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부분 중계노드 선택 기반의 협력 네트워크에서 증폭 후 전송 방식에 대한 성능분석

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Performance Analysis of Amplify-and-Forward Relaying in Cooperative Networks with Partial Relay Selection

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요 약

본 논문에서는, 증폭 후 재전송하는 부분 릴레이 선택 방식의 듀얼홉 릴레이 시스템에 대한 성능을 분석한다. 증폭 후 재전송하는 릴레이 이득은 채널과 잡음을 동시에 고려한 이득이다. 수신 SNR 모델을 이용하여 닫힌형태의 end-to-end SNR 확률밀도함수와 누적분포함수를 유도한다. 또한, 부분 릴레이 선택 방식을 갖는 듀얼홉 릴레이 시스템의 정확한 닫힌 형태의 채널 용량을 분석한다.

마지막으로 본 논문의 이론적인 분석은 Monte-Carlo 시험을 통하여 검증한다.

Key Words : Amplify-and-forward (AF) relaying, cooperative networks, ergodic capacity, relay selection

ABSTRACT

In this paper, we analyze the performance of dual-hop amplify-and-forward (AF) relaying in cooperative networks with partial relay selection. An AF relay gain considered in this paper includes channel-noise-assisted relay gain. Leveraging a received signal-to-noise ratio (SNR) model, we derive exact closed-form expressions for the probability density function (pdf) and cumulative distribution function (cdf) of the end-to-end SNR. Moreover, an exact closed-form expression of the ergodic capacity for dual-hop AF relaying with channel-noise-assisted relay gain and partial relay selection is investigated. The analytical results shown in this paper are confirmed by Monte-Carlo simulations.

Ⅰ. Introduction

Cooperative communications for wireless networks including cellular systems, sensor networks, and wireless ad-hoc networks are the objective of intensive research. One of cooperative communication schemes uses relays to enhance the wireless link quality, which is well-known cooperative diversity. Two well-known relaying protocols are decode-and-forward (DF) and amplify-and-forward (AF) relaying. In particular, AF relaying is one of the most promising technique owing to simple protocols and design simplicities. In the AF relaying, the relay receives a noisy signal transmitted at the source, amplifies it and re-transmits it to the destination[1].

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Recently, there is a lot of interest in AF relaying, where the relay nodes are selected according to some criteria, which minimizes signaling overhead of the relaying process. Bletsas et al. proposed an opportunistic relaying for a two-hop AF relaying system, where the best relay is selected before the transmission by taking into account all relay links[2]. An opportunistic relaying is time sensitive and requires perfect timing synchronization among the all nodes, as well as introduces additional network delays which is a crucial issue for practical systems.

On the other hand, partial relay selection was proposed in [3], wherein the source monitors the connectivity among the node locally rather than globally. Therefore, this approach can prolong the network lifetime for resource-constrained wireless systems such as sensor networks[4]. Although the performance of AF relaying in cooperative networks have been extensively studied in terms of error rate, there have been few results on the ergodic capacity. To the best of the authors' knowledge, there is no work in the literature studying the ergodic capacity of partial relaying systems. In [5], an upper bound for the ergodic capacity with closed-form expression was derived, however, the work in [5] did not give an exact closed-form expression of the ergodic capacity. In [6], exact closed-form expressions are derived for the probability density function (pdf) and the cumulative distribution function (cdf) of the end-to-end signal-to-noise ratio (SNR) of opportunistic dual-hop AF relaying systems with relay selection. The selection rule follows a maximum end-to-end SNR policy, based on the available channel state information. The relay selection rule in [6] is based on end-to-end SNR between source and destination.

AF relays may be categorized as fixed gain, channel assisted, and channel noise assisted, based on how source-to-relay channel information and noise statistics are used in determining the relay gains. In this paper, we derive exact closed-form expressions for the pdf and the cdf of the end-to-end SNR for dual-hop AF relaying with channel noise assisted relay gain and partial relay selection between source and relay node. Moreover, an exact closed-form expression of the ergodic capacity is investigated.

The rest of this paper is organized as follows. In Section II, we present the signal and system model used in the paper. Section III discusses the derivation of the end-to-end SNR statistics including pdf, cdf and outage probability. We also derive an exact closed-form expression for the ergodic capacity. Section IV provides simulation results verifying the derived analytical results. Finally, we provide conclusions in Section V.

Ⅱ. System Models

Consider a dual-hop cooperative network consisted of one source node S, one destination node D, and N AF relay nodes R_i for $i=1\cdots N$ shown in Fig. 1. Assume that each node is equipped with a single antenna and relays operate in a half-duplex mode. The transmission is performed in two orthogonal channels in time or frequency [7]. The source has no direct link with the destination and the communication is performed by choosing one out of the N relays R_n according to a partial relay selection strategy. For each link, the channel is assumed to be block flat-fading and modeled as zero-mean, independent, circularly-symmetric complex Gaussian random variable with each variance $\bar{\rho}_{SR_n}$ and $\bar{\rho}_{R_nD}$ for $n=1,2,\ldots,N$. We assume that all additive white Gaussian noise terms have zero mean and equal variance. Let ρ_{SR_n} and ρ_{R_nD} be the instantaneous SNRs have variance $\bar{\rho}_{SR_n}$ and $\bar{\rho}_{R_nD}$ of the links $S\rightarrow R_n$ and $R_n\rightarrow D$, respectively.

According to the partial relay selection criterion given in [3], the source monitors the quality of its connectivity with the relays via the transmission of local feedback, and selects a single nth relay among the N relay nodes having the maximum $S\rightarrow R_n$ hop instantaneous SNR ρ_{SR_n}.

Hereafter, for simplicity, let ρ_1 be the instantaneous SNR with variance $\bar{\rho}_1=1/\alpha$ for the link $S \rightarrow R_n$. Similarly, ρ_2 is the instantaneous SNR with variance $\overline{\rho}_2 = 1/\beta$ for the link $R_n \rightarrow D$. Then, the instantaneous end-to-end SNR from S to D via R_n is given by [8,9].

$$\rho = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2 + 1} \tag{1}$$

From the relay selection criterion in [3], ρ_1 is given by $\rho_1 = \max\{\rho_{SR_n}\}, n = 1, \cdots, N$.

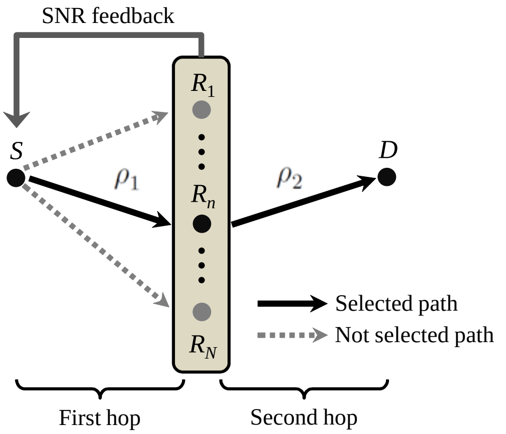

Fig. 1. Dual-hop cooperative network with relay selection.

Ⅲ. Performance Analysis

The source monitors the quality of its connectivity with the relays via the transmission of local feedback, and selects a single nth relay among the N relay nodes having the maximum $S \rightarrow R_n$ hop instantaneous SNR ρ_1 with variance $\rho_1 = 1/\alpha$. From the order statistics [10], the pdf of ρ_1 can be expressed as

$$\begin{aligned} f_{\rho_1}(\rho) &= N\alpha e^{-\alpha\rho}(1-e^{-\alpha\rho})^{N-1} \\ &= \sum_{n=1}^{N} \binom{N}{n} (-1)^{n-1} n\alpha e^{-n\alpha\rho} \end{aligned} \tag{2}$$

The second part of (2) can be obtained by using the binomial expansion. Also, the pdf of ρ_2 with variance $\overline{\rho}_2 = 1/\beta$ is given by

$$f_{\rho_2}(\rho) = \beta e^{-\beta\rho} \tag{3}$$

3.1 Statistical Analysis

We now present the main results on the statistics of the instantaneous end-to-end SNR in (1).

Theorem 1: The cdf of ρ in (1) is given by

$$\begin{aligned} F_\rho(\rho) = 1 - 2\sum_{n=1}^{N} \binom{N}{n} &(-1)^{n-1} \\ &\cdot e^{-(n\alpha+\beta)\rho} \sqrt{n\alpha\beta\rho(\rho+1)} \\ &\cdot K_1(2\sqrt{n\alpha\beta\rho(\rho+1)}) \end{aligned} \tag{4}$$

where $K_v(\cdot)$ is the modified Bessel function of the second kind of order v

Proof : The cdf can be derived as follows:

$$\begin{aligned} F_\rho(\rho) &= \Pr\left\{\frac{\rho_1\rho_2}{\rho_1+\rho_2+1} \le \rho\right\} \\ &= \int_0^\rho \underbrace{\Pr\left\{\rho_2 \ge \frac{(x+1)\rho}{x-\rho}\right\}}_{=1} f_{\rho_1}(x)dx \\ &\quad + \int_\rho^\infty \Pr\left\{\rho_2 \le \frac{(x+1)\rho}{x-\rho}\right\} f_{\rho_1}(x)dx \\ &= 1 - \int_\rho^\infty \left(1 - F_{\rho_2}\left(\frac{(x+1)\rho}{x-\rho}\right)\right) f_{\rho_1}(x)dx \end{aligned} \tag{5}$$

We also know that $F_{\rho_2}(x) = 1 - e^{-\beta x}$ and make the change of variables $u = x - \rho$ in (5), then, (5) can be rewritten as

$$\begin{aligned} F_\rho(\rho) = 1 - \sum_{n=1}^{N} &\binom{N}{n} (-1)^{n-1} n\alpha \\ &\cdot \int_0^\infty e^{-n\alpha(u+\rho)} e^{-\beta\rho(u+\rho+1)/u} du \end{aligned} \tag{6}$$

Finally, using [11, eq. (3.471.9)], we can get the cdf ρ of as shown in (4).

Theorem 2: The pdf of ρ in (1) is given by

$$\begin{aligned} f_\rho(\rho) = 1 - 2\sum_{n=1}^{N} &\binom{N}{n} (-1)^{n-1} e^{-(n\alpha+\beta)\rho} \\ &\cdot [n\alpha\beta(2\rho+1)K_0(2\sqrt{n\alpha\beta\rho(\rho+1)}) \\ &+ (n\alpha+\beta)\sqrt{n\alpha\beta\rho(\rho+1)} \\ &\cdot K_1(2\sqrt{n\alpha\beta\rho(\rho+1)})] \end{aligned} \tag{7}$$

Proof: By differentiating (4) with respect to and using the identity [11, eq. (8.486.12)] that gives the derivative $dK_v(x)/dx = -K_{v-1}(x) - vK_v(x)/x$.

Closed-form expressions of the cdf and pdf of the end-to-end SNR are also calculated in [5]. However, our results are more concise than the results in [5], allowing for simpler analysis.

3.2 Outage Probability

The outage probability is a meaningful performance measure in wireless communication systems over fading channels. For good codes and long block lengths, outage probability gives an approximation of the frame error rate [12]. We consider the outage probability that corresponds to the probability that the SNR falls below the specific SNR threshold ρ_{Th}. Thus, the outage probability for the AF relay channel can be easily obtained as

$$P_{out}(\rho_{Th}) = \Pr[\rho \le \rho_{Th}] = F_\rho(\rho_{Th}) \tag{8}$$

where $F_\rho(\cdot)$ is given in (4).

To get a further insight on the outage probability, we develop a simple outage probability expression for high SNR region. In the asymptotic SNR region $(\alpha, \beta \to 0)$, the Bessel function can be approximated by $K_v(x) \simeq 2^{v-1}\Gamma(v)/x^v$ for small x. By applying this approximation to (4) and simplifying further using the first order terms of an exponential approximation $e^{-\mu x} \simeq 1 - \mu x$, the outage probability can be simplified as follows

$$P_{out}(\rho_{Th}) \approx \begin{cases} (\alpha+\beta)\rho_{Th} & \text{for } N=1 \\ \beta\rho_{Th} & \text{for } N>1 \end{cases} \tag{9}$$

Therefore, for $N>1$ relays, the relaying link $S \to R_n \to D$ asymptotically approaches the performance of the single link $R_n \to D$. Furthermore, it can be seen from (9) that the asymptotic performance of the cooperative network with partial relay selection does not depend on the quality of source-relay links and thus it is equivalent for any choice of α. Furthermore, we can observe that the performance asymptotically approaches faster to the performance of the single link as N increases to infinity. The observation of this asymptotic results will be shown in Section IV.

For example, if the two links are identical, i.e., $\bar{\rho}_1 = \bar{\rho}_2 = \bar{\rho}$, the outage probability in (9) can be reduced to $2\rho_{Th}/\bar{\rho}$ and $\rho_{Th}/\bar{\rho}$ for $N=1$ and $N>1$, respectively. For this special case, we know that the outage performance of cooperative network with partial relay selection has better performance than conventional relay $(N=1)$ system by 3.01 dB at high SNR region. For unbalanced link, $5\bar{\rho}_1 = \bar{\rho}_2$ and $9\bar{\rho}_1 = \bar{\rho}_2$, the gain of partial relay selection over conventional relay system is 7.8 dB and 10 dB, respectively.

3.3 Ergodic Capacity Analysis

In communications over fading channels, the maximum error-free data rate that a channel can support is called the channel capacity. The work in [5] derived an upper bound for the ergodic capacity by using the Jensen' inequality, however, an exact closed-form of the ergodic capacity is not shown in [5]. In this section, we derive an exact ergodic capacity of AF relaying system with partial relay selection.

Theorem 3: An exact closed-form expression for the ergodic capacity is given by

$$\begin{aligned} C = \frac{1}{2\ln(2)}[\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}n\alpha}{n\alpha-\beta} \\ \cdot \{e^{n\alpha}\Gamma(0,n\alpha) - e^{\beta}\Gamma(0,\beta)\} \\ + e^{\beta}\Gamma(0,\beta)] \end{aligned} \tag{10}$$

where $\Gamma(x,y)$ is the upper incomplete Gamma function [11, eq. (3.381.3)].

Proof : Given the pdf of ρ, the ergodic capacity is

$$C = \frac{1}{2}E[\log_2(1+\rho)] \tag{11}$$

where the factor $1/2$ is due to the fact that the transmission is performed in 2 time slots or two orthogonal channels. Therefore, based on (1), we have

$$C=\frac{1}{2\ln(2)}E\left[\ln\left(1+\frac{\rho_1\rho_2}{\rho_1+\rho_2+1}\right)\right] \\ =\frac{1}{2\ln(2)}E\left[\ln(1+\rho_1)\right]+E\left[\ln(1+\rho_2)\right] \\ -E\left[\ln(1+\rho_1+\rho_2)\right] \tag{12}$$

To evaluate (12), we need the pdfs of ρ_1, ρ_2 and $\rho_1+\rho_2$. The pdf of ρ_1 and ρ_2 are given by (2) and (3), respectively. Then, using the moment generating function (MGF) definition expression, i.e., $M_\rho(s)=\int_0^\infty e^{-s\rho}f_\rho(\rho)d\rho$, the MGFs for ρ_1 and ρ_2 are given by $\sum_{n=1}^{N}\binom{N}{n}(-1)^{n-1}n\alpha(s+n\alpha)^{-1}$ and $\beta(s+\beta)^{-1}$, respectively. Given that ρ_1 and ρ_2 are independent, the MGF of $\rho_1+\rho_2$ can be written as $M_{\rho_1+\rho_2}(s)=M_{\rho_1}(s)M_{\rho_2}(s)$. Therefore, the MGF of $\rho_1+\rho_2$ is given by

$$M_{\rho_1+\rho_2}(s)=\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}n\alpha\beta}{n\alpha-\beta} \cdot\left(\frac{1}{s+\beta}-\frac{1}{s+n\alpha}\right) \tag{13}$$

To find the pdf of $\rho_1+\rho_2$, we use $f_{\rho_1+\rho_2}(\rho)=\mathcal{L}^{-1}\left[M_{\rho_1+\rho_2}(s)\right]$, where $\mathcal{L}^{-1}[\cdot]$ is the inverse Laplace transform operator, hence we can get the following

$$f_{\rho_1+\rho_2}(\rho)=\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}n\alpha\beta}{n\alpha-\beta} \cdot\left(e^{-\beta\rho}-e^{-n\alpha\rho}\right) \tag{14}$$

Let us define the integral , $F(c)=\int_0^\infty\ln(1+x)e^{-cx}dx$, which has the closed-form solution $F(c)=e^c\Gamma(0,c)/c$ where $\Gamma(x,y)$ is the upper incomplete Gamma function [13, eq. (79)]. Finally, an exact closed-form expression for the ergodic capacity in (12) can be written by

$$C=\frac{1}{2\ln(2)}\Big[\sum_{n=1}^{N}\binom{N}{n}(-1)^{n-1}n\alpha F(n\alpha) \\ +\beta F(\beta)-\sum_{n=1}^{N}\binom{N}{n}\frac{(-1)^{n-1}n\alpha\beta}{n\alpha-\beta} \\ \cdot\{F(\beta)+F(n\alpha)\}\Big] \tag{15}$$

After some mathematical manipulation, we can get the ergodic capacity in (10). To the best of author' knowledge, (10) is novel. Note that the upper incomplete Gamma function can be evaluated in commercial mathematics software package, such as Mathematica and Maple.

Ⅳ. Numerical Results

In this section, we present numerical results to verify Theorems 1 - 3 through Monte-Carlo simulations. For Monte-Carlo simulations, we simulate the performance index which we average over 10^4 data block realizations at each SNR. The data block length is set to 10^6.

Fig. 2 shows the pdf of the end-to-end SNR $f_\rho(\rho)$ for the AF relaying with partial relay selection for $N=2,4,8,12$ when $\bar{\rho}_1$ and $\bar{\rho}_2$ are assumed to be 3 dB and 9 dB, respectively. As the number of relays N increases, the pdf values shift to the right and the peak value becomes smaller implying that the end-to-end SNR becomes more distributed. Fig. 2 confirms that our analytical results correspond exactly to the simulation results.

Fig. 3 shows the outage probability as a function

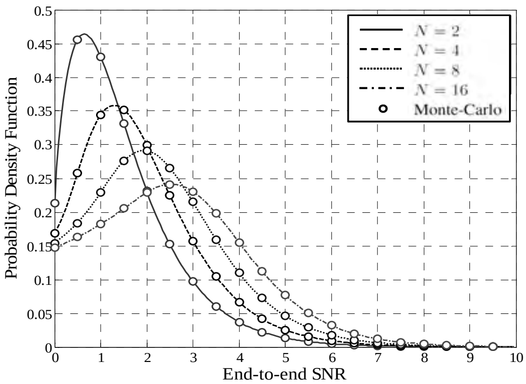

Fig. 2. Probability density function for unbalanced links, $\bar{\rho}_1$ = 3 dB and $\bar{\rho}_2$ = 9 dB.

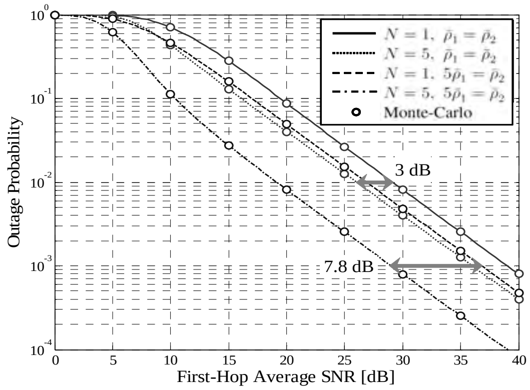

Fig. 3. Outage probability versus $\bar{\rho}_1$ for balanced and unbalanced links.

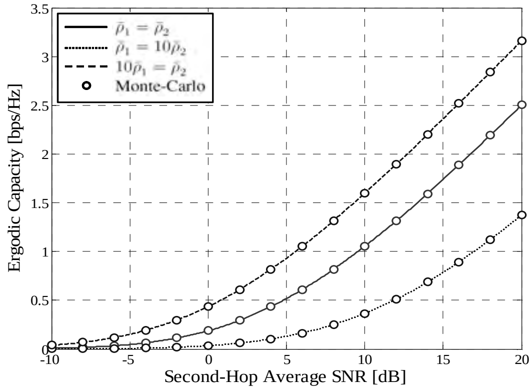

Fig. 5. Ergodic Capacity versus $\bar{\rho}_2$ for balanced and unbalanced links.

of the first-hop average SNR $\bar{\rho}_1$ (for a threshold ρ_{Th} = 6 dB) for balanced ($\bar{\rho}_1=\bar{\rho}_2$) and unbalanced links ($5\bar{\rho}_1=\bar{\rho}_2$) and for different number of relays N. Moreover, as discussed in the previous section, the outage probability performance of AF relaying with partial relay selection has better performance than the conventional relaying($N=1$) by 3.01 dB and 7.78 dB for balanced ($\bar{\rho}_1=\bar{\rho}_2$) and unbalanced ($5\bar{\rho}_1=\bar{\rho}_2$) links, respectively at high SNR. From Fig. 3, it can be seen that the partial relay selection system improves the outage performance of the conventional relaying link ($N=1$) and the resulting gain is equal to 3 dB at the high SNR region for balanced link.

Fig. 4 gives the outage performance as a function of the number of relay N for unbalanced link with $9\bar{\rho}_1=\bar{\rho}_2$ at threshold ρ_{Th} = 9 dB. Exact and asymptotic results are given by (8) and (9), respectively. From Fig. 3, we can observe that the exact result given by (8) corresponds exactly to the simulations results, and the asymptotic result in (9) corresponds closely to the exact result in (8) at high SNR region. Also, it can be observed that the outage performance converges into the same value of $\beta\rho_{Th}$ irrespective of N as SNR goes to high. As described in Section III, the performance asymptotically approaches faster to the performance of the single link as N increases, and it can be verified in Fig. 5.

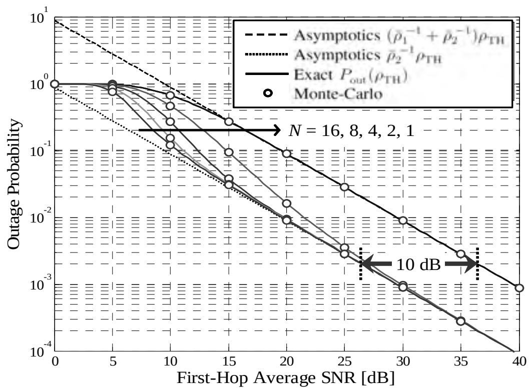

Fig. 4. Outage probability versus $\bar{\rho}_1$ for unbalanced links $9\bar{\rho}_1=\bar{\rho}_2$.

In Fig. 5, the ergodic capacity in terms of the second-hop average SNR $\bar{\rho}_2$ assuming $N=2$ and balanced or unbalanced links. It is clear that (10) in Theorem 3 provides the exact results.

All numerical results confirm our analyzes.

V. Conclusions

This paper has presented performance analysis of AF relaying in cooperative networks with partial relay selection. New closed-form expressions for the statistical behavior including pdf, cdf and outage probability have been derived. Through some approximation of the outage probability at high SNR region, we give some insight for impact of the relay location on the system performance. The asymptotic performance of partial relay selection does not depend on the quality of source-relay links. The analysis in this paper is useful for practical AF relaying systems where the relay selection based on the first hop channel state information. Moreover, we have derived the ergodic channel capacity with closed-form solution. An exact agreement between analytical and Monte-Carlo simulation results is observed.

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