영확률 최대화에 근거한 결정궤환 등화

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Decision Feedback Equalizer based on Maximization of Zero-Error Probability

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요 약

이 논문에서는 심각한 다중경로 페이딩 왜곡과 충격성 잡음을 극복하기 위해 결정궤환 구조를 갖추고 오차의 영확률을 최대화하는 비선형 알고리듬(MZEP-DF)를 제안하였다. 제안된 MZEP-DF 알고리듬은 충격성 잡음에 강인성을 보였고 궤환구조 영역에서 잔여 심볼간 간섭을 제거하는 능력을 보였다. 기존 LMS 알고리듬이 정상상태 오차 전력에 대해 -3 dB 이하로 수렴하지 못했던 심각한 다중경로 페이딩과 충격성 잡음 환경에 대해 제안한 알고리듬은 선형 MZEP 알고리듬과 비교하였을 때, 10 dB 이상 성능향상을 보였다.

Key Words : Zero-error probability, Decision Feedback, Impulsive noise, Gaussian kernel, ITL

ABSTRACT

In this paper, a nonlinear algorithm that maximizes zero-error probability (MZEP) with decision feedback (DF) is proposed to counteract both of severely distorted multi-path fading effect and impulsive noise. The proposed MZEP-DF algorithm has shown the immunity to impulsive noise and the ability of the feedback filter section to cancel the remaining intersymbol interference as well. Compared with the linear MZEP algorithm, it yields above 10 dB enhancement of steady state MSE performance in severely distorted multipath fading channels with impulse noise where the least mean square (LMS) algorithm does not converge below -3dB of MSE.

Ⅰ. Introduction

Various adaptive equalizer structures and coefficient-adjustment algorithms have been developed based on minimum squared error (MSE) criterion in order to cancel intersymbol interference (ISI) induced by multipath phenomena. The least mean square (LMS) algorithm[1] as a typical algorithm employing the MSE criterion has been being widely used due to its simplicity in realization. One drawback of the LMS algorithm is that its convergence is strongly affected by large error values since it is based on the minimization of instant error power.

Unlike the MSE criterion that utilizes error power, the information-theoretic learning (ITL) method, based on a combination of a nonparametric probability density function (PDF) estimator and a procedure to compute information potential, has been introduced and well developed[2]. As a robust ITL-type algorithm, the PDF matching algorithm has been introduced by Jeong et al. and applied successfully to the classification problem with a real biomedical data set[3] and blind equalization for multipoint communication systems[4]. In [3], the authors proposed to reuse the previously acquired

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training‐phase output samples in the test phase so that the test-phase output PDF follows the training-phase output PDF. In the research [4], the authors studied the PDF matching method using signal power for blind equalization. As a tributary version of PDF matching method, an algorithm that maximizes zero-error probability (MZEP) has been introduced in the process of developing its blind version[5].

In this paper, we study the performance of supervised MZEP algorithm for multipath fading channels contaminated with strong impulsive noise, and propose a nonlinear MZEP algorithm with decision feedback to counteract both of severely distorted multi-path fading and impulsive noise as appeared in underwater channels.

Ⅱ. LMS Algorithm based on MSE Criterion

In case of FIR linear filter, a tapped delay line (TDL) with L taps can be used for input vector $X_k = [x_k, x_{k-1}, x_{k-2}, \cdots, x_{k-L+1}]^T$ and output sample $y_k = W_k^T X_k$, where W_k is the weight vector at time k. Let us define the error $e_k = d_k - y_k$ where d_k is the desired value or training symbol, then its MSE is derived as

$$MSE = E[e_k^2] \tag{1}$$

Instead of estimating the expected value of error power, we can use the instant error power e_k^2 and apply steepest descent method with step-size or convergence parameter μ_{LMS} to obtain the weight update equation[1].

$$\boldsymbol{W}_{K+1} = \boldsymbol{W}_k - \mu_{LMS}\frac{\partial e_k^2}{\partial \boldsymbol{W}} = \boldsymbol{W}_k + 2\mu_{LMS} e_k \boldsymbol{X}_k \tag{2}$$

Ⅲ. Linear MZEP Algorithm based on ITL

In this section we introduce a linear equalizer algorithm that tries to create a concentration of error samples near zero by minimizing quadratic distance $QD[f_E(e), \delta(e)]$ between the PDF of error signal $f_E(e)$ and Dirac-delta function of error $\delta(e)$, so that error PDF forms a sharp spike at zero. Rearranging the distance, we have

$$\begin{aligned} QD[f_E(e),\delta(e)] &= \int (f_E(\xi) - \delta(\xi))^2 d\xi \\ &= \int f_E^2(\xi) d\xi + \int \delta^2(\xi) d\xi - 2\int f_E(\xi)\delta(\xi) d\xi \end{aligned} \tag{3}$$

The term $\int \delta^2(\xi)d\xi$ can be treated as a constant c since it does not depend on the weights of the adaptive system. Substituting IP_e for $\int f_E^2(\xi)d\xi$ in (3), where IP_e is defined as information potential in [2], we have

$$QD[f_E(e),\delta(e)] = IP_e + c - 2f_E(0) \tag{4}$$

Minimization of $QD[f_E(e), \delta(e)]$ in (4) induces minimization of IP_e and maximization of $f_E(0)$, simultaneously since they have opposite signs. Noticing that minimization of IP_e indicates maximization of error entropy, having error samples spread, we see that this is in discord with MEE criterion that maximizes IP_e in [6].

To avoid this conflict, the work [5] proposed to maximize only the third term $f_E(0)$ while omitting the error information potential IP_e from (4) as well as the constant term. From this process, maximization of zero-error probability criterion has been obtained as:

$$\max_W f_E(0) \tag{5}$$

By way of Parzen window method with Gaussian kernel[2], the zero-error probability $f_E(0)$ becomes

$$f_E(0) = \frac{1}{N}\sum_{i=k-N+1}^{k} G_\sigma(-e_i) \tag{6}$$

where $G_\sigma(\cdot)$ is Gaussian kernel with kernel size σ, and N is the number of error sample points.

For the maximization of the cost function (5), a gradient ascent method can be employed.

$$\boldsymbol{W}_{new} = \boldsymbol{W}_{old} + \mu_{MZEP}\frac{\partial f_E(0)}{\partial \boldsymbol{W}} \tag{7}$$

With step-size μ_{MZEP} and the gradient evaluated from

$$\frac{\partial f_E(0)}{\partial \boldsymbol{W}} = \frac{1}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i) \cdot \frac{\partial y_i}{\partial \boldsymbol{W}} \tag{8}$$

MZEP algorithm can be expressed as

$$\boldsymbol{W}_{k+1} = \boldsymbol{W}_k + \frac{\mu_{MZEP}}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i) \cdot \boldsymbol{X}_i \tag{9}$$

Ⅳ. MZEP Algorithm with Decision Feedback

The decision feedback equalizer (DFE) comprises a feed-forward filter with weight vector W_k^F and a feedback filter with weight vector W_k^B for producing corresponding decisions $\hat{d}_k$ from input x_k. The feed-forward filter is identical to the TDL which is adopted in LMS in (2) and MZEP algorithm in (9). The feedback filter receives the sequence of decisions on previously detected symbols.

The feedback filter is used to remove the part of ISI from the present estimate which is caused by previously detected symbols[7]. That is, if the values of the symbols already detected are known and past decisions are assumed to be correct, then the ISI contributed by these symbols can be canceled exactly by subtracting past symbol values with appropriate weighting from the equalizer output. It is noticeable that since the output of the feedback filter section is a weighted sum of noise-free past decisions, the feedback weights play no part in determining the noise power at the equalizer output. The ability of the feedback filter section to cancel the remaining ISI, because of a number of past samples, allows more freedom in the choice of the weights of the forward filter section. One of main drawback of DFE is that incorrect decisions can cause error propagation. Fortunately it is not catastrophic and AWGN related errors in common communication channels degrade performance only slightly. In multipath fading channels contaminated with strong impulsive noise, large errors make weight adjustment unstable and error propagation is not negligible. So decision feedback equalizers require robust DFE algorithms immune to impulsive noise.

The output of DFE with P weights in feed-forward filter section and Q weights in feedback filter section can be expressed as

$$y_k = \sum_{p=0}^{P-1} w_{k,p}^F x_{k-p} + \sum_{q=0}^{Q-1} w_{k,q}^B \hat{d}_{k-q-1} \tag{10}$$

where $\hat{d}_k$ is an estimate of the desired value at time k.

Elements of feed-forward weight vector W_k^F are $\{w_{k,0}^F, w_{k,1}^F, w_{k,2}^F, \ldots, w_{k,P-1}^F\}$ and elements of feed-back-ward weight vector W_k^B are $\{w_{k,0}^B, w_{k,1}^B, w_{k,2}^B, \ldots, w_{k,Q-1}^B\}$. The elements $\{\hat{d}_{k-1}, \hat{d}_{k-2}, \ldots, \hat{d}_{k-Q-2}\}$ of vector $\hat{D}_{K-1}$ are previously detected symbols. In an adaptive mode, the filter weights are adjusted recursively in order to maximize zero-error probability according to the gradient descent method. $f_E(0)$

$$\boldsymbol{W}_{new}^F = \boldsymbol{W}_{old}^F + \mu_{MZEP-DF}\frac{\partial f_E(0)}{\partial \boldsymbol{W}^F} \tag{11}$$

$$\boldsymbol{W}_{new}^B = \boldsymbol{W}_{old}^B + \mu_{MZEP-DF}\frac{\partial f_E(0)}{\partial \boldsymbol{W}^B} \tag{12}$$

The gradients are evaluated from

$$\begin{aligned}\frac{\partial f_E(0)}{\partial \boldsymbol{W}^F} &= \frac{1}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \frac{\partial y_i}{\partial \boldsymbol{W}^F} \\ &= \frac{1}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \boldsymbol{X}_i\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial f_E(0)}{\partial \boldsymbol{W}^B} &= \frac{1}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \frac{\partial y_i}{\partial \boldsymbol{W}^B} \\ &= \frac{1}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \hat{\boldsymbol{D}}_{i-1}\end{aligned} \tag{14}$$

where $N \geq P$ and $N \geq Q$.

Now MZEP algorithm for DFE (MZEP-DF) can be summarized as

$$\boldsymbol{W}_{k+1}^{F} = \boldsymbol{W}_{k}^{F} + \frac{\mu_{MZEP-DF}}{\sigma^2 N} \sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \boldsymbol{X}_i \qquad (15)$$

$$\boldsymbol{W}_{k+1}^{B} = \boldsymbol{W}_{k}^{B} + \frac{\mu_{MZEPDF}}{\sigma^2 N} \sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \hat{\boldsymbol{D}}_{i-1} \qquad (16)$$

In the expression of weight elements, we have

$$w_{k+1,p}^{F} = w_{k,p}^{F} + \frac{\mu_{MZEP-DF}}{\sigma^2 N} \sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot x_{i-p} \qquad (17)$$

$$w_{k+1,q}^{B} = w_{k,q}^{B} + \frac{\mu_{MZEP-DF}}{\sigma^2 N} \sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \hat{d}_{i-1-q} \qquad (18)$$

Though it is not proved theoretically in this paper that the MZEP-DF results in a mathematically tractable optimization of the equalizer weights, we apply the MZEP criterion to decision feedback equalization through simulation for performance and investigate possibilities or capabilities of the proposed algorithm.

V. Results and Discussion

In this section we present and discuss simulation results that illustrate the comparative performance of the proposed MZEP-DF algorithm versus LMS and LMS-DF in the environment of multipath channel with impulsive noise. Both cases are studied for the channel models in [7]. The transfer functions of each channel models are

$$H_1(z) = 0.304 + 0.903z^{-1} + 0.304z^{-2} \qquad (19)$$

$$H_2(z) = 0.407 + 0.815z^{-1} + 407z^{-2} \qquad (20)$$

These channel models are typical multipath channel models and result in severe inter-symbol interference. Especially the channel model $H_2(z)$ poses worst spectral nulls in spectral characteristics. Then the channel output signal is added with a zero-mean white impulsive noise generated according to the following noise PDF expression.

$$f_{NOISE}(n) = \frac{1-\varepsilon}{\sigma_1\sqrt{2\pi}} \exp[\frac{-n^2}{2\sigma_1^2}] + \frac{\varepsilon}{\sigma_2\sqrt{2\pi}} \exp[\frac{-n^2}{2\sigma_2^2}] \qquad (21)$$

where $\varepsilon = 0.03$, $\sigma_1^2 = 0.001$, and $\sigma_2^2 = 50.001$. The value σ_1^2 indicates the variance of background AWGN and σ_2^2 - σ_1^2 is the variance of impulse noise only. This noise model is widely used as an impulsive noise model[8][9].

The number of weights in the linear algorithms is set to 11. For DFE algorithms, the numbers of feed-forward and feedback filter weights are $P = 7$ and $Q = 4$, respectively. As measures of equalizer performance, we use MSE convergence, probability densities for errors.

The 4 PAM random symbol {-3, -1, 1, 3} is transmitted to the channel. The step-sizes which control convergence speed are all $\mu_{MZEP} = \mu_{MZEP-DF} =$ 0.04, and $\mu_{LMS} = \mu_{LMS-DF} = 0.0002$ for both channel models. All these step-sizes and kernel sizes were selected to have the lowest minimum MSE values. We use a common data-block size $N = 20$ and the kernel size $\sigma = 0.7$.

We see the MSE convergence performance, error-PDF for $H_1(z)$ in Fig. 1 and 2, respectively. The MSE performance in Fig. 1 shows that in impulsive noise environments LMS algorithm does not reach acceptable steady state MSE regardless of DFE. The MZEP and MZEP-DF, however, show very raid convergence and also reach about -23 dB and -25 dB of steady state MSE, respectively. Considering that the variance of impulse noise is 50 and that of AWGN is 0.001, the two algorithms proves to have powerful immunity to impulsive noise.

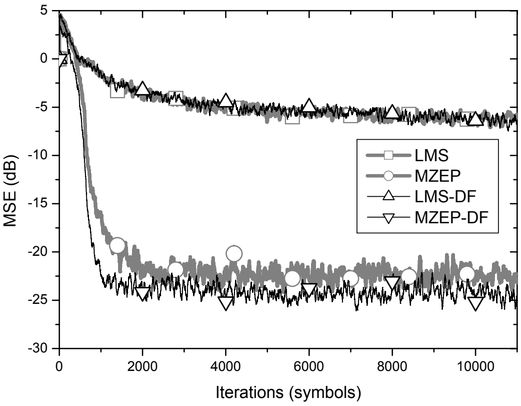

Fig. 1. MSE convergence performance for $H_1(z)$

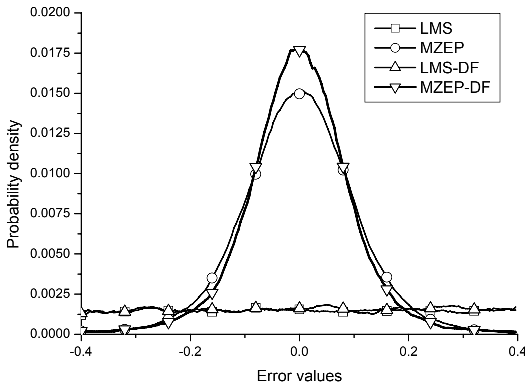

Fig. 2. Probability density for errors in $H_1(z)$

From the error PDF estimates in Fig. 2, we see their performance difference more apparently. The error distribution of MZEP-DF is shown to be the most concentrated around zero. It may be viewed that the performance difference between MZEP with DF and MZEP without DF is slight. However, in the severer channel model, $H_2(z)$ with impulsive noise, we can observe more prominently the performance improvement caused by employing DF for the compensation of residual ISI in Fig. 3. LMS and LMS-DF show severe performance degradation remaining above -3dB of steady state MSE, and linear MZEP stays around -5 dB. On the other hand, the steady-state MSE performance of MZEP-DF reaches around – 15 dB. The proposed strategy of employing DF in MZEP algorithm yields above 10 dB of performance enhancement. Fig. 4 depicts error probability performance of algorithms for $H_2(z)$

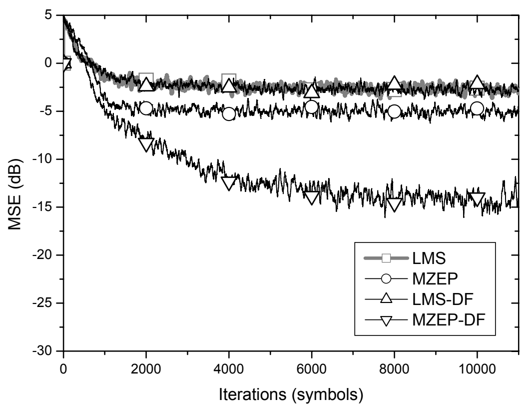

Fig. 3. MSE convergence performance for $H_2(z)$

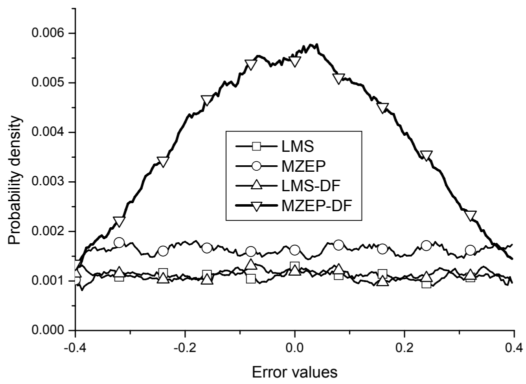

Fig. 4. Probability density for errors for $H_2(z)$.

and their performance differences are shown more clearly. The error values of LMS, LMS-DF, and even linear MZEP appear not to be gathered around zero, but MZEP-DF produces error distribution still concentrated around zero.

Ⅵ. Conclusion

In this paper, a nonlinear MZEP algorithm with decision feedback is proposed to counteract both of severely distorted multi-path fading effect and strong impulsive noise.

The conventional MSE-based algorithms like LMS-type algorithms produce enhanced ISI-cancelling performance when equipped with DF, but have no ability to cope with impulsive noise and the incorrect decisions of the algorithms induced by impulsive noise can cause severe error propagation. So DFE algorithms immune to impulsive noise are in great demand. The proposed MZEP-DF algorithm has shown the immunity to impulsive noise and the ability of the feedback filter section to cancel the remaining ISI as well. It yields above 10 dB enhancement of steady state MSE performance in severely distorted multipath fading channels with impulse noise when compared to the linear MZEP algorithm. It may be concluded that the proposed MZEP-DF can be a successful candidate for supervised equalization in impulsive noise and severe channel fading environments.

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