논문 12-37A-12-04

한국통신학회논문지 '12-12 Vol.37A No.12
http://dx.doi.org/10.7840/kics.2012.37A.12.1038

편이 확률밀도함수 사이의 거리측정 기준과 비 가우시안 잡음 환경을 위한 등화 알고리듬

김 남 용◆

Distance Measure for Biased Probability Density Functions and Related Equalizer Algorithms for Non-Gaussian Noise

Namyong Kim◆

요 약

이 논문에서는 편이된 확률밀도함수 간 거리 측정이라는 새로운 거리 측정 기준을 제안하고 이에 관련된 등화 알고리듬을 도출하여 충격성 잡음과 시변 직류 잡음이 있는 다경로 채널에 적용하였다. 이러한 비 가우시안 잡음 환경에서 시행한 시뮬레이션의 결과로부터, 제안한 알고리듬이 충격성 잡음에 강인성을 보일 뿐 아니라 시변 직류 잡음도 제거하는 탁월한 능력을 가짐을 입증하였다.

Key Words : Distance measure, Biased, Probability density function, DC bias, impulsive noise, Supervised equalization

ABSTRACT

In this paper, a new distance measure for biased PDFs is proposed and a related equalizer algorithm is also derived for supervised adaptive equalization for multipath channels with impulsive and time- varying DC bias noise. From the simulation results in the non-Gaussian noise environments, the proposed algorithm has proven not only robust to impulsive noise but also to have the capability of cancelling time-varying DC bias noise effectively.

Ⅰ. Introduction

Non-Gaussian noise such as impulsive noise and direct current noise (DC bias noise) are often present in many types of communication environments such as power line communication systems, digital subscriber line systems, mobile radio systems and also satellite communication links[1-4]. In the case of optical fiber transmission where DC bias noise is commonly present, the signal recovery is done by the received optical power whose current is proportional to the desired signal plus a large constant[5,6].

A constant DC bias noise can be removed by simple analogue or digital filters, but when the bias noise changes with time, specific adaptive techniques are required to cancel the noise. When zero-mean noise and DC bias noise are mixed, the DC bias noise makes the probability distribution of the zero mean noise shifted by the amount of the DC bias.

Recently, probability density functions (PDFs) are utilized extensively in the information theoretic learning methods (ITL)[7]. The ITL methods es-

◆ 주저자 : 강원대학교 전자정보통신공학부, namyong@kangwon.ac.kr, 종신회원
논문번호 : KICS2012-09-440, 접수일자 : 2012년 9월 16일, 최종논문접수일자 : 2012년 11월 27일

tablished based on the well known Parzen's windowing method (Kernel density estimation)[8] have not dealt with biased distributions. As one of the ITL measures, the Euclidian distance between two PDFs has been one of the main approaches in ITL[9]. Based on the Euclidian distance measure, the PDF matching algorithm using the PDF of desired symbols and the PDF of equalizer output has recently been introduced and proved to have superior performance for adaptive equalization applications[10]. Minimization of the cost function of the algorithm forces the shape of output PDF to follow the shape of the desired PDF. In non-Gaussian noise environments yielding dislocated output PDFs, algorithms based on the simple PDF distance measure cannot cope with the non-Gaussian noise problems.

In this paper, aiming at exploiting the usefulness of the bias of samples in distributions, we propose a new distance measure between biased PDFs. And then, by minimizing the proposed distance, we introduce some algorithms for adaptive equalization in non-Gaussian noise environments.

This paper is organized as follows. In Section II, we introduce the definition of biased PDF distance and propose a new cost function. In Section III, a bias control method is described. Section IV reports simulation results and discussions. Finally, concluding remarks are presented in Section V.

Ⅱ. Biased PDF Distance and Proposed Cost Function

Defining the desired PDF of an adaptive system as $f_D(\alpha)$ and the output PDF biased by the amount of τ on the α axis as $f_Y(\alpha+\tau)$,we propose the biased PDF distance (BPD) as

$$BPD = \int (f_D(\alpha) - f_Y(\alpha+\tau))^2 d\alpha \qquad (1)$$

Given M training symbols $D_M = \{d_1, d_2, ..., d_M\}$, the PDF based on Parzen window method [7] can be approximated by

$$f_D(\alpha) \cong \frac{1}{M}\sum_{i=1}^{M} G_\sigma(\alpha - d_i) \qquad (2)$$

With a block of N output samples $Y_N = \{y_1, y_2, ..., y_N\}$, the PDF forced to be shifted by τ can be expressed as

$$f_D(\alpha+\tau) \cong \frac{1}{N}\sum_{i=1}^{N} G_\sigma(\alpha + \tau - y_i) \qquad (3)$$

where $G_\sigma(\cdot)$ is typically a zero-mean Guassian kernel with standard deviation σ . Since the integrals of the multiplication of two PDFs comes from (1) can be rewritten as

$$\int f_D^2(\alpha)d\alpha = \frac{1}{M^2}\sum_{i=1}^{M}\sum_{j=1}^{M} G_{\sigma\sqrt{2}}(d_j - d_i) \qquad (4)$$

$$\int f_Y^2(\alpha+\tau)d\alpha = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} G_{\sigma\sqrt{2}}(y_j - y_i) \qquad (5)$$

$$\int f_D(\alpha) f_Y(\alpha+\tau)d\alpha = \frac{1}{MN}\sum_{i=1}^{N}\sum_{j=1}^{M} G_{\sigma\sqrt{2}}(d_j - y_i + \tau) \qquad (6)$$

Discarding (4) which is not a function of weight, we obtain the cost function to be minimized as

$$Cost = \frac{1}{MN}\sum_{i=1}^{N}\sum_{j=1}^{M}[G_{\sigma\sqrt{2}}(y_j - y_i) - 2G_{\sigma\sqrt{2}}(d_j - (y_i - \tau))] \qquad (7)$$

where y_i is a function $g(\cdot)$ of weight $\mathbf{W}^T$ and input $\mathbf{X}_i$ as $y_i = g(\mathbf{W}^T, \mathbf{X}_i)$, where the system $g(\mathbf{W}^T, \mathbf{X}_i)$ could be any type of filter structures.

Ⅲ. Bias Control based on System Expansion and the Proposed Cost Function

Employing the tapped delay line (TDL) structure with L taps, the output of the adaptive system $y_i = g(\mathbf{W}^T, \mathbf{X}_i)$ at the symbol time

k becomes $y_k = \mathbf{W}^T\mathbf{X}_k$,in which the system weight is $\mathbf{W} = [w_0, w_1, ..., w_{L-1}]^T$ and $\mathbf{X}_k = [x_k, x_{k-1}, ..., x_{k-L+1}]^T$ is the system input.

By expanding the size of $\mathbf{W}$ and $\mathbf{X}_k$,we can introduce an expanded system that consists of an expanded weight vector $\breve{\mathbf{W}} = [w_0, w_1, w_2, ..., w_L]^T$ and an expanded input vector $\breve{\mathbf{X}}_k = [x_k, x_{k-1}, ..., x_{k-L+1}, \quad b]^T$ with a constant b as

$$\breve{y}_k = \breve{\mathbf{W}}^T \breve{\mathbf{X}}_k \tag{8}$$

Then the output of the expanded system becomes $\breve{y}_k = y_k - \tau$ and the bias term τ can be expressed as

$$\tau = y_k - \breve{y}_k = \mathbf{W}^T\mathbf{X}_k - \breve{\mathbf{W}}^T \breve{\mathbf{X}}_k \qquad = -w_L \cdot b \tag{9}$$

This indicates that the bias τ can be controlled by the added weight element w_L .

The modified cost function from (7) for the expanded system is

$$Cost(\breve{\mathbf{W}}) = \frac{1}{MN}\sum_{i=1}^{N}\sum_{j=1}^{M}[G_{\sigma\sqrt{2}}(\breve{y}_j - \breve{y}_i) - 2G_{\sigma\sqrt{2}}(d_j - \breve{y}_i)] \tag{10}$$

For minimization of the cost function with respect to $\breve{\mathbf{W}}$, the gradient $\partial Cost(\breve{\mathbf{W}})/\partial\breve{\mathbf{W}}$ is calculated from

$$\frac{\partial Cost(\breve{\mathbf{W}})}{\partial \mathbf{W}} = \frac{1}{MN}\sum_{i=k-N+1}^{k}\sum_{j=k-M+1}^{k}[\frac{\partial G_{\sigma\sqrt{2}}(\breve{y}_j - \breve{y}_i)}{\partial(\breve{y}_j - \breve{y}_i)}\frac{\partial(\breve{y}_j - \breve{y}_i)}{\partial\breve{\mathbf{W}}}$$
$$-2\frac{\partial G_{\sigma\sqrt{2}}(d_j - \breve{y}_i)}{\partial(d_j - \breve{y}_i)}\frac{\partial(d_j - \breve{y}_i)}{\partial\breve{\mathbf{W}}}]$$
$$= \frac{1}{MN}\sum_{i=k-N+1}^{k}\sum_{j=k-M+1}^{k}[\frac{G_{\sigma\sqrt{2}}(\breve{y}_j - \breve{y}_i)}{-2\sigma^2}(\breve{y}_j - \breve{y}_i)(\breve{\mathbf{X}}_j - \breve{\mathbf{X}}_i)$$
$$-2\frac{G_{\sigma\sqrt{2}}(d_j - \breve{y}_i)}{-2\sigma^2}(d_j - \breve{y}_i)(-\breve{\mathbf{X}}_i)] \tag{11}$$

Using the steepest descent method to update the augmented system weights for the maximization of $Cost(\breve{\mathbf{W}})$,the expanded system weight vector with the time index k , $\breve{\mathbf{W}}_k$ can be updated as

$$\breve{\mathbf{W}}_{k+1} = \breve{\mathbf{W}}_k + \mu\frac{1}{MN}\sum_{i=k-N+1}^{k}\sum_{j=k-M+1}^{k}$$
$$[\frac{G_{\sigma\sqrt{2}}(\breve{y}_j - \breve{y}_i)}{-2\sigma^2}(\breve{y}_j - \breve{y}_i)(\breve{\mathbf{X}}_j - \breve{\mathbf{X}}_i) - \frac{G_{\sigma\sqrt{2}}(d_j - \breve{y}_i)}{\sigma^2}(d_j - \breve{y}_i)\breve{\mathbf{X}}_i] \tag{12}$$

For convenience's sake, this proposed algorithm (12) for supervised systems will be referred to in this paper as BPD algorithm (BPDA). In (12), the output $\breve{y}_k = y_k - \tau$ contains the bias term τ and the distance between the desired symbol and its output is minimized as the weight is updated according to algorithm (12). This implies both ISI and bias noise can be cancelled simultaneously.

Ⅳ. Simulation Results in Non-Gaussian Noise Environments and Discussion

The non-Gaussian noise model in this paper is composed of the background Gaussian noise, the impulse noise and additional DC bias noise. The background noise is additive white Gaussian noise (AWGN) of which variance is σ_{GN}^2 . The impulse noise occurs according to a Poisson process and the average number of impulses per information symbol duration is defined as ε . The amplitude distribution of impulse noise has a Gaussian with variance σ_{IN}^2 . Then the PDF expression of the impulsive noise n_{Im} (background Gaussian noise + impulse noise) is

$$f_{IM}(n_{\mathrm{Im}}) = \frac{1-\varepsilon}{\sigma_{GN}\sqrt{2\pi}}\exp[\frac{-n_{\mathrm{Im}}^2}{2\sigma_{GN}^2}] + \frac{\varepsilon}{\sqrt{2\pi(\sigma_{GN}^2+\sigma_{IN}^2)}}\exp[\frac{-n_{\mathrm{Im}}^2}{2(\sigma_{GN}^2+\sigma_{IN}^2)}]$$

as in [10] and [11]. With DC bias noise added to the channel, the total noise at time k can be expressed as $n_k = n_{DC,k} + n_{\mathrm{Im},k}$ in which $n_{DC,k}$ is time-varying DC bias noise.

In this section the performance of the PDF matching algorithm (PMA) in [10] and the proposed BPDA is experimented for fading channels contaminated with DC bias and impulsive noise. The performance is investigated in 4 PAM with $d_k = \{\pm 3, \pm 1\}$ and all values are equi-probable. For fair comparison of performance, the channel model and impulsive noise are the same ones used in [10], except that the non-Gaussian noise n_k for this simulation is composed of the impulsive noise $n_{\mathrm{Im},k}$ and DC bias noise $n_{DC,k}$. The DC bias noise is generated as $n_{DC,k} = -\sin(2\pi f_o k)$ & $f_o = 0.0005$.

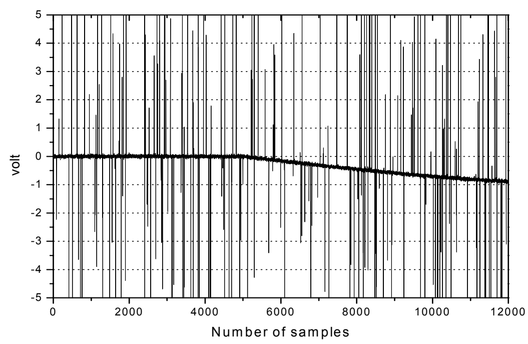

Fig. 1. Impulsive noise with time-varying DC bias.

The noise parameters are chosen to be observable as depicted in Fig.1. For the assessment of the potential usefulness of the proposed algorithm, the DC bias noise is added from the mean time at 5000 symbol time after all algorithms have converged.

In fig. 2 and 3, MSE performance and error distribution are shown, respectively. The error distribution reveals how frequently the error occurs, where the error is defined as $e_k = d_k - \tilde{y}_k$. The equalizer structure is a tapped delay line with 11 weights and the output signal is $y_k = \mathbf{W}^T \mathbf{X}_k$,but the output of the proposed BPDA is $\tilde{y}_k = \tilde{\mathbf{W}}^T \tilde{\mathbf{X}}_k$. The constant b in the expanded input vector $\tilde{\mathbf{X}}_k$ is set to 2. Data-block size $M = N = 4$. The kernel size σ and step-size for the algorithms are commonly 1.0, 0.01, respectively. All these parameters are selected in the case that the algorithms have the lowest steady state MSE values.

In Fig. 1, we can observe time varying DC bias that starts to be added from 5000 samples and many impulse spikes that frequently occur (some impulses reach over 20 volts). Both algorithms show the same convergence speed proving highly immune to impulsive noise. Though the PMA has the merits of robustness to impulsive noise, MSE of the PMA begins to increase after the time-varying DC bias is added. On the other hand, the proposed BPDA stays

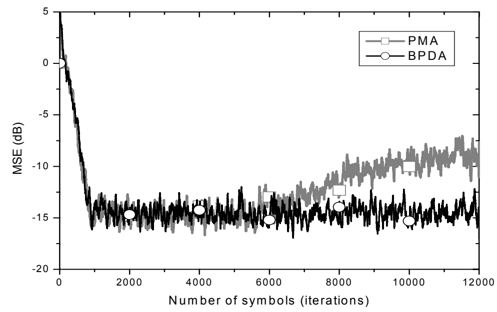

Fig. 2. MSE performance for the channel model with impulsive noise and time-varying DC bias.

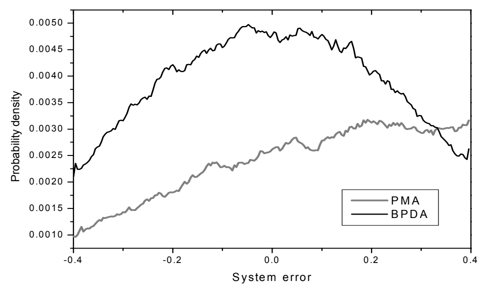

Fig. 3. Error distribution comparison in the environments with impulsive noise and time-varying DC bias.

showing the same steady state MSE.

The performance against time-varying DC bias and impulsive noise can be observed more apparently in the error distribution comparison shown in Fig. 3. The error samples of BPDA form a concentrated distribution centered around zero. However, the ones of PMA moved to right side as the time-varying DC bias noise is added. This indicates that the conventional PMA cannot compensate the bias completely yielding biased error samples centered at around 0.4.

Ⅴ. Conclusions

In this paper, the biased PDF distance measure undisturbed by the time-varying DC bias noise between two different PDFs has been proposed and a related algorithm has been derived for supervised adaptive equalization. The performance of the proposed algorithm was investigated in the environment of multipath channels with impulsive and time- varying DC bias noise.

From the simulation results, the proposed and the conventional algorithm used for performance comparison has shown to eliminate outliers coming from impulsive noise, but the conventional algorithm does not have the capability of compensating DC bias efficiently.

On the other hand, the proposed algorithm has proven not only to be robust to impulsive noise but also to have the capability of cancelling time-varying DC bias noise. So we can conclude that the proposed distance measure and related algorithms can be effectively used in adaptive systems placed in inferior environments contaminated with non-Gaussian noise such as impulsive and/or time-varying DC bias noise.

References

[1] L. M. Garth, "A dynamic convergence analysis of blind equalization algorithms," *IEEE Trans. on Commun.*, vol. 49, pp. 624-634. Apr. 2001.

[1] K. Blackard, T. Rappaport, and C. Bostian, "Measurements and models of radio frequency impulsive noise for indoor wireless communications," *IEEE J. Select. Areas Commun.*, vol. 11, pp. 991-1001, Sept. 1993.

[2] K. Koike and H. Ogiwara, "Application of Turbo TCM codes for impulsive noise channel," *IEICE Trans. Fund. Electr.*, vol. E81-A, no. 10, pp. 2032-2039, Oct. 1998.

[3] M. Button, J. Gardiner, and I. Glover, "Measurement of the impulsive noise environment for satellite-mobile radio systems at 1.5 GHz," *IEEE Trans. Veh. Technol.*. vol. 51, no. 3, pp. 551-560, May 2002.

[4] M. Richharia, *Satellite communication systems: design principles*, 2nd Ed. Palgrave Macmillan Limited, 1999.

[5] H. Sedarat, and K. Fishera, "Multicarrier communication in presence of biased-Gaussian noise sources," *J. Signal Processing*, vol. 88, issue 7, pp. 1627-1635, Jul. 2008.

[6] J. Mazo, "Asymptotic distortion spectrum of clipped, dc-biased, Gaussian noise," *IEEE Trans. Commun.*, vol. 40, no. 8, pp. 1339-1344, Aug. 1992.

[7] J. Principe, D. Xu, and J. Fisher, "Information theoretic learning," in: S. Haykin, *Unsupervised Adaptive Filtering*, Wiley, New York, pp. 265-319, 2000.

[8] E. Parzen, "On the estimation of a probability density function and the mode," *J. Ann. Math. Stat*. vol. 33, issue 3, pp. 1065-1076, 1962.

[9] K. Jeong, J. Xu, D. Erdogmus, and J. Principe, "A new classifier based on information theoretic learning with unlabeled data," *Neural Networks*, vol. 18, no. 5-6, pp. 719-726, 2005.

[10] N. Kim, "Adaptive equalization using PDF matching algorithms for underwater communication channels with impulsive noise," *J. KICS*, vol. 36, no. 10, pp. 1210-1215, Oct. 2011.

[11] P. Delaney, "Signal detection in multivariate class A interference," *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp. 365-373, Feb/Mar./Apr. 1995.

김 남 용 (Namyong Kim)

1986년 2월 연세대학교 전자공학과 졸업

1988년 2월 연세대학교 전자공학과 석사

1991년 8월 연세대학교 전자공학과 박사

1992년 8월~1998년 2월 관동대학교 전자통신공학과 부교수

1998년 3월~현 강원대학교 공학대학 전자정보통신공학부 교수

<관심분야> Adaptive equalization algorithms, ITL algorithms, Odor sensing systems.