레일레이 페이딩 채널에서 2-by-N MIMO-STBC 시스템의 SER/BER 성능 해석

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SER/BER Analysis for 2-by-N MIMO-STBC Systems in Rayleigh Fading Channels

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ABSTRACT

In this paper, novel approximated closed-form expressions of the SER(Symbol Error Rate) and BER(Bit Error Rate) are presented for the multi-input multi-output space-time block coding(MIMO-STBC) system employing two transmit and N receive antenna(s) in Rayleigh fading channels. Assuming a zero-forcing equalizer and a single-symbol maximum-likelihood (ML) decoder, the formulas are derived for the SER and BER, and then their approximated counterparts are also derived. In the derivation of the approximated expressions, acurve-fitting model is adopted that can eliminate the integral operators in the exact expressions.

Key Words : BER, MIMO, QAM, SER, STBC

Ⅰ. Introduction

Multi-input multi-output space-time block coding (MIMO-STBC) is a kind of diversity technique that can achieve spatial diversity gain in a quasi-static wireless channel environment[1-3]. Like in other wireless systems, accurate estimation of the performance measures such as SER and BER is important for the design and optimization of this system based on the underlying channel environment.

Although some previous analytical studies in [4-9] have reported elegant expressions that can be simply (but not always) extended to the MIMO-STBC systems, most of them entails the integral operators, which poses an obstacle to the low complexity evaluation of the performance measures[10-11]. It is because most of the SER and BER expressions from the previous studies is written as a linear combination of Q functions with different arguments. Thus, the elimination of the integral operators in the expressions has been of great interest in this field.

In this paper, novel approximated closed-form expressions of the SER and BER are presented for the MIMO-STBC system employing two transmit and N receive antenna(s) in Rayleigh fading channels. Assuming a zero-forcing equalizer and a single-symbol ML decoder, the formulas are derived for the SER and BER, and then their approximated counterparts are also derived. In the derivation of the approximated expressions, a curve-fitting model is adopted that can eliminate the integral operators in the exact expressions. Simulation results show that the proposed curve-fitting method for the SER and BER successfully eliminates the integral

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operators while achieving almost the same accuracy as the exact formulas. Even though this paper focuses on the Alamouti code[1] employing two transmit antennas, the proposed idea can be simply applied to other STBCs from complex orthogonal designs (COD)[2].

The remainder of this paper is organized as follows. Section II describes the MIMO-STBC system considered in this paper. In Section III, the SER and BER is analyzed. Section IV verifies the accuracy of the proposed performance expressions. Finally, Section V contains the conclusion.

Ⅱ. System Model

Fig. 1 shows the system model considered in this paper.

In a (2 Tx, N Rx) STBC system, when $(x_1,x_2) \in \{s_m, m=1,2,\cdots,M\}$ are transmitted, the received signal at the ith receive antenna at contiguous time instances t_1 and t_2 is expressed as

$$\begin{pmatrix} y_{1i} \\ y_{2i} \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \begin{pmatrix} h_{1i} \\ h_{2i} \end{pmatrix} + \begin{pmatrix} n_{1i} \\ n_{2i} \end{pmatrix}, \quad (1)$$

where n_{1i} and n_{2i} denote the additive white Gaussian noises (AWGNs) present at the ith receive antenna at time t_1 and t_2, respectively.

Define

$$\boldsymbol{z} = (y_{11}, y_{21}^*, \cdots, y_{1N}, y_{2N}^*)^T,$$
$$\boldsymbol{g}_1 = (h_{11}, h_{21}^*, \cdots, h_{1N}, h_{2N}^*)^T,$$
$$\boldsymbol{g}_2 = (h_{21}, h_{11}^*, \cdots, h_{2N}, h_{1N}^*)^T,$$

and $\boldsymbol{n_p} = (n_{11}, n_{21}^*, \cdots, n_{1N}, n_{2N}^*)^T$. Then, the received signal can be expressed as

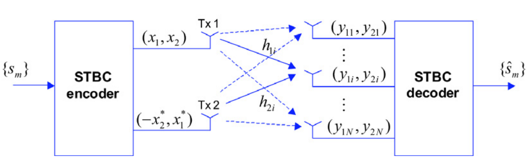


Fig. 1. System model

$$\boldsymbol{z} = \boldsymbol{g}_1 x_1 + \boldsymbol{g}_2 x_2 + \boldsymbol{n_P} \quad (2)$$

Given the signal model in (2), the equalizer that maximizes the output signal-to-interference plus noise (SINR) for $x_k, k=1,2$ is derived as follows. Let $\boldsymbol{w}_k^*$ denote the equalizer coefficient vector for x_k. Then, the equalizer output is expressed as

$$r_k = \boldsymbol{w}_k^H \boldsymbol{g}_1 x_1 + \boldsymbol{w}_k^H \boldsymbol{g}_2 x_2 + \boldsymbol{w}_k^H \boldsymbol{n_p} \quad (3)$$

and the output SINR for x_k is written as

$$\rho_{k,out} = \frac{|\boldsymbol{w}_k^H \boldsymbol{g}_k|^2 \sigma_s^2}{|\boldsymbol{w}_k^H \boldsymbol{g}_l|^2 \sigma_s^2 + \|\boldsymbol{w}_k\|^2 \sigma_n^2}, \quad (4)$$

where $l=2$ for $k=1$, and $l=1$ for $k=2$. From the Schwartz inequality of $|\boldsymbol{w}_k^H \boldsymbol{g}_k|^2 \le \|\boldsymbol{w}_k\|^2 \|\boldsymbol{g}_k\|^2$ with the equality when

$$\boldsymbol{w}_k = \alpha_k \boldsymbol{g}_k, \quad (5)$$

where α_k is an arbitrary constant, the numerator of (4) is maximized when (5) is met, which is also the condition for the denominator is minimized. This implies that a zero-forcing equalizer is optimal in respect of SINR. Therefore, using (5) in (3), we have the output of the zero-forcing equalizer as

$$r_k = \alpha_k \|\boldsymbol{g}_k\|^2 x_k + \alpha_k \boldsymbol{g}_k^H \boldsymbol{n_P}, \quad (6)$$

where $\alpha_k \boldsymbol{g}_k^H \boldsymbol{n_P}$ is a uncorrelated Gaussian random variable with zero-mean and variance of $\sigma_n^2 \alpha_k^2 \|\boldsymbol{g}_k\|^2$.

In this paper, it is assumed that h_{1i} and h_{2i} for $i \in \{1,2,\cdots,N\}$ are Gaussian distributed and symmetric with zero-mean and variance of σ_h^2; $E[\boldsymbol{n_p}\boldsymbol{n_p}^H] = \sigma_n^2 \boldsymbol{I}$; the average symbol energy of $\{s_m, m=1,2,\cdots,M\}$ equals σ_s^2. Then, the signal-to-noise ratio (SNR) of r_k equals $\|\boldsymbol{g}_k\|^2 \sigma_s^2 / \sigma_n^2$ irrespective of α_k, and since $\|\boldsymbol{g}_k\|^2 = \sum_{i=1}^{N} (|h_{1i}|^2 + |h_{2i}|^2)$, the SNR averaged over the distribution of $\|\boldsymbol{g}_k\|^2$ is written as

$$\rho_N = 2N\sigma_h^2\sigma_s^2/\sigma_n^2 = N_{\rho_1} \tag{7}$$

where $\rho_N = 2\sigma_h^2\sigma_s^2/\sigma_n^2$ is the SNR for a single antenna receiver.

Ⅲ. Derivation of SER and BER

In this section, exact closed-form expressions of the SER and BER are derived for the MIMO-STBC system, and then a curve-fitting method is presented that eliminates the integral operators in the exact formulas.

3.1 SER in a static channel

Assume that transmit signal x_k is an equiprobable M-ary square quadrature amplitude modulation (QAM) signal s_m expressed by $s_m = (A_{mc} + jA_{ms})\sqrt{E_g/2}$, where E_g is the average power of the transmit pulse waveform, and

$$A_{mc}, A_{ms} \in \{(2i - 1 + \sqrt{M})d, i = 1, 2, \cdots, \sqrt{M}\}.$$

Let $\alpha_k = 1/\|g_k\|$. Then, from (6}), we have the equalizer output for x_k given by

$$r_k = \|g_k\| s_m + \eta_k, \tag{8}$$

where $\eta_k = g_k^H n_P / \|g_k\|$ is AWGN with zero-mean and variance of σ_n^2. Thus the maximum-likelihood detector that minimizes the SER is expressed as

$$\widehat{x_k} = \arg \min (r_k - \|g_k\| x_k)^2, \quad \text{where } x_k \in \{s_1, \cdots, s_M\}. \tag{9}$$

Assuming the decision rule in (9), it is straightforward to show that the SER for an QAM signal in a given channel with its magnitude square of $\gamma = \|g_k\|^2$ is obtained by [12]

$$\begin{aligned} p_s(e|\gamma) &= \frac{1}{M}\sum_{m=1}^{M} p_s(e|\gamma, s_m) \\ &= \kappa_1 q(d) - \kappa_2 q^2(d), \end{aligned} \tag{10}$$

where

$$\kappa_1 = 4(1 - 1/\sqrt{M}),\ \kappa_2 = 4(1 - 1/\sqrt{M})^2, \text{ and } q(d) = p\left(Re(\eta_k) > d\sqrt{\frac{\gamma E_g}{2}}\right). \tag{11}$$

Define

$$u = \sqrt{\gamma d^2 E_g / \sigma_n^2}. \tag{12}$$

Then, it can be easily shown that $q(d) = Q(u)$, the Gaussian Q-function. Since the average power of the M-QAM symbols $\sigma_s^2 = (M-1)d^2E_g/3$, the term d in (12) can be eliminated as $u = \sqrt{3\sigma_s^2\gamma/(M-1)\sigma_n^2}$ and thus we have

$$p_s(e|\gamma) = \kappa_1 Q(u) - \kappa_2 Q^2(u). \tag{13}$$

3.2 SER in a Rayleigh fading channel

The SER in a Rayleigh fading channel is derived as follows. Since we have $p_s(e|\gamma)$ in (13) as a function of γ, the SER can be obtained by averaging $p_s(e|\gamma)$ over the distribution of γ as

$$\begin{aligned} p_s(e) &= \int_0^\infty p_s(e|\gamma)p(\gamma)d\gamma \\ &= \kappa_1 E[Q(u)] - \kappa_2 E[Q^2(u)]. \end{aligned} \tag{14}$$

And, using the equality in [9], we have

$$E[Q(u)] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} f_\gamma\left(\frac{-3}{2(M-1)\sin^2\phi}\frac{\sigma_s^2}{\sigma_n^2}\right)d\phi \tag{15}$$

$$E[Q^2(u)] = \frac{1}{\pi}\int_0^{\frac{\pi}{4}} f_\gamma\left(\frac{-3}{2(M-1)\sin^2\phi}\frac{\sigma_s^2}{\sigma_n^2}\right)d\phi \tag{16}$$

where $f_\gamma(s) = E[e^{s\gamma}]$. The moment generating function $f_\gamma(s)$ can be evaluated as follows. In Section I, h_{1i} and h_{2i} for $i \in \{1, 2, \cdots, N\}$ was assumed Gaussian distributed and symmetric with zero-mean and variances of σ_h^2. Under the assumption, $\gamma = \sum_{i=1}^{N}(|h_{1i}|^2 + |h_{2i}|^2)$ is χ^2-distributed with $4N$ degree of freedom, and hence it can be expressed as a sum of $4N$ i.i.d Gaussian random variables as given by $\gamma=\sum_{i=1}^{4N} x_i^2$, where $\{x_i, i=1,2,\cdots,4N\}$, have zero-mean and variances of $\sigma_h^2/2$, respectively. Thus, we have

$$f_\gamma(s)=E[e^{s\gamma}]=\prod_{i=1}^{4N} E[e^{sx_i^2}]=1/(1-\sigma_h^2 s)^{2N}, \quad (17)$$

where the third equality comes because $E[e^{sx_i^2}]=1/\sqrt{1-\sigma_h^2 s}$. Now, using (17) in (15) and (16), we get

$$E[Q(u)]=\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^{4N}\phi}{\left(\frac{3\rho_1}{4(M-1)}+\sin^2\phi\right)^{2N}} d\phi \quad (18)$$

$$E[Q^2(u)]=\frac{1}{\pi}\int_0^{\frac{\pi}{4}} \frac{\sin^{4N}\phi}{\left(\frac{3\rho_1}{4(M-1)}+\sin^2\phi\right)^{2N}} d\phi, \quad (19)$$

where ρ_1 is the SNR defined in (7).

Even though the SER in (14) can be numerically evaluated by applying the Trapezoidal rule by using (18) and (19), a closed-form SER expression without the integral operators is preferred because it more clearly explains the influence of the parameters such as M, N, as well as SNR on the SER. The problem, however, is that direct calculation of the integrals in (18) and (19) is formidable for an arbitrary N. To overcome this problem, we adopt a curve-fitting model given by

$$\widetilde{p_s}(e)=\frac{\kappa_1 p_1-\kappa_2 p_2}{\left(\frac{3\rho_1}{4(M-1)}+\delta_s\right)^{2N}}, \quad (20)$$

where

$$p_1=\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sin^{4N}\phi d\phi \text{ and } p_2=\frac{1}{\pi}\int_0^{\frac{\pi}{4}} \sin^{4N}\phi d\phi.$$

In (20), p_1 and p_2 can be evaluated as shown in Table 1. Thus we need to find the optimal value of δ_s minimizing the sum of squared error of the logarithm of the SER in the SNR region of interest.

In addition, if we define the SNR region of interest as R_{ρ_1}, then the optimum value $\delta_{s,opt}$ of δ_s is expressed by

$$\delta_{s,opt}=\arg\ \min\int_{R_{\rho_1}} |\log p_s(e)-\log\widetilde{p_s}(e)|^2 d\rho_1.$$

For instance, when the SNR region of interest $R_{\rho_1}=[0,30]dB$, the optimum value $\delta_{s,opt}$ for the number of receive antennas $N\in[1,4]$ is estimated as shown in Table 2.

Table 1. p_1 and p_2 for $N\in[1,4]$

N	p_1	p_2
1	$3/16$	0.0141725
2	$35/2^8$	0.0547857
3	$235/2^{11}$	0.0549438
4	$6435/2^{16}$	0.0488467

Table 2. The optimum value of δ_s for $N\in[1,4]$

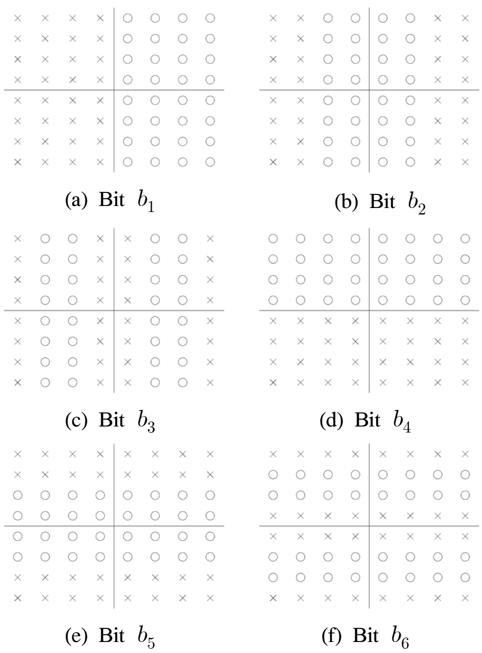
N	QPSK	16-QAM	64-QAM
1	0.811	0.811	0.818
2	0.942	0.935	0.933
3	0.820	0.794	0.784
4	0.856	0.832	0.821

3.3 BER in a Rayleigh fading channel

Assume that a transmit bit sequence $(b_1 b_2 \cdots b_{\log_2 M})$ is Gray coded to an M-ary QAM symbol as shown in Fig. 2. Now, calculating $p_b(e|\gamma)$ [7] and then taking expectation over the distribution of γ, we get the closed form expressions of the BER for QPSK, and 64-QAM symbols respectively as

$$\text{QPSK: } p_b(e)=E[Q(u)] \quad (21a)$$

$$\text{64-QAM: } p_b(e)=\frac{7E[Q(u)]}{12}+\frac{E[Q(3u)]}{2}-\frac{E[Q(5u)]}{12}+\frac{E[Q(9u)]}{12}-\frac{E[Q(13u)]}{12} \quad (21b)$$



(a) Bit b_1 (b) Bit b_2

(c) Bit b_3 (d) Bit b_4

(e) Bit b_5 (f) Bit b_6

Fig. 2. Constellation of Gray-coded 64-QAM

where

$$E[Q(u)] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^{4N}\phi}{\left(\frac{3\kappa^2\rho_1}{4(M-1)}+\sin^2\phi\right)^{2N}} d\phi \quad .$$

Given the exact BER expressions in (21), an approximate expression of the BER without the integral operators can be obtained by adopting a curve-fitting model for $p_b(e)$ as given by

$$\widetilde{p_b}(e) = \sum_{k=1}^{K} \frac{a_k p_1}{\left(\frac{3\kappa^2\rho_1}{4(M-1)}+\delta_b\right)^{2N}}, \qquad (22)$$

where p_1 is evaluated as shown in Table 1, and then by finding the optimum value of δ_b expressed by

$$\delta_{b,opt} = \arg min \int_{R_{\rho_1}} |\log p_b(e) - \log \widetilde{p_b}(e)|^2 d\rho_1. \qquad (23)$$

In (22), K equals 1, 5, 13 for QPSK, 16-QAM, and 64-QAM, respectively, and for non-zero a_k, $a_1 = 1$ for QPSK, (a_1, a_3, a_5) = (3/4, 1/2, 1/4) for 16-QAM, and $(a_1, a_3, a_5, a_9, a_{13})$ = (7/12, 1/2, -1/12, 1/12, -1/12) for 64-QAM symbols. For the SNR region of interest $R_{\rho_1} = [0,30]dB$, the optimum value $\delta_{b,opt}$ is estimated as shown in Table 3.

In addition, if $E[Q(u)]$ in the SNR region of interest is small enough (viz. below 10^{-2}), the following approximate formula written by

$$\widetilde{p_b}(e) = \frac{a_1 p_1}{\left(\frac{3\rho_1}{4(M-1)}+\delta_b\right)^{2N}} \qquad (24)$$

can be adopted instead of (22).

Table 3. The value of $\delta_{b,opt}$ by the models in (22) and (24)

N	QPSK	16-QAM		64-QAM	
		(24)	(26)	(24)	(26)
1	0.777	0.742	0.694	0.721	0.623
2	0.858	0.830	0.820	0.812	0.777
3	0.896	0.871	0.868	0.856	0.839
4	0.917	0.896	0.895	0.882	0.873

Ⅳ. Simulation Results

Computer simulations were conducted to confirm the validity of the SER/BER expressions derived in this paper. The signal model used in the simulation is as follows. Random bit sequences were Gray coded to QPSK, 16-QAM, and 64-QAM symbols as shown in Fig. 2, and then transmitted through the MIMO flat Rayleigh fading channels. The channel coefficients $\{h_{ij}, i=1,2, j=1,\cdots,N\}$ were generated to have $\sigma_h^2 = 1$. In generating the AWGN, the SNR range of $[0,30]dB$ for a single antenna receiver (ρ_1) was assumed. Note that if $\sigma_h^2 = 1$, we have $\rho_1 = \log_2 M \times E_b/N_0$, where $N_0 = \sigma_n^2$. In order to focus on the verification of the analytical results, it was assumed that the perfect channel knowledge is available at the receiver. The SER and BER of the MIMO-STBC system were empirically estimated with 10^6 symbol receptions.

Fig. 3 compares the simulated SERs with the analytical SERs. In the figure, the dash-dotted lines denote the exact analytical SER in (14) and the solid lines the approximated SER in (20), while the symbols (◦,∗,◇,□) represent the simulation results with the different number of receiver antennas. A remarkable agreement is observed between the exact analytical SERs and the simulation results, and the approximate SERs (i.e., solid lines) coincide with the corresponding simulation results (i.e., dash-dotted lines) as well.

Fig. 4 compares the simulated BER with the analytical counterparts. In the figure, the dash-dotted lines denote the exact BERs in (21), the solid lines the approximated BERs in (22), and the dashed lines the approximated BERs in (24). Likewise, The symbols (◦,∗,◇,□) represent the simulated

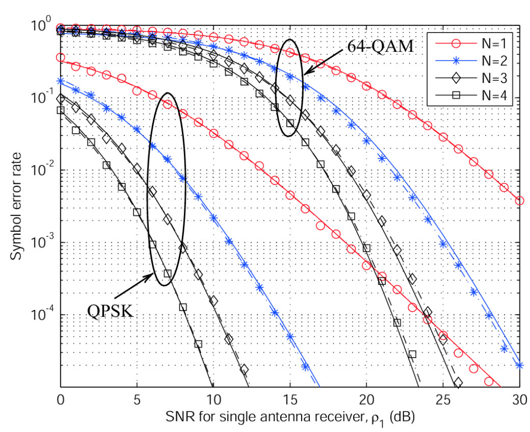


Fig. 3. SER versus SNR with (14) and (20).

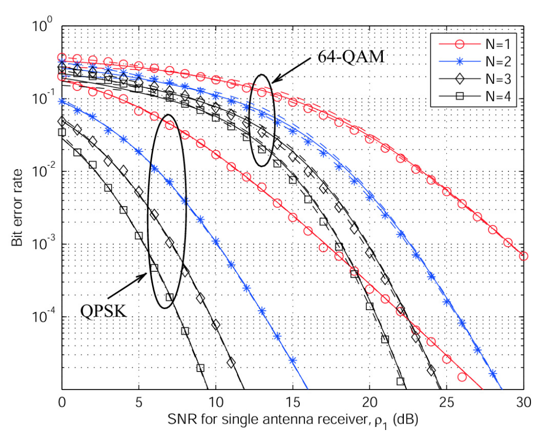


Fig. 4. BER versus SNR with (21), (22), and (24)

BERs with the different number of receiver antennas. As the case of the SERs in Fig. 3, it is observed that the simulated BERs agree with the analytical results in (21), and that the approximated BERs in (22) (i.e., the solid line) also agree with the exact BERs as well as the simulation results without noticeable deviation. Particularly, it is noted that the single term approximate BERs in (24) also agree with the simulated BERs without large approximation error.

Ⅳ. Conclusions

In this paper, we have analyzed the performance of a MIMO-STBC system adopting a zero-forcing equalizer and single-symbol ML decoders in Rayleigh fading channels. The exact formulas were derived for the SER and BER, which can be considered a simple extension of the previous studies to a MIMO-STBC system, and then by using a curve fitting model approximate formulas were derived that successfully eliminated the integral operators in the exact formulas. Through computer simulation, it was shown that the proposed SER and BER expressions well coincide with the simulated counterparts. The agreement between the analytical and simulation results proves the validity of the analytical results derived in this paper.

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