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Performance Analysis on Non-SIC ML Receiver for NOMA Strong Channel User

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ABSTRACT

We analyze the performance of the maximum likelihood (ML) receiver for the stronger channel user of non-orthogonal multiple access (NOMA), when no successive interference cancelation (SIC) is performed. This paper compares the non-SIC ML receiver to the standard receiver with the SIC. It is shown that the performance of the non-SIC ML receiver is almost the same as that of the standard receiver with the SIC for the operating range of the power allocation factor less than 20 %. In result, the non-SIC ML receiver could be a promising scheme for the NOMA stronger channel user, reducing the SIC complexity.

Key Words : Non-orthogonal multiple access, successive interference cancelation, maximum likelihood receiver, binary phase shift keying, power allocation

Ⅰ. Introduction

Non-orthogonal multiple access (NOMA) has become a new paradigm in place of conventional orthogonal multiple-access (OMA) concepts for fifth generation (5G) mobile networks due to its superior spectral efficiency[1-5]. In NOMA, the user with the better channel condition employs successive interference cancelation (SIC) to remove the signals of users with the worse channel conditions. In this paper, the performance of the maximum likelihood (ML) receiver for the NOMA stronger channel user is analyzed when no SIC is performed. The paper is organized as follows. Section II defines the system and channel model. In Section III, the non-SIC ML receiver is derived for the user with the stronger channel condition. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, and the channel gains are h_1 and h_2 with $|h_1|>|h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{1}$$

If no SIC is performed on the user-1 with the better channel condition, the received signal of the strong channel user is given by

$$\begin{aligned} r_1 &= |h_1| x + n_1 \\ &= |h_1| \left(\sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 \right) + n_1 \\ &= |h_1| \sqrt{\alpha P} s_1 + |h_1| \sqrt{(1-\alpha)P} s_2 + n_1 \end{aligned} \tag{2}$$

where $n_1 \sim \mathcal{N}(0, N_0/2)$ is additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ and N_0 is one-sided power spectral density. After the SIC is performed on the user-1, the received signal is given by

$$y_1 = |h_1| \sqrt{\alpha P} s_1 + n_1 . \tag{3}$$

Ⅲ. Non-SIC ML Receiver

We design the maximum likelihood (ML) receiver when no SIC is performed on the user-1. The performance of the non-SIC ML receiver is

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compared to that of the standard receiver with the SIC. We consider the binary phase shift keying (BPSK), with $s_1, s_2 \in \{+1, -1\}$. For the standard receiver with SIC, the decision region for $s_1 = +1$ $s_1 = +1$ is simply given by

$$y_1 > 0, \qquad \text{for all } \alpha. \tag{4}$$

The probability of error $P_e^{(standard;\, SIC)}$ for all α is calculated as,

$$P_e^{(standard;\, SIC)} = Q\left(\frac{|h_1|\sqrt{\alpha P}}{\sqrt{N_0 / 2}}\right) \tag{5}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz$.

Now, we derive the non-SIC ML receiver. The likelihoods $p_{R_1|S_1}(r_1 \mid s_1 = +1)$ and $p_{R_1|S_1}(r_1 \mid s_1 = -1)$ are expressed as

$$\begin{aligned} p_{R_1|S_1}(r_1 \mid s_1) &= \int_{-\infty}^{\infty} p_{R_1,S_2|S_1}(r_1, s_2 \mid s_1)\, ds_2 \\ &= \int_{-\infty}^{\infty} p_{R_1|S_1,S_2}(r_1 \mid s_1, s_2) p_{S_2}(s_2)\, ds_2 \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} s_1 - |h_1|\sqrt{(1-\alpha)P} s_2\right)^2}{N_0}} \\ &\quad \times \left(\frac{1}{2}\delta(s_2 - 1) + \frac{1}{2}\delta(s_2 + 1)\right) ds_2 \\ &= \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} s_1 - |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \\ &\quad + \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} s_1 + |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \end{aligned} \tag{6}$$

where $p_X(x)$ is the probability density function (PDF) and $\delta(x)$ is the Dirac delta function. The ML detection is made as

$$s_1 = \underset{s_1 \in \{+1,-1\}}{\arg\max}\; p_{R_1|S_1}(r_1 \mid s_1). \tag{7}$$

The equal likelihood equation is given by

$$p_{R_1|S_1}(r_1 \mid s_1 = +1) = p_{R_1|S_1}(r_1 \mid s_1 = -1), \tag{8}$$

which is

$$\begin{aligned} &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \\ &+ \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} = \\ &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \\ &+ \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}}. \end{aligned} \tag{9}$$

If $\alpha > 0.5$, the equal likelihood equation (9) has the one exact decision boundary, $r_1 = 0$, which is obtained directly from the equation (9). If $\alpha < 0.5$, however, the equal likelihood equation (9) has the three decision boundaries. The first exact decision boundary, $r_1 = 0$, is the same as that in case of $\alpha > 0.5$. The second approximate decision boundary, $r_1 \simeq |h_1|\sqrt{(1-\alpha)P}$, is obtained from

$$\begin{aligned} &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} = \\ &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \end{aligned} \tag{10}$$

where we use the following observation, at $r_1 = |h_1|\sqrt{(1-\alpha)P}$, for $\alpha < 0.5$,

$$\begin{aligned} &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \simeq 0 \\ &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \simeq 0. \end{aligned} \tag{11}$$

Similarly, the third approximate decision boundary, $r_1 \simeq -|h_1|\sqrt{(1-\alpha)P}$, is obtained from

$$\begin{aligned} &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} = \\ &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \end{aligned} \tag{12}$$

where we use the following observation, at $r_1 = -|h_1|\sqrt{(1-\alpha)P}$, for $\alpha < 0.5$,

$$\begin{aligned}&\frac{1}{2}\frac{1}{\sqrt{\pi N_0}}e^{-\frac{\left(r_1-|h_1|\sqrt{\alpha P}-|h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \simeq 0\\&\frac{1}{2}\frac{1}{\sqrt{\pi N_0}}e^{-\frac{\left(r_1+|h_1|\sqrt{\alpha P}-|h_1|\sqrt{(1-\alpha)P}\right)^2}{N_0}} \simeq 0.\end{aligned} \tag{13}$$

Then, the decision region for $s_1 = +1$ is given by

$$r_1 > 0, \quad \text{if } \alpha > 0.5. \tag{14)}$$

and

$$\begin{cases}-|h_1|\sqrt{(1-\alpha)P} < r_1 < 0\\ |h_1|\sqrt{(1-\alpha)P} < r_1\end{cases}, \quad \text{if } \alpha < 0.5. \tag{15}$$

The probability of error $P_e^{(ML;\,non\text{-}SIC)}$ for $\alpha > 0.5$ is calculated as

$$\begin{aligned}P_e^{(ML;\,non\text{-}SIC)} &= \frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}+\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right)\\&+\frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right)\end{aligned} \tag{16}$$

and for $\alpha < 0.5$,

$$\begin{aligned}P_e^{(ML;\,non\text{-}SIC)} \simeq\ & Q\left(\frac{|h_1|\sqrt{\alpha P}}{\sqrt{N_0/2}}\right)\\&+\frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}+2\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right)\\&+\frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}-2\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right)\\&-\frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}+\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right)\\&-\frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right).\end{aligned} \tag{17}$$

Ⅳ. Results and Discussions

Assume that the channel gain of the user-1 is $|h_1| = 1.7$. The total transmit signal power to one-sided power spectral density ratio is $P/N_0 = 30$. Then we define the signal-to-noise ratio (SNR) as $\gamma \triangleq |h_1|^2 \alpha P/N_0$. The probabilities of error $P_e^{(standard;\,SIC)}$ and $P_e^{(ML;\,non\text{-}SIC)}$ are shown in Fig. 1, with different power allocations, $0 \le \alpha \le 1$. As shown in Fig. 1, The performance of the non-SIC ML receiver is almost the same as that of the standard receiver with the SIC for $\gamma < 12.4$ dB, ($\alpha < 0.2$, up to about 10^{-8} in the probability of error), which is calculated by

$$\begin{aligned}12.4\text{ dB} &= 10\log_{10}\gamma\\&=10\log_{10}\left(|h_1|^2\alpha P/N_0\right)\\&=10\log_{10}\left(|1.7|^2\,0.2\cdot 30\right)\\&=10\log_{10}(17.34).\end{aligned} \tag{18}$$

Note that the normal operating range of α is much less than 0.5. Usually, the power allocated to the stronger user is often less than 20 %. In this case, the non-SIC ML receiver works well, compared to the standard receiver with the SIC, especially without the SIC complexity. So, the non-SIC ML scenario looks feasible. The local minimum of $P_e^{(ML;\,non\text{-}SIC)}$ is at $\alpha = 1/5$, which is obtained from

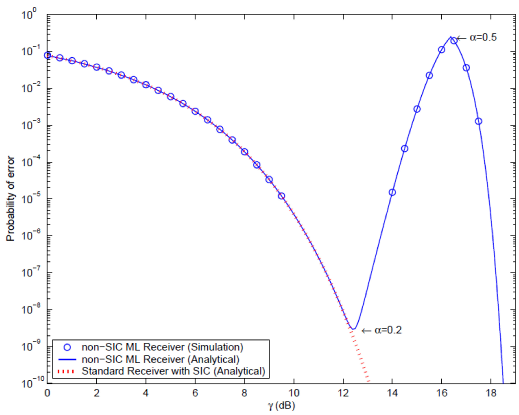

Fig. 1. Probabilities of error for the standard receiver with SIC and the non-SIC ML receiver.

$$\begin{aligned} &|h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P} \\ &= (-|h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P} \\ &\quad - |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}) \;/\; 2. \end{aligned} \tag{19}$$

The local maximum of $P_e^{(ML;\,non\text{-}SIC)}$ is at $\alpha = 0.5$, which is obtained from

$$\begin{aligned} &-|h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P} \\ &= |h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P}. \end{aligned} \tag{20}$$

We also show simulation results up to 10^{-5} in the probability of error, which are in good agreement with analytical results.

Ⅴ. Conclusion

We analyzed the performance of the ML receiver for the stronger channel user of NOMA, when no SIC was performed. This paper compared the non-SIC ML receiver to the standard receiver with the SIC. It was shown that the performance of the non-SIC ML receiver is almost the same as that of the standard receiver with the SIC for the operating range of the power allocation factor less than 20 %. In result, the non-SIC ML receiver could be a promising scheme for the NOMA stronger channel user, reducing the SIC complexity.

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