [z|b] &= e^{-z} z^b \\ G_{2,2}^{1,2} \left[ z \middle| \begin{matrix} a, a \\ a, a-1 \end{matrix} \right] &= z^{a-1} \ln(1+z). \end{aligned} \quad (5)$$

By using (5) and<sup>[21]</sup>, we obtain the following useful representation<sup>[15]</sup>

$$\begin{aligned} &\int_0^\infty \alpha^{a-1} \ln(1+b\alpha) e^{-\alpha} d\alpha \\ &= \int_0^\infty \alpha^{a-1} G_{2,2}^{1,2} \left[ b\alpha \middle| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \right] G_{0,1}^{1,0} [ca|0] d\alpha \\ &= \frac{b^{-a}}{\Gamma(a)} G_{2,3}^{3,1} \left[ \frac{c}{b} \middle| \begin{matrix} -a, 1-a \\ 0, -a, -a \end{matrix} \right]. \end{aligned} \quad (6)$$

We note that the Meijer's G-function is implemented in most of the well-known mathematical software packages, such as MATHEMATICA, MAPLE, etc.

### IV. Ergodic Capacity of STBCs in Nakagami- $m$ Fading Channels

In this section, we derive closed-form expressions for the ergodic capacity of STBCs from GLCQDs and GCIODs in the presence of quasi-static frequency-nonselective i.i.d. Nakagami- $m$

fading. In addition, we also derive a closed-form ergodic capacity formula for ABBA/MDC-ABBA.

#### 4.1 Ergodic Capacity Derivation of STBCs from GLCODs

The decoupling property of the signals transmitted from different antennas enables the STBCs from GLCODs to transform a MIMO channel into an equivalent Gaussian single-input single-output (SISO) channel with a channel gain  $\|\mathbf{H}\|_F^2$  as  $\mathbf{r} = R_c^{-1} \|\mathbf{H}\|_F^2 \mathbf{s} + \mathbf{v}$ , where  $\mathbf{r}$  is the  $N_R \times 1$  received signal vector after decoding from the received matrix  $\mathbf{R}$ ,  $\mathbf{s}$  is the  $N_T \times 1$  transmitted signal vector,  $\mathbf{v}$  is the  $N_R \times 1$  noise vector,  $R_c$  denotes the code rate, and  $\|\mathbf{H}\|_F^2 = \sum_{l=1}^{N_T} \sum_{n=1}^{N_R} |h_{nl}|^2$ . Letting  $\rho = E_s / \sigma_v^2$  denote the average SNR at the receiver, where  $E_s$  is the total transmitted power on the  $N_T$  transmit antennas per symbol duration, the ergodic capacity of STBCs from GLCODs in bps/Hz is given by [5], [7], [8].

$$\begin{aligned} C_{\text{GLCOD}}(N_T, N_R, R_c, \rho) &= \mathbb{E} \left[ R_c \log_2 \left( 1 + \frac{\rho}{R_c N_T} \|\mathbf{H}\|_F^2 \right) \right] \\ &= \int_0^\infty R_c \log_2 \left( 1 + \frac{\rho}{R_c N_T} \zeta \right) f_\zeta(\zeta) d\zeta, \end{aligned} \quad (7)$$

where  $\zeta = \|\mathbf{H}\|_F^2 \sim G(1/m, mN_T N_R)$ . Thus, by replacing the pdf of a Gamma random variable  $\zeta$  into (7) and using (6), the ergodic capacity of STBCs from GLCODs in quasi-static frequency-nonselective i.i.d. Nakagami- $m$  fading channels can be evaluated in closed-form as follows [13].

$$\begin{aligned} C_{\text{GLCOD}}(N_T, N_R, R_c, \rho) &= \int_0^\infty R_c \log_2 \left( 1 + \frac{\rho}{R_c N_T} \zeta \right) \frac{\zeta^{mN_T N_R - 1} m^{mN_T N_R} e^{-m\zeta}}{\Gamma(mN_T N_R)} d\zeta \\ &= \frac{R_c m^{mN_T N_R}}{\Gamma(mN_T N_R) \ln 2} \int_0^\infty \zeta^{mN_T N_R - 1} \\ &\quad \times G_{2,2}^{1,2} \left[ \frac{\rho}{R_c N_T} \zeta \middle| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \right] G_{0,1}^{1,0} [m\zeta | 0] d\zeta \end{aligned} \quad (8)$$

$$= \frac{R_c}{\Gamma(mN_T N_R) \ln 2} \left[ \frac{mR_c N_T}{\rho} \right]^{mN_T N_R} \\ \times G_{2,3}^{3,1} \left[ \frac{mR_c N_T}{\rho} \middle| \begin{matrix} -mN_T N_R, 1 - mN_T N_R \\ 0, -mN_T N_R, -mN_T N_R \end{matrix} \right].$$

Here, we note that (8) is valid for any real valued Nakagami fading parameter  $m \geq 1/2$ . When  $m$  is an integer, however, the numerical evaluation of the Meijer's G-function, especially for MATHEMATICA, fails. This problem can be easily resolved by setting  $m$  to a real number (but very close to the original integer). For example, when  $m = 1.001$  is used (instead of  $m = 1$ ), the corresponding Meijer's G-function still provides a very accurate result.

#### 4.2 Ergodic Capacity Derivation of STBCs from GCIODs

The ergodic capacity of STBC from GCIOD can be represented by the ergodic capacities of its two component GLCODs as [5]

$$\begin{aligned} C_{\text{GCIOD}}(N_T, N_R, R_c, \rho) &= \frac{1}{T} \left\{ T_1 C_{\text{GLCOD}}(N_{T_1}, N_R, R_{c1}, \rho) \right. \\ &\quad \left. + T_2 C_{\text{GLCOD}}(N_{T_2}, N_R, R_{c2}, \rho) \right\} \\ &= \frac{R_c}{2} \left\{ \mathbb{E} \left[ \log_2 \left( 1 + \frac{\rho \|\mathbf{H}_1\|_F^2}{R_{c1} N_{T_1}} \right) \right] \right. \\ &\quad \left. + \mathbb{E} \left[ \log_2 \left( 1 + \frac{\rho \|\mathbf{H}_2\|_F^2}{R_{c2} N_{T_2}} \right) \right] \right\} \\ &= \frac{R_c}{2} \left\{ \mathbb{E} \left[ \log_2 \left( 1 + \frac{2T_1 \rho \|\mathbf{H}_1\|_F^2}{R_c N_{T_1} T} \right) \right] \right. \\ &\quad \left. + \mathbb{E} \left[ \log_2 \left( 1 + \frac{2T_2 \rho \|\mathbf{H}_2\|_F^2}{R_c N_{T_2} T} \right) \right] \right\}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} \|\mathbf{H}_1\|_F^2 &= \sum_{l=1}^{N_T} \sum_{n=1}^{N_R} |h_{nl}|^2 \sim G\left(\frac{1}{m}, mN_{T_1} N_R\right), \\ \|\mathbf{H}_2\|_F^2 &= \sum_{l=N_{T_1}+1}^{N_T} \sum_{n=1}^{N_R} |h_{nl}|^2 \sim G\left(\frac{1}{m}, mN_{T_2} N_R\right). \end{aligned}$$

Hence, letting  $\xi_1 = \|\mathbf{H}_1\|_F^2$  and  $\xi_2 = \|\mathbf{H}_2\|_F^2$ , and from the fact that  $\xi_1$  and  $\xi_2$  are independent of each other, we have

$$\begin{aligned}
C_{\text{GCIOD}}(N_T, N_R, R_c, \rho) &= \int_0^\infty \frac{R_c}{2} \log_2 \left( 1 + \frac{2T_1\rho}{R_c N_{T_1} T} \xi_1 \right) f_{\xi_1}(\xi_1) d\xi_1 \\
&\quad + \int_0^\infty \frac{R_c}{2} \log_2 \left( 1 + \frac{2T_2\rho}{R_c N_{T_2} T} \xi_2 \right) f_{\xi_2}(\xi_2) d\xi_2 \\
&= \frac{R_c}{\Gamma(mN_{T_1}N_R)2\ln 2} \left[ \frac{mR_c N_{T_1} T}{2T_1\rho} \right]^{mN_{T_1}N_R} \\
&\quad \times G_{2,3}^{3,1} \left[ \frac{mR_c N_{T_1} T}{2T_1\rho} \middle| -mN_{T_1}N_R, 1-mN_{T_1}N_R \right] \\
&\quad + \frac{R_c}{\Gamma(mN_{T_2}N_R)2\ln 2} \left[ \frac{mR_c N_{T_2} T}{2T_2\rho} \right]^{mN_{T_2}N_R} \\
&\quad \times G_{2,3}^{3,1} \left[ \frac{mR_c N_{T_2} T}{2T_2\rho} \middle| -mN_{T_2}N_R, 1-mN_{T_2}N_R \right]. \tag{10}
\end{aligned}$$

In addition, the ergodic capacity of STBC from CIOD (i.e.,  $T_1 = T_2 = T/2$ ,  $N_{T_1} = N_{T_2} = N_T/2$ , and  $R_{c1} = R_{c2} = R_c$ ) can be easily derived as

$$\begin{aligned}
C_{\text{CIOD}}(N_T, N_R, R_c, \rho) &= \frac{R_c}{\Gamma\left(\frac{mN_T N_R}{2}\right)\ln 2} \left[ \frac{mR_c N_T}{2\rho} \right]^{\frac{mN_T N_R}{2}} \\
&\quad \times G_{2,3}^{3,1} \left[ \frac{mR_c N_T}{2\rho} \middle| -\frac{mN_T N_R}{2}, 1-\frac{mN_T N_R}{2} \right] \\
&\quad \times G_{2,3}^{3,1} \left[ \frac{mR_c N_T}{2\rho} \middle| 0, -\frac{mN_T N_R}{2}, -\frac{mN_T N_R}{2} \right]. \tag{11}
\end{aligned}$$

Comparing (11) with (8), we find that  $C_{\text{CIOD}}(N_T, N_R, R_c, \rho) = C_{\text{GLCOD}}(N_T/2, N_R, R_c, \rho)$ , when the CIOD is composed of the GLCODs. Furthermore, it is demonstrated in [18] that the ergodic capacity of ABBA/MDC-ABBA codes is equivalent to that of their component GLCOD (i.e.,  $C_{\text{ABBA}}(N_T, N_R, R_c, \rho) = C_{\text{MDC-ABBA}}(N_T, N_R, R_c, \rho) = C_{\text{GLCOD}}(N_T/2, N_R, R_c, \rho)$ ). Therefore, we obtain  $C_{\text{ABBA}}(N_T, N_R, R_c, \rho) = C_{\text{MDC-ABBA}}(N_T, N_R, R_c, \rho) = C_{\text{CIOD}}(N_T, N_R, R_c, \rho)$ , when they are constructed by the same components GLCODs. Furthermore, we note that as will be observed in Sec. V, GCIODs can achieve larger portion of ergodic capacity than GLCODs, which is caused by the transmission code rate deficiency of GLCODs for more than two transmit antennas.

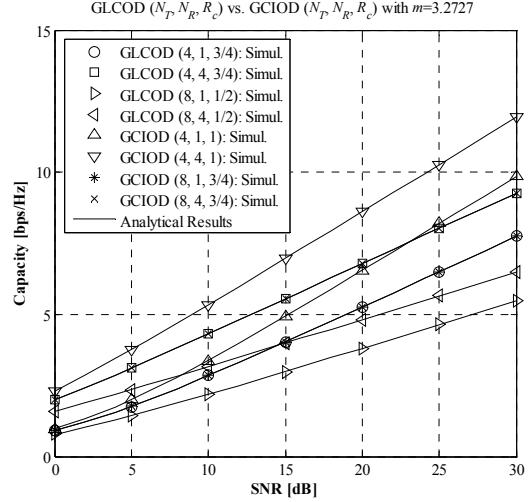


Fig. 1. Ergodic capacity versus SNR  $\rho$  for GLCOD  $(N_T, N_R, R_c)$  and GCIOD  $(N_T, N_R, R_c)$  in a quasi-static frequency-nonselective Nakagami- $m$  fading channel with  $m = 3.2727$ .

## V. Numerical Results

In this section, we provide numerical results and some comparisons to examine the exactness of the derivations given in the previous section and to show the effects of antenna configuration and Nakagami fading parameter  $m$  on the ergodic capacity of STBCs. Hereafter, for comparison, we consider the rate-3/4 and -1/2 GLCODs [2],[4] and the rate-1 and -3/4 GCIODs [5] for  $N_T = 4$  and 8.

In Fig. 1, we plot the ergodic capacity versus the average SNR at the receiver for the STBCs from GLCODs and GCIODs with  $(N_T, N_R, R_c)$  over a quasi-static frequency-nonselective i.i.d. Nakagami fading channel with  $m = 3.2727$ . From the relation between the Nakagami fading parameter  $m$  and the Rician factor  $K_R$  as  $K_R = \sqrt{m^2 - m}/(m - \sqrt{m^2 - m})$ ,  $m = 3.2727$  corresponds to  $K_R = 5$ . As seen from the figure, the analytically derived curves accurately match with the exact (via Monte-Carlo simulation) ergodic capacities of STBCs from both GLCODs and GCIODs in the channels under consideration. For the same antenna configuration, the ergodic capacity of GLCODs is inferior to those of

GCIODs, which is due to the code rate deficiency of GLCUDs. Additionally, we can also observe that  $C_{\text{GCIOD}}(8, N_R, 3/4, \rho) = C_{\text{GLCUD}}(4, N_R, 3/4, \rho)$ , which exemplifies that  $C_{\text{CIOD}}(N_T, N_R, R_c, \rho) = C_{\text{GLCUD}}(N_T/2, N_R, R_c, \rho)$ , when the CIOD consists of two component GLCUDs with the same codeword structure.

Fig. 2 and 3 illustrate the effects of antenna configuration and Nakagami fading parameter  $m$  on the ergodic capacity of STBCs from GCIODs. As shown in Fig. 2, the ergodic capacity increases as the Nakagami fading becomes less severe (i.e., as the Nakagami fading parameter  $m$  increases). However, the degree of this improvement becomes reduced with the number of receive antennas. Furthermore, we can observe that the improvement of ergodic capacity with the number of antennas is much more dominant than that with the Nakagami fading parameter  $m$ , which can be interpreted by the high diversity gain that is already achieved by STBCs from GCIODs with multiple transmit and receive antennas. This observation is more explicitly demonstrated in Fig. 3, where we depict the ergodic capacity curves of GCIOD (4,  $N_R$ , 1) for various values of  $N_R$  and  $m$ .

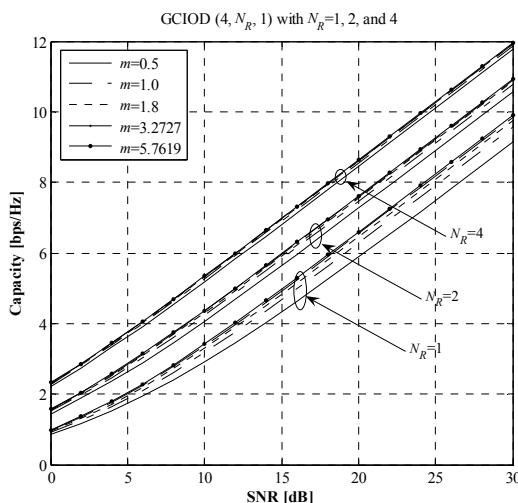


Fig. 2. Ergodic capacity for GCIOD (4,  $N_R$ , 1) in quasi-static frequency-nonselective Nakagami- $m$  fading channels.  $N_R = 1, 2, 4$  and  $m = 0.5, 1, 1.8, 3.2727, 5.7619$ .

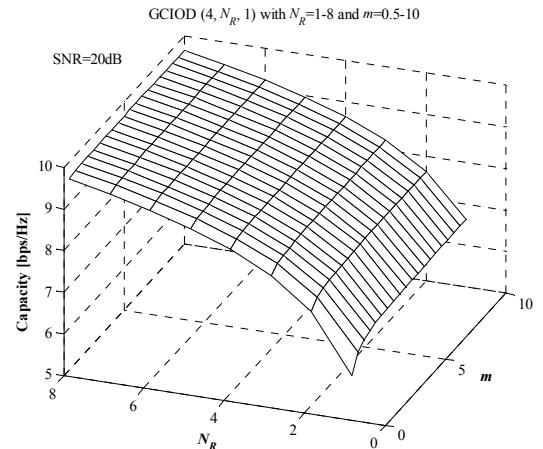


Fig. 3. Ergodic capacity for GCIOD (4,  $N_R$ , 1) in quasi-static frequency-nonselective Nakagami- $m$  fading channels.  $N_R = 1-8$  and  $m = 0.5-10$  at  $\rho = 20\text{dB}$ .

## VI. Conclusions

Exact closed-form ergodic capacity formulas have been derived for STBCs from both GLCUDs and GCIODs in quasi-static frequency-nonselective i.i.d. Nakagami- $m$  fading channels. We demonstrated that when STBCs from CIODs, ABBA codes, and MDC-ABBA codes are constructed by two component GLCUDs with the same codeword structure, their ergodic capacities are equivalent to that of the component GLCUD. For the system employing more than two transmit antennas, however, the codes provide a higher achievable capacity than GLCUDs. In addition, we also presented some numerical results, showing the exactness of the derived formulas for various antenna configurations and channel conditions. Throughout this paper, we have considered the i.i.d. fading channel model. In realistic wireless fading environments, however, there exists spatial correlation between the antenna elements, which significantly affects the performance of a MIMO system. Thus, our future research aims for assessing the effects of spatial correlation on the performance measures (e.g., ergodic capacity, error rates, etc.) of STBCs from GCIODs.

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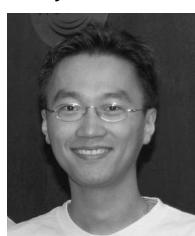
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