

# Error-Correcting 7/9 Modulation Codes For Holographic Data Storage

Kyoungoh Lee<sup>\*</sup>, Byungsun Kim<sup>\*</sup>, Jaejin Lee<sup>o</sup>

Abstract

Holographic data storage (HDS) has a number of advantages, including a high transmission rate through the use of a charge coupled device array for reading two-dimensional (2D) pixel image data, and a high density capacity. HDS also has disadvantages, including 2d intersymbol interference by neighboring pixels and interpage interference by multiple pages stored in the same holographic volume. These problems can be eliminated by modulation codes. We propose a 7/9 error-correcting modulation code that exploits a Viterbi - trellis algorithm and has a code rate larger (about 0.778) than that of the conventional 6/8 balanced modulation code. We show improved performance of the bit error rate with the proposed scheme compared to that of the simple 7/9 code without the trellis scheme and the 6/8 balanced modulation code.

**Key Words** : Holographic Data Storage, Two-Dimensional Intersymbol Interference, Modulation Code

## I. Introduction

Holographic data storage (HDS) systems meet new-generation data storage system demands<sup>[1-3]</sup>. These systems have two advantages—a high density capacity and a high transmission rate because the charge coupled device (CCD) array is exploited when simultaneously reading and writing two-dimensional (2D) pixel image data. Also, by using holographic technology, many pages can be stored in the same volume. Even though HDS shows better capacity and transmission rates compared to other data storages, it has a number of problems, including 2D intersymbol interference (ISI), interpage interference (IPI), and misalignment<sup>[4-5]</sup>. ISI occurs between neighboring pixels in all directions because of the 2D image data. Misalignment occurs during the data reading and writing procedure as a result of the holographic system's physical beam errors.

To solve these problems, two-dimensional modulation codes are usually applied<sup>[5-12]</sup>. One scheme removes the isolated ON or OFF pixel patterns that are the main source of the ISI problem<sup>[5-6]</sup>, resulting in a better performance of the modulation code by removing the isolated patterns. Another scheme incorporates an error-correcting capability. Burr et al. proposed an 8:12 code to improve robustness against noise; this is considered an error-correcting code because the minimum Hamming distance of the code is greater than 3<sup>[8-11]</sup>. Kim and Lee introduced a trellis modulation structure for error-correction capability<sup>[10]</sup>. They assigned appropriate codewords to each branch by selecting codewords with a minimum Hamming distance greater than or equal to 2; the trellis modulation then increases the distance of the shortest error event, which creates the error-correction capability. As the minimum Hamming distance increases, the error-correction

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• First Author : 숭실대학교 정보통신전자공학부 정보저장 및 통신 연구실, kasinamooth@naver.com, kimbs@ssu.ac.kr, 학생회원

◦ Corresponding Author : 숭실대학교 정보통신전자공학부 정보저장 및 통신 연구실, zlee@ssu.ac.kr, 중신회원

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also increases.

The encoding procedure of trellis modulation coding starts from setting the initial state as  $S_0$ . For each input block, the encoder outputs a codeword corresponding to the input and moves to the next state. This is repeated  $m$  times. A parity symbol is then inserted for returning to the initial state  $S_0$ . This procedure is repeated to the end of the input data.

To extend the previous work, we propose a 7/9 error-correcting modulation code that has a higher code rate and error-correcting capability compared with prior modulation codes<sup>[9-11]</sup>. In Section 2, we introduce codewords of a 7/9 error-correcting modulation code for an HDS system. This modulation code has a minimum Hamming distance greater than or equal to 2. We also explain the encoding procedure of the trellis modulation scheme that has an error-correcting capability. In Section 3, we compare the simulation results to other modulation codes – a no error-correcting 7/9 modulation code, an error-correcting 6/8 modulation code, and a conventional 6/8 balanced modulation

code, which has an equal ratio of ON to OFF pixels. We present our conclusions in Section 4.

## II. 7/9 Error-Correcting Modulation Code

Figure 1 shows 128 symbols of the 7/9 error-correcting modulation code. The symbols have some patterns with a minimum Hamming distance greater than or equal to 2. This code performs better than a minimum Hamming distance of 2 since approximately 55–60% of codewords have a distance of 4. The encoding is done by a trellis modulation scheme for an error-correcting capability by using the symbols. The decoding of the modulation code is done by a soft-output Viterbi-algorithm (SOVA).

### 2.1 Codewords for the 7/9 error-correcting modulation code

The 7/9 error-correcting modulation code has 128 codewords, and each codeword is composed of a  $3 \times 3$  nine-pixel array. First, we selected 256 symbols having an odd number of ON pixel patterns and a

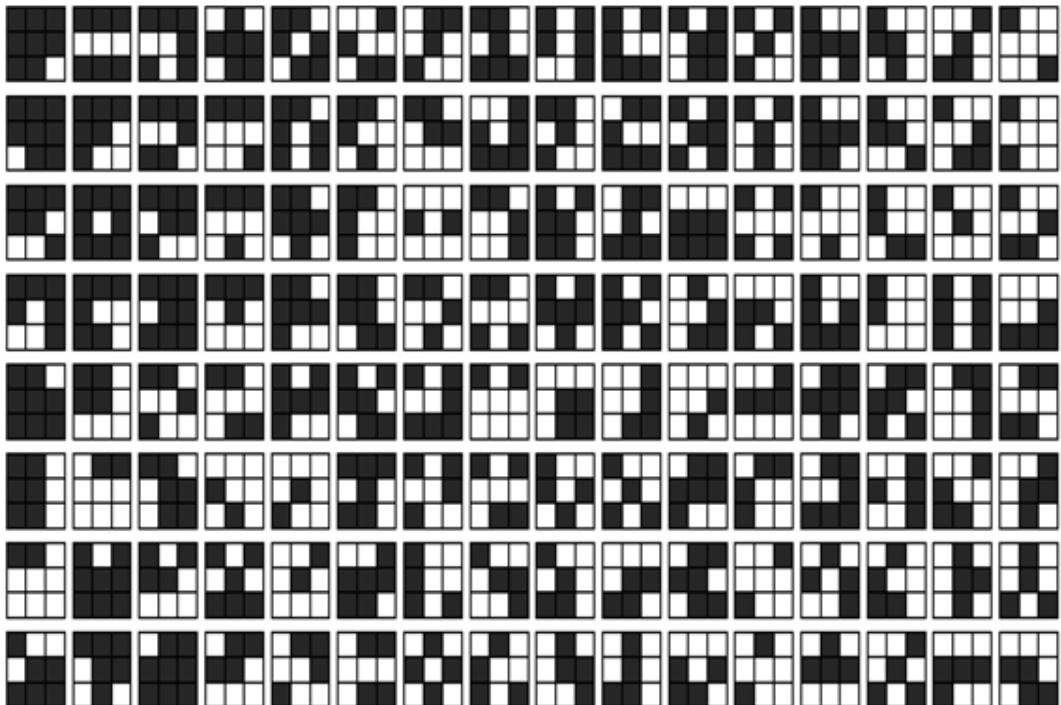


Fig. 1. List of 128 symbols

Table 1. Encoding table of modulation method (128 states).

Input	S <sub>0</sub>		S <sub>1</sub>		S <sub>2</sub>		...	S <sub>125</sub>		S <sub>126</sub>		S <sub>127</sub>	
	o.s.	n.s.	o.s.	n.s.	o.s.	n.s.		o.s.	n.s.	o.s.	n.s.	o.s.	n.s.
0000001	0	S <sub>0</sub>	1	S <sub>0</sub>	2	S <sub>0</sub>		125	S <sub>0</sub>	126	S <sub>0</sub>	127	S <sub>0</sub>
0000010	1	S <sub>1</sub>	2	S <sub>1</sub>	3	S <sub>1</sub>		126	S <sub>1</sub>	127	S <sub>1</sub>	0	S <sub>1</sub>
0000011	2	S <sub>2</sub>	3	S <sub>2</sub>	4	S <sub>2</sub>		127	S <sub>2</sub>	0	S <sub>2</sub>	1	S <sub>2</sub>
0000100	3	S <sub>3</sub>	4	S <sub>3</sub>	5	S <sub>3</sub>		0	S <sub>3</sub>	1	S <sub>3</sub>	2	S <sub>3</sub>
0000101	4	S <sub>4</sub>	5	S <sub>4</sub>	6	S <sub>4</sub>		1	S <sub>4</sub>	2	S <sub>4</sub>	3	S <sub>4</sub>
0000110	5	S <sub>5</sub>	6	S <sub>5</sub>	7	S <sub>5</sub>	...	2	S <sub>5</sub>	3	S <sub>5</sub>	4	S <sub>5</sub>
⋮			⋮							⋮			
1111101	125	S <sub>125</sub>	126	S <sub>125</sub>	127	S <sub>125</sub>		122	S <sub>125</sub>	123	S <sub>125</sub>	124	S <sub>125</sub>
1111110	126	S <sub>126</sub>	127	S <sub>126</sub>	0	S <sub>126</sub>		123	S <sub>126</sub>	124	S <sub>126</sub>	125	S <sub>126</sub>
1111111	127	S <sub>127</sub>	0	S <sub>127</sub>	1	S <sub>127</sub>		124	S <sub>127</sub>	125	S <sub>127</sub>	126	S <sub>127</sub>

o.s.: output symbol, n.s.: next state

minimum Hamming distance of 2. Second, we found a subset with a minimum distance of 4 (about 55 - 60%). Third, we added the other patterns to fit the 128 codewords. This set has a minimum distance of 2 but more than half the codewords have a distance of 4. Finally, we selected 128 symbols that have some patterns with a minimum Hamming distance greater than 2 (Fig. 1).

2.2 Encoding procedure of the error-correcting modulation code

The encoding of the no error-correcting 7/9 modulation code is a simple one-to-one mapping between input and codeword (here in referred to as No ECC 7/9 code). When the modulation code is decoded, the demodulation procedure follows the minimum Euclidean distance rule<sup>[12]</sup>. The code rate of the No ECC 7/9 code is 0.778, which is larger than the code rate of the 6/8 balanced modulation

code. The No ECC 7/9 modulation code has a Hamming distance equal to 2 or greater than or equal to 4. Meanwhile, the error-correcting 7/9 modulation coding scheme exploits the trellis modulation scheme. The trellis structure makes the shortest error event distance be greater than 3. Because the input symbol is a 7-bit input, the trellis has 128 states. Table I shows the trellis encoding procedure. This method has a terminating parity symbol, which makes the trellis go back to S0 for a coding gain. Therefore, the real code rate becomes 7m/9(m+1). The value m is the interval between the terminating parity symbols.

The encoding procedure is as follows: First, set the initial state to S0. Then, assign 128 symbols to the corresponding 7-bit input so that the encoder outputs a symbol corresponding to the input on the current state and moves to the next state. This is repeated m times. After that, the parity symbol is

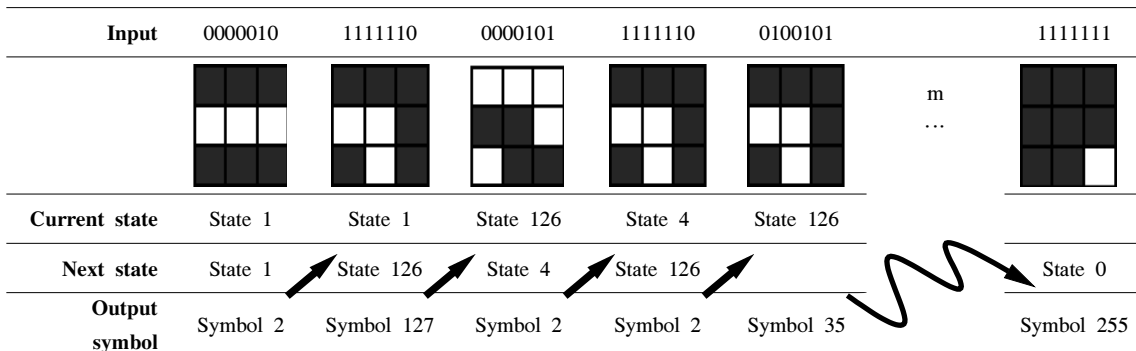


Fig. 2. Example of the encoding procedure.

inserted for going back to state S0. These steps are repeated until there is no input. Figure 2 shows an example of the encoding procedure.

### III. Simulation Results

#### 3.1 Experimental environment

We evaluated each modulation code's performances by simulation. For the HDS channel model, we used the continuous point-spread function.

$$h(x,y) = \frac{1}{\sigma^2} \text{sinc}^2\left(\frac{x}{\sigma}, \frac{y}{\sigma}\right) \quad (1)$$

where  $\sigma_b$  represents the grade of blur in the resultant diffracted signals. We can store a lot of data when the value of  $\sigma_b$  is large, but interference between adjacent symbols increases and bit error rate (BER) performance becomes worse.

We simulated 1000 data pages with a  $1024 \times 1024$  array size for each page and a blur level of 1.85. We defined the channel signal-to-noise ratio (SNR) as the ratio of signal power and the additive white Gaussian noise (AWGN) power, and the channel noise is AWGN. The detection process uses a 2D SOVA.<sup>[13-14]</sup> We chose the interval between the terminating parity symbols to be 30 ( $m=30$ ). We compared three modulation codes according to the grade of blur, SNR, misalignment level, and interval of parity symbols ( $m$ ).

#### 3.2 Simulation results

Figure 3 shows the BER performance with AWGN. No ECC 7/9 code shows a better performance than the conventional 6/8 balanced modulation code because the decoder of the No ECC 7/9 code has 9 pixels for decoding information. In addition, the No ECC 7/9 code corrects all errors at 18 dB. However, the best performance is shown by the error-correcting 7/9 modulation code. After 14 dB, the modulation code corrects all errors.

Performance depending on the grade of blur is shown in Fig. 4. In the simulation, the SNR

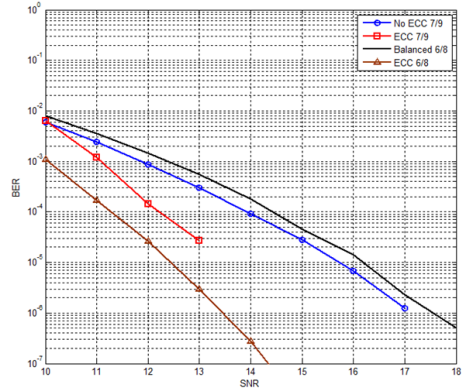


Fig. 3. Comparison of BER performance.

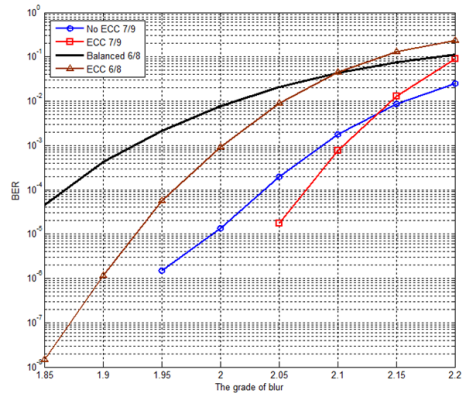


Fig. 4. BER performance in relation to blur.

condition is 15 dB. The error-correcting 7/9 modulation code showed the best performance when the blur is less than 2.1. However, as the grade of blur increases, which means worse channel conditions, the performance of the No ECC 7/9 modulation code showed better performance because under unfavorable channel conditions, the trellis scheme has an error propagation that negatively affects the performance.

Figure 5 shows the BER performance based on the horizontal and vertical misalignment level. In the simulation, the grade of blur is 1.85. No ECC 7/9 code shows better performance than the conventional 6/8 balanced modulation code. The BER performance of the error-correcting 7/9 modulation code decreased in relation to increasing misalignment because the trellis scheme is unable to function under bad channel conditions. Generally, the error-correcting 7/9 modulation (ECC 7/9) code

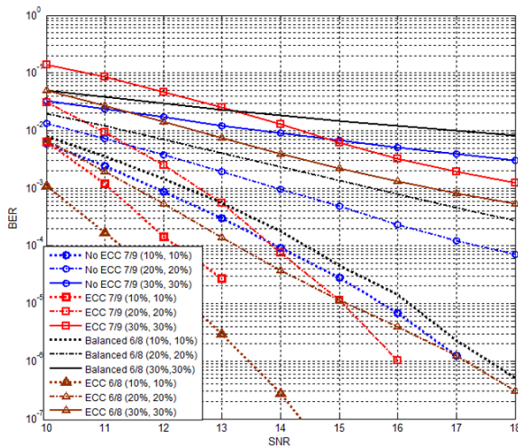


Fig. 5. BER performance with the 6/8 balanced code, 6/8 ECC modulation code, No ECC 7/9 modulation code, and 7/9 ECC modulation code in horizontal and vertical misalignments.

performs better than the 6/8 balanced and No ECC 7/9 codes. Furthermore, if the channel condition is favorable, it outperforms the error-correcting 6/8 modulation (ECC 6/8) code. When there are (10%, 10%) misalignments, ECC 7/9 has no error at 14dB while ECC 6/8 code does not. Also, when there are (20%, 20%) misalignments, the performance graphs of ECC 6/8 and ECC 7/9 are crossed at 15dB.

Figure 6 illustrates the BER performance in relation to the interval of parity symbols. Obviously, there is a trade-off between the BER and the code rate, which means that a better BER requires a lower code rate, which is a short interval of parity

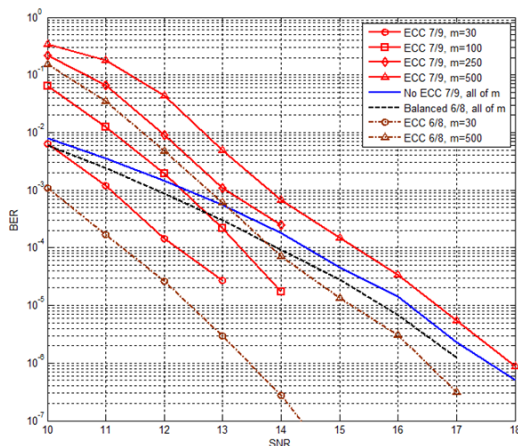


Fig. 6. BER performance with different parity interval (m).

symbols. From the simulation, ECC 7/9 code has no error at 15dB when the terminating parity symbol interval is less than or equal to  $m=250$ .

#### IV. Conclusions

In this paper, we proposed an error-correcting 7/9 modulation code for an HDS system. Our proposed modulation code increases the code rate compared to the conventional balanced 6/8 modulation code. In addition, by using a trellis structure, error-correcting capability is improved. When the channel is favorable condition, the proposed modulation code with a trellis structure shows better BER performance than the 6/8 balanced and error-correcting modulation codes.

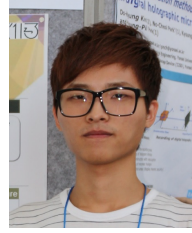
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**이 경 오 (Kyungoh Lee)**



2008년 3월 : 송실대학교 정보통신전자공학부 학사 입학  
 2014년 2월 : 송실대학교 정보통신전자공학부 학사 졸업  
 <관심분야> 스토리지 시스템

**김 병 선 (Byungsun Kim)**



2012년 2월 : 정보통신공학과 학사  
 2012년 9월~현재 : 송실대학교 정보통신전자공학부 석사 재학중  
 <관심분야> 스토리지 시스템, 오류정정부호, 채널코딩

**이 재 진 (Jaejin Lee)**



1983년 2월 : 연세대학교 전자공학과 학사  
 1984년 12월 : U. of Michigan Dept. of EECS 석사  
 1994년 12월 : Georgia Tech. Sch. of ECE 박사  
 1995년 1월~1995년 12월 : Georgia Tech. 연구원

1996년 1월~1997년 2월 : 현대전자 정보통신연구소 책임연구원  
 1997년 3월~2005년 8월 : 동국대학교 전자공학과 부교수  
 2005년 9월~현재 : 송실대학교 정보통신전자공학부 교수  
 <관심분야> 통신이론, 채널코딩, 정보저장시스템