

## 인지 무선 시스템에서 스펙트럼 센싱을 위한 이항 필터

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## Binomial Filters for Spectrum Sensing in Cognitive Radio System

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## 요약

본 논문에서는 인지 무선통신에서 스펙트럼 센싱을 위해 사용할 수 있는 세 가지 형태의 이항 필터를 제안하였다. 세 가지 필터는 이항 필터 및 부정 이항필터, 복합 이항 필터이며 이들 전달함수의 주파수 응답을 각각 분석하였으며 필터에 요구되는 감쇠를 위해 필요한 단계 수를 도출하였다. 또한 각 필터에 대해 요구되는 감쇠에 필요한 단계수를 성능으로 하여 분석한 결과 부정 이항 필터 및 복합 이항 필터가 기본 이항 필터에 비해 성능이 더 우수한 것으로 나타났다. 제안된 세 가지 필터는 통합된 직렬 구조를 가지고 있고 곱셈기가 필요 없어 구현에 용이하므로 광범위한 응용이 가능하다.

**Key Words** : Binomial, multiplier, base, frequency, cascaded, filter

## ABSTRACT

In this paper, we proposed three types of binomial filter for spectrum sensing in cognitive radio system. Three filters are binomial, negative binomial and composite binomial filters and the frequency responses of their transfer functions are analyzed and the numbers of stages to meet the required attenuation are driven. As a result of performance analysis in terms of the number of stages, negative and composite binomial filters are superior to the binomial filter. Since the proposed three filters have a unified cascaded structure and are easy to be implemented without any multiplier, it is expected that they will have wide applications.

## I. Introduction

In cognitive radio (CR) systems, the transmitter should hear the channel state to recognize if the interested channel is available or not. In order to hear the channel exactly it is essential to protect any interference from adjacent channels and from signal sources generated by its own transmitter. Therefore, filters to separate highly accurately the interested

channel from interference sources are required in CR system. Recently, wavelet multi-tone based filters are proposed to obtain high separation gain against conventional OFDM scheme<sup>[1]</sup> and a cascaded integration and comb (CIC) filter structure was proposed<sup>[2-3]</sup>, and an sensing algorithm utilizing DFT was also proposed to reduce computation for signal detection<sup>[4]</sup>. Although they are all good separation characteristics, they require high sampling

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rates more than symbol ones. A cooperative compressed sensing technique was proposed to recover signals sampled below the Nyquist rate but there are some issues on accurate recovery from compressed signals<sup>[5]</sup>.

Binomial filters form a compact rapid finite impulse response (FIR) approximation of the discretized Gaussian. An appealing implementation feature of these filters is that they do not require multiplications and thus are amenable to hardware implementation<sup>[6-9]</sup>. A key application of these filters in computer vision is in the construction of multi-scale image/volume representations<sup>[10,11]</sup>.

In this paper, we developed and analyzed binomial, negative binomial and composite binomial filters to utilize as channel separation filters. We obtained the transfer function for each filter, its frequency response, the number of cascaded stages to meet the required attenuation and its coefficients.

## II. Binomial filters for Spectrum Sensing

### 2.1 Binomial filter

The basic equation for the binomial filter is

$$y_+(n) = \frac{1}{2}(x(n) + x(n-1)). \quad (1)$$

The related circuit diagram is depicted in Fig. 1.

The circuit consists of an adder to add two value  $x(n)$  and  $x(n-1)$ , a register and a shifter which denotes as  $1/2$ . So there is no multiplier in (1). Its transfer function is given by

$$H_+(z) = \frac{Y_+(z)}{X(z)} = \frac{1}{2}(1 + z^{-1}). \quad (2)$$

The frequency response is

$$H_+(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega}). \quad (3)$$

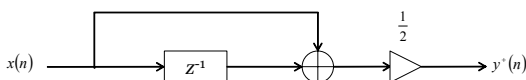


Fig. 1. Basic circuit for binomial filter

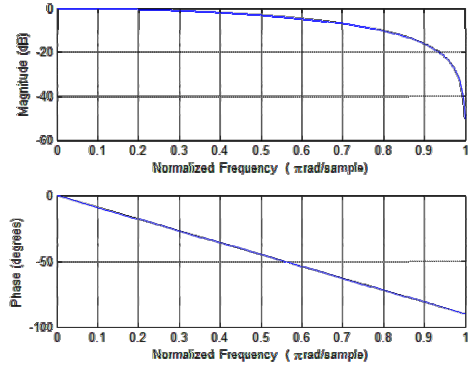


Fig. 2. Frequency response of single stage binomial filter

The magnitude and phase characteristics are as depicted in Fig. 2 as below.

As shown in Fig. 2, we know that the magnitudes of the transfer function have 0 [dB] at  $\omega = 0$  and  $-\infty$  [dB] at  $\omega = \pi$ , and the phase of that has linear characteristic. Therefore this filter does not have any group delay. We can have a cascaded form by placing the basic circuit in serial as

$$H_+^N(z) = \frac{1}{2^N}(1 + z^{-1})^N = \frac{1}{2^N} \sum_{k=0}^N \binom{N}{k} z^{-k}. \quad (4)$$

As shown in (4), the cascaded filter has coefficients in binomial form. The frequency response is given by

$$H_+^N(e^{j\omega}) = \left( e^{-\frac{j\omega}{2}} \cos \frac{\omega}{2} \right)^N = e^{-\frac{jN\omega}{2}} \cos^N \frac{\omega}{2}. \quad (5)$$

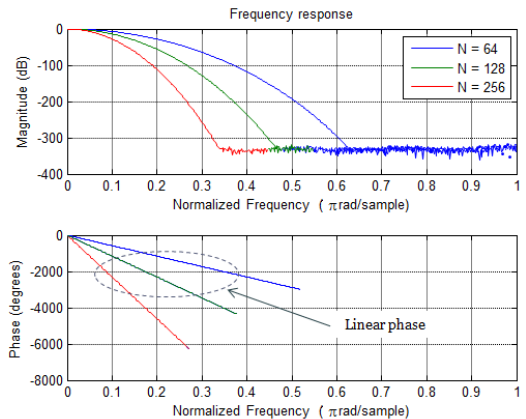


Fig. 3. Frequency response of binomial filters

As shown in Fig. 3, we know that the more number of stages the narrower the bandwidth of the filter and the linearity of filters are maintained irrespective of the number of stages.

Taking the absolute value, taking the log of (5) and letting the term a value  $x$  that is a required attenuation, we have

$$10N \log \left( \cos \frac{\omega}{2} \right) = x \text{ [dB]}. \tag{6}$$

The needed number of stages is

$$N = \left\lceil \frac{x}{10 \log \left( \cos \frac{\omega}{2} \right)} \right\rceil. \tag{7}$$

The table 1 and Fig. 4 show the number of stages to achieve the required attenuation.

Table 1. Number of stages of binomial filter

Normalized Frequency ( $\times \pi$ )	Attenuation[dB]				
	-100	-80	-60	-40	-20
0.02	46653	37322	27992	18661	9331
0.04	11658	9326	6995	4663	2332
0.06	5177	4142	3107	2071	1036
0.08	2909	2327	1746	1164	582
0.1	1859	1487	1116	744	372

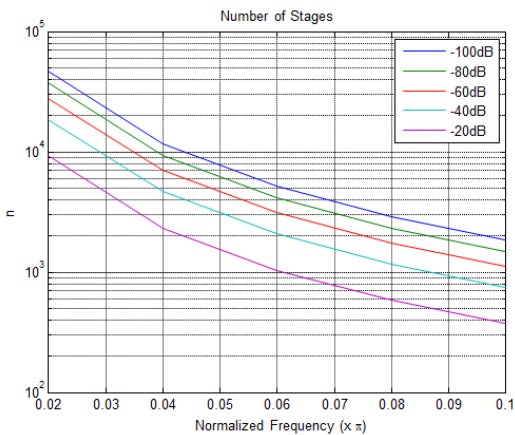


Fig. 4 The number of stages versus the needed attenuations for binomial filters

## 2.2 Negative Binomial filter

The basic equation for the negative binomial filter is

$$y_-(n) = \frac{1}{2} (y_-(n-1) + x(n)). \tag{8}$$

The related circuit diagram is depicted in Fig.5

The circuit consists of an adder to add two value  $x(n)$  and  $x(n-1)$ , a register and a shifter which denotes as  $1/2$ . So there is no multiplier in (8). The transfer function is given by

$$H_-(z) = \frac{1}{2} \left( 1 - \frac{1}{2} z^{-1} \right)^{-1}. \tag{9}$$

The frequency response is

$$H_-(e^{j\omega}) = \frac{1}{2} \left( \frac{1}{1 - \frac{1}{2} e^{-j\omega}} \right). \tag{10}$$

The magnitude and phase characteristics are depicted in Fig. 6.

As shown in Fig. 6, we know that the magnitudes of the transfer function have 0 [dB] at  $\omega = 0$  and -

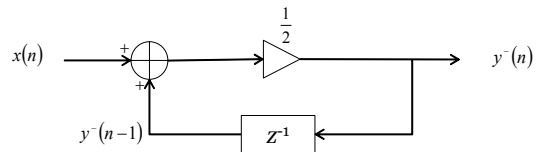


Fig. 5. Basic circuit for negative binomial filter

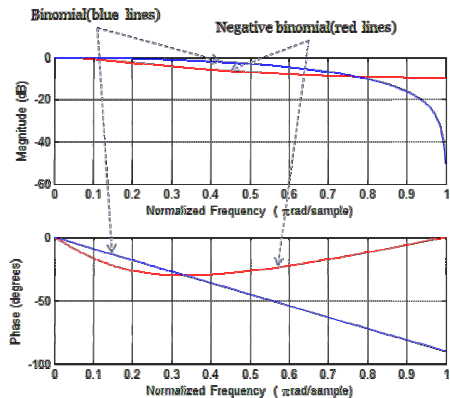


Fig. 6. Frequency response of single stage negative binomial filter

infinity [dB] at  $\omega = \pi$ , and the phase of that has nonlinear characteristic. Therefore there may be some group delay in (10). We can have a cascaded form by placing the basic circuit in serial as

$$H_-^N(z) = \frac{1}{2^N} \left(1 - \frac{1}{2} z^{-1}\right)^{-N} = \frac{1}{2^N} \sum_{k=0}^{\infty} \binom{-N}{k} \left(-\frac{1}{2} z^{-1}\right)^k$$

$$= \frac{1}{2^N} \sum_{k=0}^{\infty} \frac{1}{2^k} \binom{N+k-1}{k} z^{-k}. \quad (11)$$

As shown in (11), the cascaded filter has coefficients in negative binomial form. The frequency response is given by

$$H_-^N(e^{j\omega}) = 2^{-N} \left(1 - \frac{1}{2} \cos \omega + j \frac{1}{2} \sin \omega\right)^{-N}. \quad (12)$$

As shown in Fig. 7, we know that the more the number of stages the narrower the bandwidth of the filter and the linearity of filters are improved as the number of stages increments. The improvement of linearity is caused by approaching this filter to Gaussian filter as the number of stages increments.

Taking the absolute, taking the log for (12) and letting the term a value  $x$  that is a required attenuation, we have

$$10N \log \left( \frac{1}{\sqrt{(2 - \cos \omega)^2 + (\sin \omega)^2}} \right) = x. \quad (13)$$

The needed number of stages is

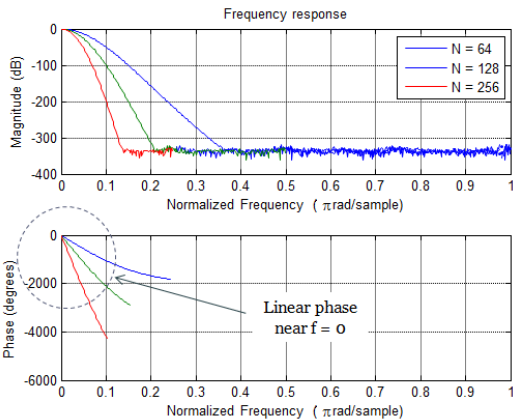


Fig. 7. Frequency response of negative binomial filters

$$\therefore N = \left\lceil \frac{x}{10 \log \left( \frac{1}{\sqrt{(2 - \cos \omega)^2 + (\sin \omega)^2}} \right)} \right\rceil. \quad (14)$$

The table 2 and Fig. 8 show the number of stages to achieve the required attenuation.

Table 2. Number of stages of negative binomial filter

Normalized Frequency (* π)	Attenuation[dB]				
	-100	-80	-60	-40	-20
0.02	5858	4686	3515	2343	1172
0.04	1483	1187	890	594	297
0.06	673	539	404	270	135
0.08	390	312	234	156	78
0.1	258	207	155	104	52

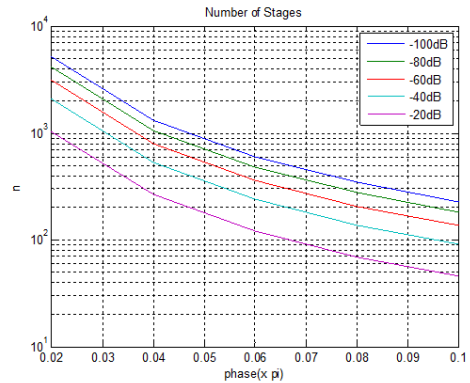


Fig. 8 The number of stages versus the needed attenuations for negative binomial filters

### 2.3 Composite binomial filter

The basic equations for the composite binomial filter are

$$p(n) = \frac{1}{2} (x(n) + q(n)),$$

$$q(n) = p(n-1), \quad (15)$$

$$y_c(n) = \frac{1}{2} (p(n) + q(n)).$$

The related circuit diagram is depicted in Fig. 9. The circuit consists of one adder to add two value  $x(n)$  and  $q(n)$ , another adder to sum two value  $p(n)$

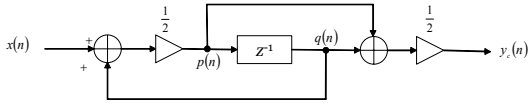


Fig. 9. Basic circuit for composite binomial filter

and  $q(n)$ , a register and two shifters which denotes as  $1/2$ . So there is no multiplier in (15).

The transfer function is given by

$$H_c(z) = H_+(z)H_-(z) = \frac{1}{2}(1+z^{-1}) \cdot \frac{1}{2}\left(1-\frac{1}{2}z^{-1}\right)^{-1} \quad (16)$$

The frequency response is

$$H_c(e^{j\omega}) = \frac{1}{4} \left( \frac{1+e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}} \right) \quad (17)$$

The magnitude and phase characteristics are depicted in Fig. 10. As shown in Fig. 10, we know that the magnitudes of the transfer function have 0 [dB] at  $\omega = 0$  and  $-\infty$  [dB] at  $\omega = \pi$ , and the phase of that has nonlinear characteristic. Therefore there may be some group delay in (17). We can have a cascaded form by placing it in serial as

$$H_c^N(z) = \frac{1}{4^N} \left( \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}} \right)^N = \frac{1}{2^{2N}} \sum_{k=0}^{\infty} \sum_{s=0}^k \frac{1}{2^s} \binom{N+s-1}{s} \binom{N}{k-s} z^{-k} \quad (18)$$

$k = 0, 1, 2, \dots$

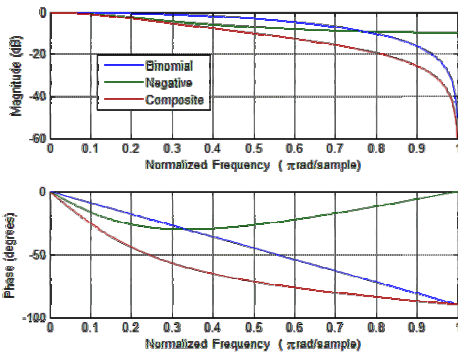


Fig. 10. Frequency response of single stage composite binomial filter

As shown in (18), the cascaded filter has coefficients in binomial and negative binomial forms. The frequency response is given by

$$H_c^N(e^{j\omega}) = \frac{e^{-\frac{jN\omega}{2}} \cos^N \frac{\omega}{2}}{(2 - e^{-j\omega})^N} \quad (19)$$

As shown in Fig. 11, we know that the more the number of stages the narrower the bandwidth of the filter and the linearity of filters are improved as the number of stages increments. The improvement of linearity is caused by approaching this filter to Gaussian filter as the number of stages increments like the case of the negative filter.

Taking the absolute, taking the log for (19) and letting the term a value  $x$  that is a required attenuation, we have

$$10N \log \left( \frac{\cos \frac{\omega}{2}}{\sqrt{(2 - \cos \omega)^2 + \sin^2 \omega}} \right) = x \quad (20)$$

The needed number of stages is

$$\therefore N = \left\lceil \frac{x}{10 \log \left( \frac{\cos \frac{\omega}{2}}{\sqrt{(2 - \cos \omega)^2 + \sin^2 \omega}} \right)} \right\rceil \quad (21)$$

The table 3 and Fig. 12 show the number of stages to achieve the required attenuation.

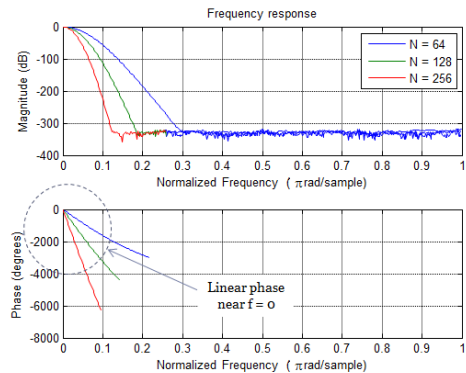


Fig. 11. Frequency response of composite binomial filters

Table 3. Number of stages of composite binomial filter

Normalized Frequency (* π)	Attenuation[dB]				
	-100	-80	-60	-40	-20
0.02	5205	4146	3123	2082	1041
0.04	1316	1053	790	527	264
0.06	596	477	358	239	120
0.08	344	275	206	138	69
0.1	227	181	136	91	46

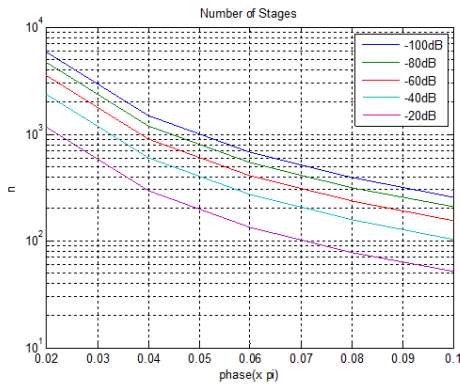


Fig. 12. The number of stages versus the needed attenuations for composite binomial filters

### III. Performance

The performance for each type of filter is defined as the number of stages to meet the required attenuation. The performance ratios of negative and composite binomial filters versus binomial filter are

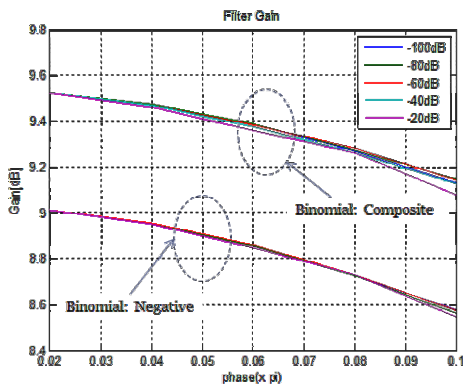


Fig. 13. Performance comparison of negative and composite filters versus binomial filter

depicted in Fig. 13. The negative binomial filter is superior to the binomial filter by above 8 [dB] and the composite binomial filter is superior to the binomial filter by above 9 [dB] in the practical range of frequency, [0 0.1 π]. Especially, as mentioned above, although the complexities of the binomial and negative binomial filters are the same, negative binomial filter shows the better performance than binomial filter.

### IV. Conclusion

We proposed binomial, negative binomial and composite binomial filters to utilize as channel separation filters for CR systems. We also obtained the transfer function for each filter, its frequency response, the number of cascaded stages to meet the required attenuation and its coefficients. Since the proposed filters have simple structure, do not require any multiplier and can be implemented by addition and shift operations, it is expected to be utilized in high-speed communications and signal processing areas in wireless sensor network to monitor various ocean environmental parameters as well as in spectrum sensing one.

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