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디콘볼루션을 이용한 시간지연추정

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Time Delay Estimation Using De-Convolution

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ABSTRACT

This paper deals with the problem of time delay estimation using de-convolution. Two approaches, conjugate gradient method and the total lease square method have been presented to solve the de-convolution problem. Numerical simulation demonstrates the superior performance of the proposed methods over the conventional GCC based algorithms and FIR filter method.

Key Words : Time delay, De-convolution, Conjugate gradient, Total least square

Ⅰ. Introduction

The estimation of time delay between two sensors is important in many applications of sonar, radar, geophysics, acoustic localizations, etc. In the last two decades, several methods have been proposed for solving the time delay problems. Generalized Cross Correlation (GCC) proposed by Knapp and Carter[1] is the most commonly used method for TDE. In this technique, the delay estimate is obtained as the time lag that maximizes the cross correlation between the filtered versions of the received signals. Several windows have been proposed to improve the GCC in the presence of noise [1,2,3]. These windows depend on input power spectral density of the signal that are generally unknown and need to be estimated[4]. Chan[5] used a discrete Wiener filter approach to obtain time delay but this does not work well in a noisy environment. Techniques using de-convolution have been introduced for acoustic applications[6].

In this paper, we present a novel approach to estimate the time delay based on de-convolution for highly ill conditioned matrix. De-convolution is the process of finding the impulse response from the known values of the input and output. Two very efficient linear equation solvers, conjugate gradient method and total least square method[7,8,9], are presented and are used to get an accurate estimate of the time delay parameter in a noisy environment. Our numerical simulation represent that the proposed method performs better than the conventional methods with the margin of 10dB in SNR to have the same output error.

This work is organized as follows. Section 2 describes the conventional GCC based windowing and FIR filtering approach. In section 3, the time delay estimation technique using de-convolution is developed. Section 4 gives some numerical examples and provides comparison with different algorithms. The conclusion of our work is presented in section 5.

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Ⅱ. Generalized Cross Correlation (GCC)

Assume a linear time invariant causal system of input $x(t)$. $y(t)$ is the time domain signal measured at the output of the system that satisfy following equations;

$$x(t)=\alpha_1 s[t-\tau_i]+N_1(t) \quad (1)$$

$$y(t)=\alpha_2 s[t-\tau_i+D]+N_2(t) \quad (2)$$

where $s(t)$ is an unknown signal, $\alpha_i s(t-D)$ is a shifted and scaled version of $s(t)$, τ_i is the initial time delay, and $N_1(t)$ and $N_2(t)$ are unknown noise sources. The problem is to estimate the time delay *D* from the finite length of discrete measurement of $x(k)$ and $y(k)$; $k=1,2,...,N$. The simplest and the most popular way to estimate *D* will be to use of the cross correlation between $x(k)$ and $y(k)$, i.e.,

$$c_{xy}(\tau)=E[x(t)y(t+\tau)];\ -\infty<\tau<\infty, \quad (3)$$

where *E*[] is the expectation operator in time. Note that $N_1(t)$ and $N_2(t)$ are zero mean stationary signals independent with each other and $s(t)$. $c_{xy}(\tau)$ will have the maximum at $\tau=D$. However, in real situations, due to noise and reverberation as well as with finite number of observations, $c_{xy}(\tau)$ doest not always give a peak at the time delay *D*. Knapp and Carter[1,4] used a smoothed cross-correlation so called “Generalized Cross Correlation(GCC)” to estimate τ. That is

$$C_{xy\,GCC}(\omega)=C_{xy}(\omega)\,W(\omega), \quad (4)$$

$$c_{xy\,GCC}(\tau)=c_{xy}(\tau)*w(\tau), \quad (5)$$

where $C_{xy\,GCC}$, $C_{xy}(\omega)$, $W(\omega)$ are the Fourier transform of $c_{xy\,GCC}(\tau)$, $c_{xy}(\tau)$, $w(\tau)$ respectively, $W(\omega)$ is a window function, and * is the convolution operator.

Various window functions have been suggested to smooth the cross-correlation in order to sharpen the maximum peak and suppress the minor peaks[1,2,3]. They are summarized as:

2.1 Roth window

$$W(\omega)=\frac{1}{C_{xx}(\omega)}, \quad (6)$$

where $C_{xx}(\omega)$ is the power spectral density of $x(t)$.

2.2 SCOT (smoothed Coherence Transform) window

The smoothed Coherence Transform has following form;

$$W(\omega)=\frac{1}{\sqrt{C_2^x(\omega)\,C_2^y(\omega)}} \quad (7)$$

where $C_2^x(\omega)$ and $C_2^y(\omega)$ are the power spectral density of $x(t)$ and $y(t)$.

2.3 PHAT (The Phase Transform)

The smoothing window is inversely proportional to the magnitude of $C_{xy}(\omega)$, i.e.,

$$W(\omega)=\frac{1}{|C_{xy}(\omega)|}\ . \quad (8)$$

In an ideal case, the resulting cross correlation will be

$$c_{xy}(\tau)=IFT\left(\frac{C_{xy}(\omega)}{|C_{xy}(\omega)|}\right)=IFT(\exp^{j\omega D})=\delta(t-D) \quad (9)$$

2.4 Wiener filtering approach

Another interesting approach is to use a discrete FIR filter proposed by Chan[5]. This method is parametric in the sense that it estimates the parameters of the FIR filter from signals $x(k)$, and $y(k)$. The main idea is to pass the input $x(k)$ through a FIR filter with coefficients, $a(n)$, and then choose the optimum value of the parameters such that the filter output $y(k)$ is close to the desired output $x(t+D)$ in the mean square error sense. The filter output $z(k)$ is related to the input by;

$$z(k)=\sum_{n=0}^{P}a(n)x(k-n). \tag{10}$$

We truncate the summation to include $a(n)$, where *P* is large enough to guarantee $D\leq P\Delta t$, Δt is the sampling interval, and the time delay parameter *D* is assumed to be positive. The Wiener problem is to minimize the mean square error with respect to the filter coefficients an, i.e.,

$$\begin{aligned} J&=E\{[y(k)-z(k)]^2\} \\ &=E\left\{\left[y(k)-\sum_{n=0}^{P}a(n)x(k-n)\right]^2\right\} \end{aligned}, \tag{11}$$

$$\frac{dJ}{da(i)}=2E\left\{\left[y(k)-\sum_{n=0}^{P}a(n)x(k-n)\right]x(k-i)\right\}=0. , \tag{12}$$

One can form this into a matrix equation

$$\begin{bmatrix} c_{xy}(0) \\ c_{xy}(1) \\ 5 \\ c_{xy}(p) \end{bmatrix}=\begin{bmatrix} c_2(0) & 0 & 4 & 0 \\ c_2(1) & c_2(0) & & 5 \\ 5 & 5 & 7 & 0 \\ c_2(p) & c_2(p-1) & 4 & c_2(0) \end{bmatrix}\begin{bmatrix} a(0) \\ a(1) \\ 5 \\ a(p) \end{bmatrix}, \tag{13}$$

where

$$c_{xy}(i)=E\{y(k)x(k-i)\}, \tag{14}$$

$$c_2(i)=E\{x(k)x(k-i)\}. \tag{15}$$

The matrix equation can be solved using various linear solvers. The time delay *D* corresponds to the index of the coefficient $a(n)$ which has the maximum value. In case, when the time delay is not an integer, the interpolation can be applied to the problem.

Ⅲ. TDE using De-convolution

Here we use the principle of de-convolution to estimate the delay. Consider a linear, time invariant, causal system with zero initial state. The relation between the input $x(k); k=1,2,...,N$, the impulse response $h(k); k=1,2,...,P$, and the output $y(k); k=1,2,...,N$, is given by

$$y(k)=\sum_{p=1}^{P}x(k-p+1)h(p). \tag{16}$$

If the input $x(k)$, and output $y(k)$ of a system are known, then one can compute its impulse response $h(k)$ from (16). Equivalently, it can be cast into the following matrix form;

$$AB=C, \tag{17}$$

where

$$A=\begin{bmatrix} x(1) & 0 & 4 & 0 \\ x(2) & x(t_1) & 4 & 0 \\ 5 & 5 & 7 & 5 \\ x(P) & x(n-1) & 4 & x(1) \\ 5 & 5 & & 5 \\ x(N) & x(N-1) & 4 & x(N-P+1) \end{bmatrix}_{N\times P},$$

$$B=\begin{bmatrix} h(1) \\ h(2) \\ 5 \\ h(P) \end{bmatrix}_{P\times 1},\quad C=\begin{bmatrix} y(1) \\ y(2) \\ 5 \\ y(N) \end{bmatrix}_{N\times 1}. \tag{18}$$

De-convolution is the process of finding the impulse response vector *B* from the known values of the input $x(k)$ and the output vector *C*. The discrete de-convolution problem is essentially reduced to the problem of solving a set of linear equations given by (17). Typically *N* is larger than *P* and therefore (17) needs to be solved in a least squares fashion. The matrix *A* tends to be highly ill-conditioned. To deal with this problem, we proposed two approaches to solve this linear equation which will be briefly explained below.

3.1 Conjugate gradient approach

For the solution of $AB = C$ in (17), the conjugate gradient method starts with an initial guess B_0 for the solution and lets[7]

$$P_0 = -b_{-1}A^H R_0 = -b_{-1}A^H (AB_0 - C), \tag{19}$$

where superscript H denotes the conjugate transpose of a matrix. At the r-th iteration the conjugate gradient method develops the following:

$$t_r = \frac{1}{\|AP_r\|^2}, \tag{20}$$

$$B_{r+1} = B_r + t_r P_r, \tag{21}$$

$$R_{r+1} = R_r + t_r AP_r, \tag{22}$$

$$b_r = \frac{1}{\|A^H R_{r+1}\|^2}, \tag{23}$$

$$P_{r+1} = P_r - b_r A^H R_{r+1}. \tag{24}$$

The norm is defined by

$$\|AP_r\|^2 = P_r^H A^H A P_r. \tag{25}$$

The above equations are applied in a routine fashion till the desired error criterion for the residuals $\| R_r \|$, is satisfied. In our case, the error criterion is defined as

$$\frac{\|AB_r - C\|}{\|C\|} \le 10^{-6}. \tag{26}$$

The iteration is stopped when the above criterion is satisfied.

3.2 Total lease square method

Given a data matrix $A \in R^{N\times P} (N > P)$ and an observation vector $C \in R^P$ which are contaminated by error, one can find the Total Least Square (TLS) solution of the following equation,

$$AB = C. \tag{27}$$

by applying the following algorithm.

We first find the singular values $D = [A : C] \in R^{N\times(P+1)}$ by using the Singular Value Decomposition (SVD).

$$\begin{gathered} U^T DV = \Sigma = diag[\sigma_1, \cdots, \sigma_r, \sigma_{r+1}, \cdots, \sigma_{P+1}] \in R^{N\times(P+1)} \\ \sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_r \ge \sigma_{r+1} \ge \cdots \ge \sigma_{P+1} \ge 0 \end{gathered}, \tag{28}$$

then try to find a good approximation of the matrix D by looking at its condition number, and comparing the small singular values to the maximum singular value σ_1 and then setting the small singular values beyond a priori fixed threshold to zero. The SVD of the approximated matrix $\hat{D} = [\hat{A} : \hat{C}] \in R^{N\times(P+1)}$ with the rank of r can be written as

$$\begin{gathered} U^T \hat{D} V = \hat{\Sigma} = diag[\sigma_1, \cdots, \sigma_r, 0, \cdots, 0] \in R^{N\times(P+1)} \\ \sigma_1 > \sigma_2 > \cdots > \sigma_r > 0 \end{gathered} \tag{29}$$

and in this algorithm we use the condition number $\frac{\sigma_1}{\sigma_r}$ to be greater than a priori threshold value.

For the orthonormal matrix V, we may write

$$V = [v_1 \cdots v_r \; v_{r+1} \cdots v_{P+1}] \in R^{(P+1)\times(P+1)}, \tag{30}$$

such that $v_i \in R^{n+1}$. Then the TLS solution of the system (27) is given by

$$x_{TLS} = \frac{-\left(\sum_{i=r+1}^{P+1} v_{P+1,i}[v_{1,i} \; v_{2,i} \cdots v_{P,i}]^H\right)}{\sum_{i=r+1}^{P+1} {v_{P+1,i}}^2}, \tag{31}$$

or

$$x_{TLS} = \frac{\sum_{i=1}^{r} v_{P+1,i}[v_{1,i} \; v_{2,i} \cdots v_{P,i}]^H}{1 - \sum_{i=1}^{r} {v_{P+1,i}}^2}. \tag{32}$$

The detailed derivation of the formulation can be found in the references[9,10].

Ⅳ. Numerical simulations

As a first example, consider a signal of three sinusoids of frequencies [2, 3, 5]Hz with magnitudes [1, 2, 0.5], i.e.,

$$x(t) = \sin(4\pi t) + 2\sin(6\pi t) + 0.5\sin(10\pi t),$$

The output will be a delayed version of $x(k)$ and the time delay D equals 0.152sec.

$$y(t) = \sin[4\pi(t-0.152)] + 2\sin[6\pi(t-0.152)] + 0.5\sin[10\pi(t-0.152)].$$

We sampled the signals with a sampling interval $\Delta t = 0.01\sec$ during the time interval of [0, 2] seconds and padded with zeros in the interval from (2, 4]. The input $x(k)$ and output $y(k)$ are shown in Fig. 1. All the approaches described in the previous section had been simulated and the results are shown in Fig. 2. One can observe that for GCC with windows, Fig. 2-(e)(f) have smaller minor peaks than the case of GCC without windows, Fig. 2-(d). All the conventional approaches, Wiener filtering, GCC-Roth window, GCC-SCOT window, and GCC-PHAT window, have a peak at the desired time delay at t=0.152sec, but the main peaks are not sharp as the proposed methods. Fig. 2-(a)(b) shows

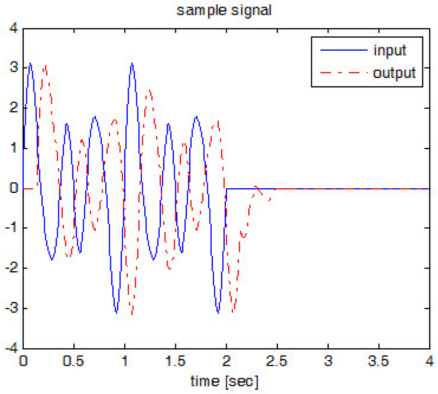

Fig. 1. Sample signal

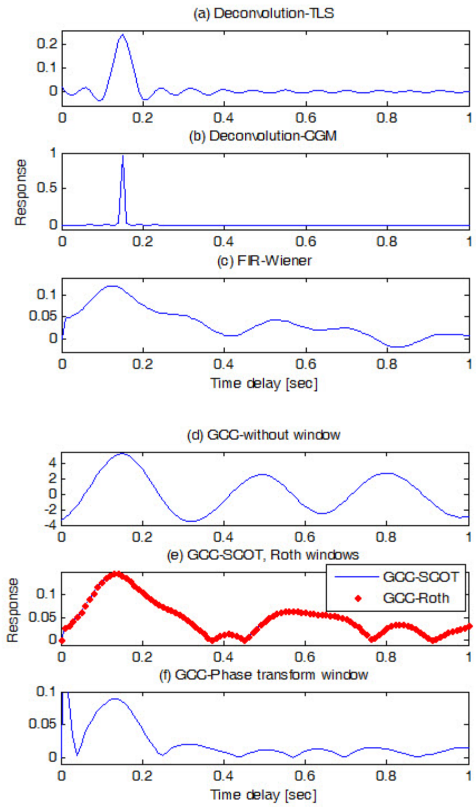

Fig. 2. Responses of conventional approaches and proposed approaches

that the de-convolution using CGM and TLS offer better resolutions and less values for the minor peaks.

For the next example, the input and output was modeled by the following relationships;

$$x(t) = \sin(10\pi t) + 2\sin(18\pi t) + 0.5\sin(50\pi t) + N_1(t),$$

$$y(t) = \sin[10\pi(t-0.152)] + 2\sin[18\pi(t-0.152)] + 0.5\sin[50\pi(t-0.152)] + N_2(t),$$

where $N_i(t) \approx N_{mag} N(0, N_{mag}^{\ 2})$; i=1,2 are independent and identically distributed (i.i.d.) noise sequences following normal distribution with zero mean and variance of N_{mag}^2. Fig. 3 shows the Monte Carlo simulation for the SNR versus the error in the estimation which is defined by $\frac{|D_{est} - D|}{D}$, where

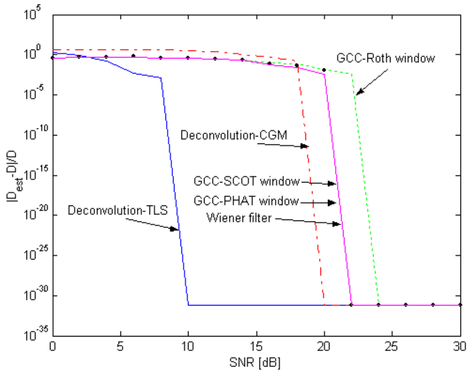

Fig. 3. SNR versus error in the estimation

D_{est} is an estimation of *D*. The sampling interval was $\Delta t = 0.01\text{sec}$ and the other condition was same as that of the first example. As shown in Fig. 3, the superior performance of the proposed methods is demonstrated in comparison with conventional GCC based algorithms. Observe that the TLS approach generates better noise immunity than the other methods even though the CGM method offers better resolution than TLS method. Usually, the CGM approach converges much faster than the other iterative linear equation solver with a certain threshold of error bound. Fig. 4 shows the condition number, defined as a ratio of the maximum eigenvalue and the minimum eigenvalue, of the matrix *A* with changing SNR. The condition number tends to get worse as the SNR get high.

For the third example, consider a signal of three sinusoids of frequencies $[0.4f_{\max}, 0.6f_{\max}, f_{\max}]$Hz with magnitudes [1, 2, 0.5], i.e.,

$$x(t) = \sin(2\pi 0.4 f_{\max} t) + 2\sin(2\pi 0.6 f_{\max} t) + 0.5\sin(2\pi f_{\max} t) + N_1(t),$$

$$y(t) = \sin[2\pi 0.4 f_{\max}(t-0.152)] + 2\sin[2\pi 0.6 f_{\max}(t-0.152)] + 0.5\sin[2\pi f_{\max}(t-0.152)] + N_2(t),$$

where $N_i(t) \approx N_{mag} N(0, N_{mag}{}^2)$; i=1,2.

SNR was equal to 18dB which is equivalent to $N_{mag} = 0.1448$ in this example. Fig. 5 plots the maximum bandwidth of the signal, $f_{\max}$, versus the error in the estimation, $\frac{|D_{est} - D|}{D}$. The x-axis is the maximum frequency in the signal $f_{\max}$, which is not actual highest frequency in the signal because of the zero padding. $f_{\max}$ was scanned from $0.2Hz(=0.004f_{Nyquist})$ to $50Hz(=f_{Nyquist})$, where $f_{Nyquist}$ is the Nyquist sampling criteria. Both de-convolution - TLS method and GCC-without window work better at low frequency than de-convolution - CGM approach. The CGM approach converges fast but need a threshold of error bound. The fundamental limitation of the conjugate gradient method is that it requires, in general, *N* cycles to reach the minimum. We need a procedure which will perform most of the function minimization in the first few cycles.

For the forth example, multipath environment signals has been introduced to the first example. The signal is same as the first example and we set the multipath delay as 0.25sec, 0.3sec, 0.5sec with the

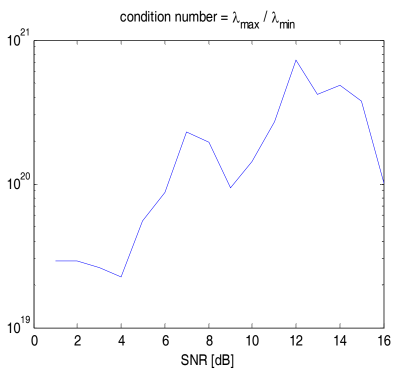

Fig. 4. Condition number of matrix A with SNR

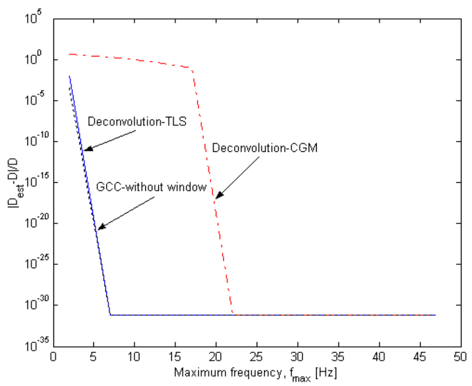

Fig. 5. Condition number of matrix A wBandwidth of the signal versus error in the estimation

magnitude 0.3, 0.2, 0.1 respectively. That is;

$$x(t) = \sin(4\pi t) + 2\sin(6\pi t) + 0.5\sin(10\pi t)$$
$$y(t) = x(t-0.15) + 0.3x(t-0.25) + 0.2x(t-0.3) + 0.1x(t-0.5) + N(t),$$

Fig. 6 shows the result with multipath signal. Only CGM and TLS approach can estimate the time delay. Fig.7 respesents the Monte Carlo simulation for the SNR versus the error in the estimation which is defined by $\frac{|D_{est} - D|}{D}$. The other condition was same as that of the first example.

The superior performance of the proposed methods is demonstrated in comparison with conventional GCC based algorithms.

The constraints of this research can be summarized as follows;

- If the multipath components become large, the performance severely decreases and none of the time delay method can be applied.
- Generally, including the proposed approach, time delay estimation requires much faster sampling rate than the maximum frequency in the signal.
- The proposed method works well for the wideband signal like voice signal, not for the narrowband signal like radar signal.
- In the future research, various types of signal would be examined for a better analysis of the work.

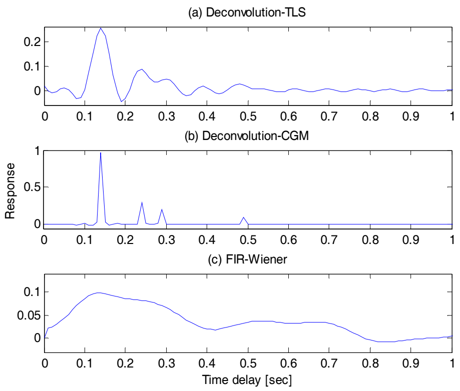

Fig. 6. Simulation with multipath signal

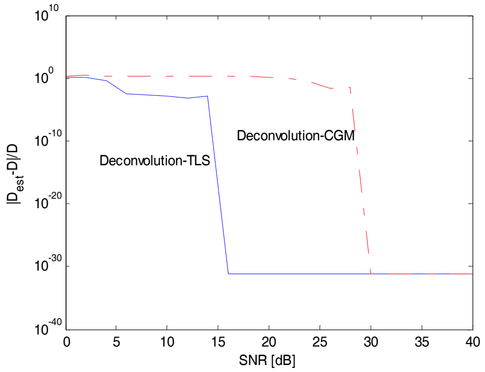

Fig. 7. SNR performance of multipath signal

Ⅴ. Conclusions

This paper deals with the problem of time delay estimation using deconvolution. Two approaches, conjugate gradient method and the total lease square method have been presented to solve the de-convolution problem. Numerical simulation demonstrated the superior performance of the proposed methods over the conventional GCC based algorithms and FIR filter method.

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