Non-SIC NOMA: Gaussian Mixture Channel Capacity under BPSK and QPSK Modulations

Kyuhyuk Chung◆

ABSTRACT

We propose non-orthogonal multiple access (NOMA) without the successive interference cancelation (SIC). This paper compares the channel capacity of the non-SIC NOMA to that of the standard NOMA. It is shown that the channel capacity of the non-SIC NOMA is almost the same as that of the standard NOMA for the operating range of the power allocation factor less than 20 % or greater than 80 %. In result, the non-SIC NOMA could be one of promising schemes, without the SIC complexity.

Key Words : Non-orthogonal multiple access, successive interference cancelation, channel capacity, binary phase shift keying, quadrature phase shift keying, power allocation

Ⅰ. Introduction

Non-orthogonal multiple access (NOMA) is one of promising technologies, which provides high system capacity and low latency, to become a new paradigm for fifth generation (5G) mobile networks [1-5]. In NOMA, the user with the better channel condition employs the successive interference cancelation (SIC) to remove the signals of users with the worse channel conditions. In this paper, NOMA without the SIC is proposed and the channel capacity of the non-SIC NOMA is compared to that of the standard NOMA. The paper is organized as follows. Section II defines the system and channel model. In Section III, the channel capacity for the non SIC NOMA is calculated. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{1}$$

If no SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} r_1 &= |h_1|\sqrt{\alpha P} s_1 + \left(|h_1|\sqrt{(1-\alpha)P} s_2 + n_1\right) = |h_1|\sqrt{\alpha P} s_1 + n_4 \\ r_2 &= |h_2|\sqrt{(1-\alpha)P} s_2 + \left(|h_2|\sqrt{\alpha P} s_1 + n_2\right) \\ &= |h_2|\sqrt{(1-\alpha)P} s_2 + n_3 \end{aligned} \tag{2}$$

where n_1 and $n_2 \sim \mathcal{N}(0, N_0/2)$ are additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ. In addition, n_3 and n_4 are defined as

$$\begin{aligned} n_3 &= |h_2|\sqrt{\alpha P} s_1 + n_2 \\ n_4 &= |h_1|\sqrt{(1-\alpha)P} s_2 + n_1 . \end{aligned} \tag{3}$$

If we assume the binary phase shift keying (BPSK) modulation, with $s_1, s_2 \in \{+1, -1\}$ then the probability density functions (PDFs) of n_3 and n_4 are given by

◆ First Author : (ORCID:0000-0001-5429-2254)Department of Software Science, Dankook University, khchung@dankook.ac.kr, 종신회원
논문번호 : 201901-418-A-LU, Received January 22, 2019; Revised February 26, 2019; Accepted February 26, 2019

$$p_{N_3}(n_3)=\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(n_3-|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}}+\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(n_3+|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \quad (4)$$

and

$$p_{N_4}(n_4)=\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(n_4-|h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}}+\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(n_4+|h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}}. \quad (5)$$

In the standard NOMA, the SIC is performed only on the user-1. Then the received signals are given by

$$\begin{aligned} y_1 &= |h_1|\sqrt{\alpha P}s_1+n_1 \\ y_2 &= r_2. \end{aligned} \quad (6)$$

Ⅲ. Non-SIC NOMA Channel Capacity

We calculate the channel capacity for the non SIC NOMA. Note that if we use the channel capacity with the ideal Gaussian modulation for both the signal and the inter user interference, the non-SIC NOMA looks not working[6]. However, when we use the channel capacity with the practical modulations, such as the BPSK and quadrature phase shift keying (QPSK) modulations, for both the signal and the inter user interference, the non-SIC NOMA is now unveiled and looks working[7,8]. In this paper, the channel capacity is defined as the maximum mutual information, maximized with an input PDF, without the shaping, (often called as the mutual information with equiprobable M-ary constellations[7]). The capacity of equiprobable M-ary constellations asymptotically approaches a straight line parallel to the capacity of the ideal Gaussian modulation, shifted right by $\pi e/6$ (1.53 dB), which is the shaping loss[6,7]. The capacity of equiprobable M-ary constellations saturates because information cannot be sent at a rate higher than $\log_2 M$. In Table I, we give the modulations for the capacities. For the standard NOMA, the channel capacities in bit/s/Hz for the user-1 and the user-2 are calculated as, with the ideal Gaussian modulation,

$$\begin{aligned} C_1 &= \log_2\left(1+\frac{|h_1|^2\alpha P}{N_0}\right) \\ C_2 &= \log_2\left(1+\frac{|h_2|^2(1-\alpha)P}{|h_2|^2\alpha P+N_0}\right). \end{aligned} \quad (7)$$

For the non-SIC NOMA, the channel capacities with the ideal Gaussian modulation for the user-1 and the user-2 are calculated as

$$\begin{aligned} C_1^{(non-SIC)} &= \log_2\left(1+\frac{|h_1|^2\alpha P}{|h_1|^2(1-\alpha)P+N_0}\right) \\ C_2^{(non-SIC)} &= C_2 . \end{aligned} \quad (8)$$

Note that the channel capacities C_2 and $C_2^{(non-SIC)}$ for the user-2 are the same for both the standard NOMA and the non-SIC NOMA. We summarize the channel capacity with the practical modulation for both the signal and the inter user interference of the standard NOMA, which is

Table 1. Modulations for Capacities

NOMA	capacity	user-1	user-2
standard	C_1	$Gaussian$	
	C_2	$Gaussian$	$Gaussian$
non-SIC	$C_1^{(non-SIC)}$	$Gaussian$	$Gaussian$
	C_2	$Gaussian$	$Gaussian$
standard	$C_1^{(q)}$	$QPSK$	
	$C_2^{(q)}$	$QPSK$	$QPSK$
non-SIC	$C_1^{(q)(non-SIC)}$	$QPSK$	$QPSK$
	$C_2^{(q)}$	$QPSK$	$QPSK$

presented for the BPSK modulation in [5]. The channel capacity for the user-1 is given by

$$C_1^{(b)}\left(P\right) = -\int_{-\infty}^{\infty} p_{Y_1}(y_1)\log_2\left(p_{Y_1}(y_1)\right)dy_1 - \frac{1}{2}\log_2\left(2\pi e N_0 / 2\right) \quad (9)$$

where

$$p_{Y_1}(y_1) = \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(y_1-|h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(y_1+|h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}}. \quad (10)$$

Note that $C_1^{(b)}\left(P\right)$ is the capacity of equiprobable *M*-ary constellations[7]. The channel capacity for the user-2 is given by

$$C_2^{(b)}\left(P\right) = -\int_{-\infty}^{\infty} p_{Y_2}(y_2)\log_2\left(p_{Y_2}(y_2)\right)dy_2 + \int_{-\infty}^{\infty} p_{N_3}(n_3)\log_2\left(p_{N_3}(n_3)\right)dn_3 \quad (11))$$

where

$$p_{Y_2}(y_2) = \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(y_2-|h_2|\sqrt{\alpha P}-|h_2|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(y_2+|h_2|\sqrt{\alpha P}-|h_2|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(y_2-|h_2|\sqrt{\alpha P}+|h_2|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(y_2+|h_2|\sqrt{\alpha P}+|h_2|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}}. \quad (12)$$

Note that n_3 is not Gaussian, but Gaussian mixture. So, $C_2^{(b)}$ is capacity for Gaussian mixture channel, not for AWGN channel. This observation is not clearly presented in [8]. The results in [8] can be easily extended to the non-SIC user-1 as

$$C_1^{(b)(non-SIC)}\left(P\right) = -\int_{-\infty}^{\infty} p_{R_1}(r_1)\log_2\left(p_{R_1}(r_1)\right)dr_1 + \int_{-\infty}^{\infty} p_{N_4}(n_4)\log_2\left(p_{N_4}(n_4)\right)dn_4 \quad (13)$$

where

$$p_{R_1}(r_1) = \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(r_1-|h_1|\sqrt{\alpha P}-|h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(r_1+|h_1|\sqrt{\alpha P}-|h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(r_1-|h_1|\sqrt{\alpha P}+|h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}e^{-\frac{\left(r_1+|h_1|\sqrt{\alpha P}+|h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}}. \quad (14))$$

Since the QPSK modulation amounts to two BPSK constellations in quadrature with the power evenly divided between them, we calculate the capacity for the QPSK modulation with the following equation,

$$C^{(q)}\left(P\right) = 2\,C^{(b)}\left(\frac{P}{2}\right). \quad (15)$$

Ⅳ. Results and Discussions

Assume that the channel gains are $\left|h_1\right| = 1.2$ and $\left|h_2\right| = 0.6$. The total transmit signal power to one-sided power spectral density ratio is $P / N_0 = 15$, ($11.76\text{ dB} = 10\log_{10}(15)$). This total power ratio is chosen to guarantee that two users transmit their information almost at the maximum rate of the QPSK modulation, i.e., at the rate of 2 bit/s. The channel capacities of the ideal

Gaussian modulation and the BPSK/QPSK modulations for the standard NOMA and the non-SIC NOMA are shown in Fig. 1, with different power allocations, $0 \leq \alpha \leq 1$. As shown in Fig. 1, for the BPSK/QPSK modulations, the channel capacity of the non-SIC NOMA is almost the same as that of the standard NOMA for the operating range of the power allocation factor less than 20 % or greater than 80 %. Note that if we use the channel capacity with the ideal Gaussian modulation for both the signal and the inter user interference, the non-SIC NOMA looks not working. However, when we use the channel capacity with the practical modulation for both the signal and the inter user interference, the non-SIC NOMA is now unveiled and looks working. In addition, the non-SIC NOMA with the BPSK/QPSK modulations suffers the capacity loss for $20\,\% \leq \alpha \leq 80\,\%$, as shown in Fig. 1, due to the non-SIC.

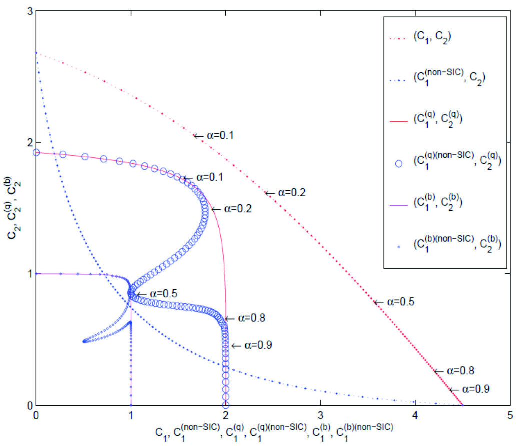

Fig. 1. Channel capacities of the ideal Gaussian modulation and the BPSK/QPSK modulations for the standard NOMA and the non-SIC NOMA with various power allocations.

V. Conclusion

We proposed the non-SIC NOMA. This paper compares the channel capacity of the non-SIC NOMA to that of the standard NOMA. It was shown that the channel capacity of the non-SIC NOMA is almost the same as that of the standard NOMA for the operating range of the power allocation factor less than 20 % or greater than 80 %. In result, the non-SIC NOMA could be one of promising schemes, without the SIC complexity.

References

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE 77th VTC Spring*, pp. 1-5, 2013.

[2] Z. Ding, P. Fan, and H. V. Poor, "Impact of user pairing on 5G nonorthogonal multiple-access downlink transmissions," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6010–6023, Aug. 2016.

[3] S. R. Islam, J. M. Kim, and K. S. Kwak, "On non-orthogonal multiple access (NOMA) in 5G systems," *J. KICS*, vol. 40, no. 12, pp. 2549-2558, Dec. 2015.

[4] M. H. Lee, V. C. M. Leung, and S. Y. Shin, "Dynamic bandwidth allocation of NOMA and OMA for 5G," *J. KICS*, vol. 42, no. 12, pp. 2383-2390, Dec. 2017.

[5] M. B. Uddin, M. F. Kader, A. Islam, and S. Y. Shin, "Power optimization of NOMA for multi-cell networks," *J. KICS*, vol. 43, no. 7, pp. 1182-1190, Jul. 2018.

[6] C. E. Shannon, "A mathematical theory of communication," *The Bell Syst. Tech. J.*, vol. 27, no. 3, pp. 379-423, Oct. 1948.

[7] G. D. Forney Jr. and G. Ungerboeck, "Modulation and coding for linear Gaussian channels," *IEEE Trans. Inf. Theory*, vol. 44, pp. 2384–2415, Oct. 1998.

[8] K. Chung (in press Fec. 2019), "On channel capacity for NOMA under BPSK modulation," *J. KICS*, vol. 44, no. 2. Accepted for publication.