

# On the Relationship Between the Performance Criteria of Unitary Space-Time Codes with Noncoherent and Coherent Decoding

Kyungwhoon Cheun\* Lifelong Member,
Jeongchang Kim\*\*, Soongyoon Choi\*\*\* Regular Members

### **ABSTRACT**

Hochwald *et al.* introduced unitary space-time codes for quasi-static Rayleigh fading channels which allows for noncoherent decoding when the channel response is not known at the receiver. However, when reliable information on the channel response is available, coherent decoding is preferable for improved performance. Here, we study the relationship between the performance criteria on the diversity and coding advantages provided by unitary space-time codes with noncoherent and coherent decoding. We show that when a unitary space-time code achieves full spatial diversity with noncoherent decoding, full spatial diversity is also guaranteed with coherent decoding.

Key Words: Unitary Space-Time Codes (STC), Noncoherent, Coherent, Diversity, Transmit Antennas

### I. Introduction

A recent approach to obtaining spatial diversity over multipath fading channels is to employ coding techniques appropriate for multiple transmit antennas, namely, space-time coding [1]-[7]. Tarokh et al. in [1] developed design criteria for space-time codes under the assumption that the channel is known at the receiver. Specifically, the rank and determinant criteria for quasi-static Rayleigh fading channels quantify the diversity and the coding gains of space-time codes, respectively. Hochwald et al. in [2], introduced unitary space-time codes for quasi-static Rayleigh fading channels which allows for noncoherent decoding for the case when the channel response is not available at the receiver.

Since the channel environment for a receiver

may change depending on time and place, the assumption that the channel information is always either known or unknown at the receiver is unrealistic. If reliable channel response becomes available at the receiver, coherent decoding may be desirable for improved performance. In this case, we would at least like to be guaranteed that the unitary space-time code in use achieving full spatial diversity and maximizing coding advantage with noncoherent decoding also achieves full spatial diversity and maximizes coding advantage with coherent decoding. In this paper, we study the relationship between the performance criteria on the diversity and coding advantages provided by unitary space-time codes under noncoherent and coherent decoding. We show that when a unitary space-time code achieves full diversity with noncoherent decoding, full spatial

<sup>\*\*</sup> This research was supported by the MKE(The Ministry of Knowledge Economy), Korea, under the ITRC(Information Technology Research Center) support program supervised by the NIPA(National IT Industry Promotion Agency) (NIPA-2010-C1090-1011-0011)

<sup>\*</sup> Department of Electronic and Electronical Engineering, Pohang University of Science and Technology (POSTECH) (cheun@postech.ac.kr)

<sup>\*\*</sup> Department of Electronics and Communications Engineering, Korea Maritime University (jchkim@hhu.ac.kr)

<sup>\*\*\*</sup> Samsung Electronics Co., Ltd. (soongyoon.choi@samsung.com) 논문번호: KICS2010-08-385, 접수일자: 2010년 8월 6일, 최종논문접수일자: 2010년 11월 26일

diversity is also guaranteed with coherent decoding.

The remainder of the paper is organized as follows. In Section II, the signal model is presented and in Section III, we briefly review the performance criteria of unitary space-time codes for the coherent and noncoherent decoding. In Section IV, we show that a unitary space-time code achieving full spatial diversity with noncoherent decoding also achieves full spatial diversity with coherent decoding and conclusions are drawn in Section V.

## II. Signal Model

We consider a communication system with M transmit and N receive antennas under quasi-static, frequency flat Rayleigh fading channels. Each receive antenna responds to each of the transmit antennas through a statistically independent channel response, assumed to be constant for T symbol periods. Using the complex baseband representation, the lowpass equivalent channel input-output relationship for such channels can be written as

$$Y = \sqrt{\frac{\rho}{M}} SH + W.$$
 (1)

Here,  $\mathbf{Y} = \{y_{tn}\}$  is the  $T \times N$  received signal matrix with  $y_{tn}$  being the signal received by the nth receive antenna at time t,  $\mathbf{S} = \{s_{tm}\}$  is the  $T \times M$  transmitted signal matrix with  $s_{tm}$  being the signal transmitted on the mth transmit antenna at time t. The channel coefficient matrix  $\mathbf{H} = \{h_{mn}\}$  is an  $M \times N$  matrix of independent and identically distributed (i.i.d.) Rayleigh fading coefficients with  $h_{mn}$  being the fading coefficient between the mth transmit antenna and the nth receive antenna. The noise matrix,  $\mathbf{W} = \{w_{tn}\}$  is the  $T \times N$  matrix representing the i.i.d. additive receiver thermal noise with  $w_{tn}$  being the thermal noise observed by the nth receive antenna at time t. Let CN(0.1/2) denote a complex Gaussian

random vsiiable with independent real and imaginsianpsits, each with zero mean and vsiiaenot 1/2. The fading coefficients  $h_{mn}$  are assumed to be CN(0,1/2) and each entian  $w_{tn}$  of  $\textbf{\textit{W}}$  are also CN(0,1/2) distributed. The average energanof the symbols transmitted from each antenna is normalized to be one, i.e.,  $\frac{1}{M}\sum_{m=1}^{M}E\{|s_{tm}|^2\}=1.$  Therefore, the average received

SNR at each of the receive antennas is  $\rho$ .

In this paper, we consider unitary space-time  $\operatorname{codes}^{[3]}$  of size L consisting of codewords  $S_{l} = \sqrt{T} \Phi_{l}, \quad l = 1, \cdots, L$ , where  $\Phi_{l}$  are  $T \times M$  complex matrices satisfying  $\Phi_{l}^{H} \Phi_{l} = I_{M}$ . Here,  $A^{H}$  denotes the conjugate transpose of A and  $I_{M}$  denotes the  $M \times M$  identity matrix. In the sequel, we will identify  $S_{l}$  with  $\Phi_{l}$  and use then interchangeably.

# III. Review of the Design Criteria of Unitary Space-Time Codes for Coherent and Noncoherent Decoding

First, let us briefly review the design criteria for unitary space-time codes for the cases when the channel coefficient matrix H is either known or unknown at the decoder. For simplicity, we will refer to the two decoding operations, as coherent and noncoherent decoding, respectively.

### 3.1 Coherent decoding

Most of the previous works on space-time codes assume that the channel coefficient matrix H is known at the receiver. The following provides a quick review of optimal receivers, pairwise error probability and the design criteria for coherent decoding of unitary space-time codes<sup>[2],[3]</sup>.

When H is known to the decoder, the maximum-likelihood (ML) decoder is given as<sup>[2]</sup>

$$\mathbf{\Phi}_{ML} = \operatorname{arg\,min}_{\mathbf{\Phi}_{l} \in U}$$

$$Tr \left\{ \left( \mathbf{Y} - \sqrt{\frac{\rho T}{M}} \mathbf{\Phi}_{l} \mathbf{H} \right) \left( \mathbf{Y} - \sqrt{\frac{\rho T}{M}} \mathbf{\Phi}_{l} \mathbf{H} \right)^{H} \right\}$$
(2)

where  $U = \{ \boldsymbol{\Phi}_1, \cdots, \boldsymbol{\Phi}_L \}$  and  $Tr\{\boldsymbol{A}\}$  denotes the trace of the matrix  $\boldsymbol{A}$ . The Chernoff bound on the pairwise error probability between  $\boldsymbol{\Phi}_l$  and  $\boldsymbol{\Phi}_l'$  takes the form<sup>[3]</sup>

$$\begin{split} P_{l,l} &\leq \left| \boldsymbol{I}_{M} + \frac{\rho T}{4M} (\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l})^{H} (\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l}) \right|^{-N} \\ &\approx \left( \Lambda_{c} (\boldsymbol{\Phi}_{l}, \boldsymbol{\Phi}_{l}) \frac{\rho T}{4M} \right)^{-\nu_{c}(\boldsymbol{\Phi}_{l}, \boldsymbol{\Phi}_{l})N}, \ \rho \gg 1 \end{split} \tag{3}$$

where  $\nu_c(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_l)$  denotes the rank of the difference matrix  $(\boldsymbol{\Phi}_l - \boldsymbol{\Phi}_l)$  and  $|\boldsymbol{A}|$  denotes the determinant of  $\boldsymbol{A}$ . The quantity  $\nu_c(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_l)$  can be interpreted as the *diversity advantage* of the corresponding codeword pair. The quantity  $\Lambda_c(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_l)$ , on the other hand, can be interpreted as the *coding advantage* of the corresponding codeword pair and is given by [1]

$$\Lambda_{c}(\boldsymbol{\Phi}_{l},\boldsymbol{\Phi}_{l}) = \left| \left( \boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l} \right)^{H} \left( \boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l} \right) \right|_{\dagger}^{1/\nu_{c}(\boldsymbol{\Phi}_{l},\boldsymbol{\Phi}_{l})} \tag{4}$$

where  $|A|_{\dagger}$  denotes the product of the nonzero eigenvalues of A. Hence, for large values of  $\rho$ , the performance of a unitary space-time code under coherent decoding is determined primarily by the minimum diversity advantage  $\nu_c^m$  given as

$$\nu_c^m = \min_{1 < l < L, l \neq l'} \nu_c(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{l'}) \tag{5}$$

and the minimum coding advantage  $\Lambda_c^m$  given by,

$$\Lambda_c^m = \min_{1 \le l \le L, \ l \ne l', \ \nu_c(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{l'}) = \nu_c^m} \Lambda_c(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{l'}).$$
 (6)

#### 3.2 Noncoherent decoding

Assuming that neither the transmitter nor the receiver knows the channel coefficient matrix H, the ML decoding rule for unitary space-time codes is given by<sup>[2]</sup>

$$\boldsymbol{\Phi}_{ML} = \arg\max_{\boldsymbol{\Phi}_{l} \in U} Tr \left\{ \boldsymbol{Y}^{H} \boldsymbol{\Phi}_{l} \boldsymbol{\Phi}_{l}^{H} \boldsymbol{Y} \right\}. \tag{7}$$

The Chernoff bound on the pairwise error probability between codewords  $\Phi_l$  and  $\Phi_{l'}$  is then given by<sup>[3]</sup>

$$P_{l,l'} \leq \left| \mathbf{I}_{M} + \frac{(\rho T/M)^{2}}{4(1 + \rho T/M)} (\mathbf{I}_{M} - \mathbf{\Phi}_{l'}^{H} \mathbf{\Phi}_{l} \mathbf{\Phi}_{l'}^{H} \mathbf{\Phi}_{l}) \right|^{-N}$$

$$\approx \left( \Lambda_{n} (\mathbf{\Phi}_{l}, \mathbf{\Phi}_{l}) \frac{\rho T}{4M} \right)^{-\nu_{n} (\mathbf{\Phi}_{l}, \mathbf{\Phi}_{l'})N}, \ \rho \gg 1.$$
(8)

The diversity advantage,  $\nu_n(\boldsymbol{\Phi}_l,\boldsymbol{\Phi}_l)$  is equal to the rank of matrix  $(\boldsymbol{I}_{\!M}\!-\!\boldsymbol{\Phi}_l^H\!\boldsymbol{\Phi}_l\!\boldsymbol{\Phi}_l^H\!\boldsymbol{\Phi}_l)$ . Hochwald and Marzetta<sup>[2]</sup> noted that the maximum value of  $\nu_n(\boldsymbol{\Phi}_l,\boldsymbol{\Phi}_l)$  is M, which is achieved when 1 is not a singular value of  $\boldsymbol{\Phi}_l^H\!\boldsymbol{\Phi}_l$ . The coding advantage  $\Lambda_n(\boldsymbol{\Phi}_l,\boldsymbol{\Phi}_l)$  is given by  $\boldsymbol{\Phi}_l^{(1)}$ 

$$\Lambda_{n}(\boldsymbol{\Phi}_{l},\boldsymbol{\Phi}_{l}) = |\boldsymbol{I}_{M} - \boldsymbol{\Phi}_{l}^{H}\boldsymbol{\Phi}_{l}\boldsymbol{\Phi}_{l}^{H}\boldsymbol{\Phi}_{l}|_{\perp}^{1/\nu_{n}(\boldsymbol{\Phi}_{l},\boldsymbol{\Phi}_{l})}.$$
 (9)

For large values of  $\rho$ , the performance of a unitary space-time code under noncoherent decoding is again determined mainly by the minimum diversity advantage given as

$$\nu_n^m = \min_{1 < l < l, l \neq l'} \nu_n(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{l'}) \tag{10}$$

and the minimum coding advantage given by

$$\Lambda_n^m = \min_{1 \le l \le L, \ l \ne l', \ \nu_n(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{l'}) = \nu_n^m} \Lambda_n(\boldsymbol{\Phi}_l, \boldsymbol{\Phi}_{l'}). \ (11)$$

Hence, for both coherent and noncoherent decoding, the important design criteria are to maximize the minimum diversity advantages  $\nu_c^m$  and  $\nu_n^m$ .

# IV. Relationship of the Performance Criteria for Coherent and Noncoherent Decoding

Even when a unitary space-time code is employed in order to allow noncoherent decoding, we may wish to perform coherent decoding at the receiver when the channel variation is slow enough to allow accurate channel estimation for an extra performance boost. Hence, we need to address some basic questions regarding the performance of unitary space-time codes optimized for noncoherent decoding under coherent decoding. We will provide the answer to the most important question, i.e., does a unitary space-time code achieving full spatial diversity and maximizing coding advantage with noncoherent decoding guarantee full spatial diversity with coherent decoding? We first establish a Lemma that is crucial in relating the minimum diversity advantages of unitary space-time codes under coherent and noncoherent decoding.

Lemma 1: The minimum diversity advantage of unitary space-time codes under noncoherent decoding can also be written as

$$\nu_c^m = \min_{1 \le l \le L, l \ne l'} rank([\boldsymbol{\Phi}_l \ \boldsymbol{\Phi}_{l'}]) - M. \tag{12}$$

*Proof:* Let us first consider the following  $2M \times 2M$  Hermitian matrix

$$\mathbf{P} = \begin{bmatrix} \mathbf{I}_{M} & \mathbf{\Phi}_{l}^{H} \mathbf{\Phi}_{l'} \\ (\mathbf{\Phi}_{l}^{H} \mathbf{\Phi}_{l'})^{H} & \mathbf{I}_{M} \end{bmatrix}, \ l \neq l'.$$

We know from [8]1) that

$$rank(\mathbf{P}) = M + rank(\mathbf{I}_{M} - \mathbf{\Phi}_{l}^{H} \mathbf{\Phi}_{l} \mathbf{\Phi}_{l}^{H} \mathbf{\Phi}_{l}).$$
 (13)

The matrix P can also be written as

$$\begin{bmatrix} \mathbf{I}_{M} & \mathbf{\Phi}_{l}^{H} \mathbf{\Phi}_{l} \\ (\mathbf{\Phi}_{l}^{H} \mathbf{\Phi}_{l})^{H} & \mathbf{I}_{M} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{l} & \mathbf{\Phi}_{l} \end{bmatrix}^{H} \begin{bmatrix} \mathbf{\Phi}_{l} & \mathbf{\Phi}_{l} \end{bmatrix}. \quad (14)$$

Moreover, since matrices A and  $A^HA$  have identical rank<sup>[6]</sup>, we have

$$rank(\mathbf{P}) = rank([\mathbf{\Phi}_l \ \mathbf{\Phi}_{l'}]). \tag{15}$$

Combining this with (13), we may rewrite the rank of the matrix  $(\mathbf{I}_{M} - \mathbf{\Phi}_{l}^{H} \mathbf{\Phi}_{\ell}^{H} \mathbf{\Phi}_{l})$  as

$$rank(\mathbf{I}_{M} - \mathbf{\Phi}_{l}^{H}\mathbf{\Phi}_{l}\mathbf{\Phi}_{l}^{H}\mathbf{\Phi}_{l}) = rank([\mathbf{\Phi}_{l} \ \mathbf{\Phi}_{l}]) - M.(16)$$

Therefore, the minimum diversity advantage of unitary space-time codes with noncoherent decoding is given by

$$\nu_n^m = \min_{1 \le l \le L, \ l \ne l'} \ rank(\left[\boldsymbol{\Phi}_l \ \boldsymbol{\Phi}_{l'}\right]) - M. \ (17)$$

From Lemma l, it is required that  $rank([\Phi_l \ \Phi_l]) = 2M$ , for all  $l \neq l'$ , in order to achieve full spatial diversity of M, which also requires  $T \geq 2M$ . Assuming that  $T \geq 2M$ , and armed with this result, the following theorem relating the minimum diversity advantages of unitary space-time codes under coherent and noncoherent decoding is easily shown.

Theorem 1: If a unitary space-time code with noncoherent decoding achieves full spatial diversity, i.e.,  $\nu_n^m = M$ , then it also guarantees full spatial diversity under coherent decoding, i.e.,  $\nu_c^m = M$ , when the channel response is available at the receiver.

Proof: Assume that a given unitary space-time code U consisting of L codewords under noncoherent decoding achieves full spatial diversity. Then,  $rank([\boldsymbol{\Phi}_l \ \boldsymbol{\Phi}_{l'}]) = 2M$ , for all  $l \neq l'$ , which requires that the columns of  $\Phi_l$  and  $\Phi_{l'}$  are linearly independent for  $l \neq l'$ . Full spatial diversity under coherent decoding, on the other hand, is guaranteed if the columns of the matrix  $\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l'} = \begin{bmatrix} \phi_{l1} - \phi_{l'1} & \cdots & \phi_{lM} - \phi_{l'M} \end{bmatrix}$ independent which is trivially satisfied if  $[\boldsymbol{\Phi}_{t} \; \boldsymbol{\Phi}_{t'}]$ has full rank. Hence, the given unitary space-time code achieves full spatial diversity under coherent decoding when the channel response is available

<sup>1)</sup> Suppose that a Hermitian matrix G is partitioned as  $G = \begin{bmatrix} A & B \\ B^H & C \end{bmatrix}$  where A and C are  $M \times M$  positive definite matrices. Then, this matrix G is positive definite and  $rank(G) = M + rank(A - BC^{-1}B^H)$ .

at the receiver.

An alternate proof of *Theorem 1* is possible following the results of [5].

Alternate proof: With noncoherent decoding, the diversity advantage is given by  $rN^{[5]}$ , where

$$r = \min_{1 \le l \le L, \ l \ne l'} \left( \dim \left( W_{\mathbf{\Phi}_{l}} \right) - \dim \left( W_{\mathbf{\Phi}_{l}} \cap W_{\mathbf{\Phi}_{l}} \right) \right). \tag{18}$$

Here,  $W_{\Phi_l}$  denotes the subspace spanned by the columns of  $\Phi_l$  and  $\dim(W_{\Phi_l})$  denotes the dimension of  $W_{\Phi_l}$ . Since  $\dim(W_{\Phi_l}) = M$ , we need  $\dim(W_{\Phi_l} \cap W_{\Phi_l}) = 0$  for r to equal M. This in turn requires the columns of  $\Phi_l$  and  $\Phi_l$  to be linearly independent which again implies linear independent of the columns of  $\Phi_l - \Phi_{l'}$ .

Next, we relate the coding advantage of a unitary space-time code under coherent and noncoherent decoding. Suppose that a given unitary space-time code U with L codewords under noncoherent decoding achieves full spatial diversity. Then, the coding advantage between codewords  $\boldsymbol{\Phi}_l$  and  $\boldsymbol{\Phi}_l'$  under noncoherent decoding is given by

$$\left| \boldsymbol{I}_{M} - \boldsymbol{\Phi}_{l}^{H} \boldsymbol{\Phi}_{l} \boldsymbol{\Phi}_{l}^{H} \boldsymbol{\Phi}_{l}^{\dagger} \right|^{1/M} = \left\{ \prod_{m=1}^{M} \left( 1 - d_{ll'm}^{2} \right) \right\}^{1/M} \tag{19}$$

where  $0 \leq d_{ll'M} \leq \cdots \leq d_{ll'1} \leq 1$  are the singular values of the  $\Phi_l^H \Phi_l^{(2)}$ . Hence, in order to maximize the coding advantage, we need to minimize the singular values  $d_{ll'm}$ ,  $m=1,\cdots,M$  which are all less than 1.

Theorem 2: Let U be a given unitary space-time code achieving full spatial diversity and maximizing the coding advantage under noncoherent decoding. Suppose that  $\mathbf{\Phi}_l^H\mathbf{\Phi}_l = Re\{\mathbf{\Phi}_l^H\mathbf{\Phi}_l\}$  for  $\mathbf{\Phi}_l, \mathbf{\Phi}_l \in \{\mathbf{\Phi}_1, \cdots, \mathbf{\Phi}_L\}$ ,  $\forall l \neq l'$ . Then, U also maximizes the coding advantage under coherent

decoding.

*Proof:* The coding advantage of the code with coherent decoding is given by

$$\begin{aligned} |(\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l})^{H} (\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l})|^{1/M} \\ &= |2\boldsymbol{I}_{M} - 2Re\left\{\boldsymbol{\Phi}_{l}^{H} \boldsymbol{\Phi}_{l}\right\}|^{1/M} \\ &= \left\{\prod_{m=1}^{M} (2 - 2\lambda_{ll'm})\right\}^{1/M} \end{aligned} (20)$$

where  $\lambda_{ll'M} \leq \cdots \leq \lambda_{ll'1}$  are the singular values of  $Re\left\{ {m \Phi}_l^H {m \Phi}_{l'} \right\}$ . Since  ${m \Phi}_l^H {m \Phi}_{l'} = Re\left\{ {m \Phi}_l^H {m \Phi}_{l'} \right\}$ ,  $\lambda_{ll'm} = d_{ll'm}$ ,  $m=1,\cdots,M$ . Then, the coding advantage of U under coherent decoding can be written as

$$|(\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l})^{H}(\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l})|^{1/M}$$

$$= \left\{ \prod_{m=1}^{M} (2 - 2d_{ll'm}) \right\}^{1/M}.$$
(21)

By Lemma 2, if  $\left\{\prod_{m=1}^{M}\left(1-d_{ll'm}^2\right)\right\}^{1/M}$  is maximized, then  $\left\{\prod_{m=1}^{M}\left(2-2d_{ll'm}\right)\right\}^{1/M}$  is also maximized. Therefore, the coding advantage of U under coherent decoding is maximized.

In addition, since  $0 \le d_{ll'M} \le \cdots \le d_{ll'1} \le 1^{[2]}$ , we have

$$\left\{ \prod_{m=1}^{M} \frac{\left(2-2d_{ll'm}\right)}{\left(1-d_{ll'm}^{2}\right)} \right\}^{1/M} = \left\{ \prod_{m=1}^{M} \frac{2}{\left(1+d_{ll'm}\right)} \right\}^{1/M} \geq 1$$

from (19) and (21). Thus, we have

$$\left\{\prod_{m=1}^{M} \left(2 - 2d_{ll'm}\right)\right\}^{1/M} \ge \left\{\prod_{m=1}^{M} \left(1 - d_{ll'm}^2\right)\right\}^{1/M}. \tag{22}$$

Therefore, the coding advantage under coherent decoding is greater than or equal to the coding advantage under noncoherent decoding.

The coding advantage of the code with coherent decoding, on the other hand, is given by

$$\left| \left( \boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l} \right)^{H} \left( \boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l} \right) \right|^{1/M} = \left\{ \prod_{m=1}^{M} \sigma_{ll'm}^{2} \right\}^{1/M} \quad (23)$$

where  $0 \leq \sigma_{ll'M} \leq \cdots \leq \sigma_{ll'1} \leq 2$  are the singular values of  $\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l}^{[2]}$ . Hence, for coherent decoding, we wish to maximizing the singular values  $\sigma_{ll'm}$ ,  $m=1,\cdots,M$ . In general, there is no direct relationship between the singular values  $d_{ll'm}$  and  $\sigma_{ll'm}$ . However, for M=1,  $d_{ll'1} = |\boldsymbol{\Phi}_{l}^{H}\boldsymbol{\Phi}_{l}|$  and  $\sigma_{ll'm} = |\boldsymbol{\Phi}_{l} - \boldsymbol{\Phi}_{l}| = \sqrt{2-2Re(\boldsymbol{\Phi}_{l}^{H}\boldsymbol{\Phi}_{l})}$  which implies that  $\sqrt{2(1-d_{ll'1})} \leq \sigma_{ll'1} \leq \sqrt{2(1+d_{ll'1})}^{[2]}$ . For the special case  $d_{ll'1} = \cdots = d_{ll'M} = 0$ , then  $\sigma_{ll'1} = \cdots = \sigma_{ll'M} = \sqrt{2}$ .

#### V. Conclusions

In this paper, we studied the relation between the performance criteria on the diversity gain with coherent and noncoherent decoding of unitary space-time codes. From our study, we knew the fact that a unitary space-time code with full spatial transmit diversity under noncoherent decoding also guarantees full spatial transmit diversity under coherent decoding.

### REFERENCES

- V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction," *IEEE Trans. Inform. Theory*, Vol.44, pp.744-765, March, 1998.
- [2] B. M. Hochwald, T. L. Marzetta, "Unitary Space-Time Modulation for Multiple-Antenna Communications in Rayleigh Flat Fading," *IEEE Trans. Inform. Theory*, Vol.46, No.2, pp.543-564, March, 2000.
- [3] B. L. Hughes, "Differential Space-Time Modulation," *IEEE Trans. Inform. Theory*, Vol.46, No.7, pp.2567-2578, Nov., 2000.
- [4] B. M. Hochwald, W. Sweldens, "Differential Unitary ST Modulation," *IEEE Trans. Commun.*, Vol.48, No.12, pp.2041-2052, Dec.,

2000.

- [5] V. Tarokh, I. Kim, "Existence and Construction of Noncoherent Unitary ST Codes," *IEEE Trans. Inform. Theory*, Vol.48, No.12, pp.3112-3117, Dec., 2002.
- [6] J. Kim, K. Cheun, "An efficient decoding algorithm for QO-STBCs based on iterative interference cancellation," *IEEE Commun. Lett.*, Vol.12, No.4, pp.292-294, April, 2008.
- [7] J. Kim, K. Cheun and S. Choi, "Unitary space-time constellations based on quasiorthogonal sequences," *IEEE Trans. Commun.*, Vol.58, No.1, pp.35-39, Jan., 2010.
- [8] M. J. Grimble, *System Identification*, Prentice Hall, 1989.

### Kyungwhoon Cheun Lifelong Member



Kyungwhoon Cheun was born in Seoul, Korea, on December 16, 1962. He received the B.A. degree in electronics engineering from Seoul National University, Seoul, Korea, in 1985, and the M.S. and Ph.D. degrees from

the University of Michigan, Ann Arbor, in 1987 1989, respectively, both in electrical engineering. From 1987 to 1989, he was a Research Assistant at the EECS Department at the University of Michigan, and from 1989 to 1991, he joined the Electrical Engineering Department at the University of Delaware, ewark, Assistant Professor. In 1991, he joined the Division of Electrical and Computer Engineering the Pohang University of Science Technology (POSTECH), Pohang, Korea, where he is currently a Professor. He also served as an engineering consultant to various industries in the area of mobile communications and modem design. His current research interests include turbo codes, RA codes, space-time codes, MIMO systems, UWB communications, cellular packet radio networks, and OFDM systems.

### Jeongchang Kim

Regular Member



Jeongchang Kim received the B.S. degree in electronics, communication and radio engineering from Hanyang University, Seoul, Korea, in 2000, and M.S. and Ph.D. degrees from Pohang University

of Science and Technology (POSTECH), Pohang, Korea, in 2002 and 2006, respectively, both in electronic and electrical engineering. From 2006 to 2008, he was a Full-Time Researcher at POSTECH, and from 2008 to 2009, he was a Research Assistant Professor. From 2009 and 2010, he was with the Broadcasting Systems Research Electronics Department at Telecommunications Research Institute (ETRI) as a senior member of engineering staff. In 2010, he Department of Electronics joined the Communications Engineering at Korea Maritime University, Busan, Korea, where he is currently an Assistant Professor. His current research interests include MIMO, space-time codes, OFDM, digital communication systems.

### Soongyoon Choi

Regular Member



Soongyoon Choi received the B.S. degree in electronic and electrical Engineering from Kyungpook National University, Daegu, Korea, in 2002, and the M.S. degree in electronic and electrical engineering from

Pohang University of Science and Technology (POSTECH), Pohang, Korea, in 2004. In 2004, she joined Samsung Electronics Co., Ltd., where she is currently a senior engineer.