논문 19-44-07-06 The Journal of Korean Institute of Communications and Information Sciences '19-07 Vol.44 No.07
https://doi.org/10.7840/kics.2019.44.7.1286

Quadrature Polar On-Off Keying in NOMA

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ABSTRACT

Beyond the perfect successive interference cancellation (SIC) performance, one of two users in non-orthogonal multiple access (NOMA) can be served orthogonally in power-domain with polar on-off keying (POOK), which is recently proposed in [6]. We extend the concept of POOK into 2-dimensional modulation constellations, just as the extension of binary phase shift keying (BPSK) into quadrature phase shift keying (QPSK). The suitable name for this modulation can be quadrature polar on-off keying (QPOOK).

Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, maximum likelihood, binary phase shift keying, quadrature phase shift keying, polar on-off keying

Ⅰ. Introduction

All communication resources have been used up except power for orthogonal multiple access (OMA). Therefore to use the power as the channel resource for multiple accesses, non-orthogonal multiple access (NOMA) [1-5] has been proposed recently to provide higher system capacity. However, the non-perfect successive interference cancellation (SIC) due to the severe effect of the superposition brings down NOMA performance. The latest solution for the non-perfect SIC performance is polar on-off keying (POOK) [6]. This paper expands POOK in 2-dimensional constellations, in quadrature, namely, quadrature polar on-off keying (QPOOK). The paper is organized as follows. Section II defines the system and channel model. In Section III, a brief review of POOK is summarized. In Section IV, the quadrature expansion of POOK is presented. In Section V, results are presented and discussed. The paper is concluded in Section VI.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{1}$$

Before the SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} z_1 &= h_1 \sqrt{\alpha P} s_1 + \left(h_1 \sqrt{(1-\alpha)P} s_2 + w_1 \right) \\ z_2 &= h_2 \sqrt{(1-\alpha)P} s_2 + \left(h_2 \sqrt{\alpha P} s_1 + w_2 \right) \end{aligned} \tag{2}$$

where w_1 and $w_2 \sim \mathcal{CN}(0, N_0)$ are complex additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The notation $\mathcal{CN}(\mu, \Sigma)$ denotes the complex circularly symmetric normal distribution with mean μ and variance Σ. In addition, if the channel gains are assumed to be Rayleigh faded, then h_1 and $h_2 \sim \mathcal{CN}(0, 1^2)$. The coherent receivers of Rayleigh fading channels construct the following metrics from the received signals;

$$\begin{aligned} h_1^* z_1 &= |h_1|^2 \sqrt{\alpha P} s_1 + \left(|h_1|^2 \sqrt{(1-\alpha)P} s_2 + h_1^* w_1 \right) \\ h_2^* z_2 &= |h_2|^2 \sqrt{(1-\alpha)P} s_2 + \left(|h_2|^2 \sqrt{\alpha P} s_1 + h_2^* w_2 \right). \end{aligned} \tag{3}$$

Furthermore, the receivers process the above metrics one step more;

$$\begin{aligned} \frac{h_1^*}{|h_1|} z_1 &= |h_1| \sqrt{\alpha P} s_1 + \left(|h_1| \sqrt{(1-\alpha)P} s_2 + \frac{h_1^*}{|h_1|} w_1 \right) \\ \frac{h_2^*}{|h_2|} z_2 &= |h_2| \sqrt{(1-\alpha)P} s_2 + \left(|h_2| \sqrt{\alpha P} s_1 + \frac{h_2^*}{|h_2|} w_2 \right). \end{aligned} \tag{4}$$

Note that the noise $\frac{h_1^*}{|h_1|} w_1$ and $\frac{h_2^*}{|h_2|} w_2$ have the same

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논문번호 : 201904-049-A-LU, Received April 19, 2019; Revised April 27, 2019; Accepted May 4, 2019

statistics as w_1 and w_2, because $\frac{h_1^*}{|h_1|} = e^{j\theta}$ with θ uniformly distributed. Moreover, if the 1-dimensional modulation constellation is considered, the following metrics are sufficient statistics;

$$\begin{aligned} r_1 &= \mathrm{Re}\left\{\frac{h_1^*}{|h_1|} z_1\right\} \\ &= |h_1|\sqrt{\alpha P}s_1 + \left(|h_1|\sqrt{(1-\alpha)P}s_2 + \mathrm{Re}\left\{\frac{h_1^*}{|h_1|} w_1\right\}\right) \\ &= |h_1|\sqrt{\alpha P}s_1 + \left(|h_1|\sqrt{(1-\alpha)P}s_2 + n_1\right) \\ r_2 &= \mathrm{Re}\left\{\frac{h_2^*}{|h_2|} z_2\right\} \\ &= |h_2|\sqrt{(1-\alpha)P}s_2 + \left(|h_2|\sqrt{\alpha P}s_1 + \mathrm{Re}\left\{\frac{h_2^*}{|h_2|} w_2\right\}\right) \\ &= |h_2|\sqrt{(1-\alpha)P}s_2 + \left(|h_2|\sqrt{\alpha P}s_1 + n_2\right) \end{aligned} \tag{5}$$

where n_1 and $n_2 \sim \mathcal{N}\left(0, N_0 / 2\right)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}\left(\mu, \Sigma\right)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signals are given by

$$\begin{aligned} y_1 &= |h_1|\sqrt{\alpha P}s_1 + n_1 \\ y_2 &= r_2. \end{aligned} \tag{6}$$

Ⅲ. Brief Review of POOK

We, now, briefly review POOK [6] in order to be ready to extend it into the 2-dimensional constellations. On-off keying (OOK) is the simplest modulation technique. The carrier is sent or not. Assume the binary phase shift keying (BPSK) modulation for the user-1, with $s_1 \in \{+1, -1\}$. Then POOK, with $s_2 \in \{+\sqrt{2}, 0, -\sqrt{2}\}$, is the inter user interference s_1 dependent OOK. The power is normalized as

$$\mathbb{E}\left[|s_2|^2\right] = \frac{1}{4}\left(+\sqrt{2}\right)^2 + \frac{1}{2}\left(\sqrt{0}\right)^2 + \frac{1}{4}\left(-\sqrt{2}\right)^2 = 1. \tag{7}$$

Compare the standard OOK, $s_{OOK} \in \{+\sqrt{2}, 0\}$ with

$$\mathbb{E}\left[|s_{OOK}|^2\right] = \frac{1}{2}\left(\sqrt{2}\right)^2 + \frac{1}{2}\left(0\right)^2 = 1. \tag{8}$$

If there exists interference, polar OOK gets away from interference in the direction from the origin to interference. Therefore we give polarity to OOK, with the information input bits for the user-1 and the user-2 being $b_1, b_2 \in \{0,1\}$, as

$$\begin{aligned} &\begin{cases} s_1(b_1 = 0) = +1 \\ s_1(b_1 = 1) = -1 \end{cases} \\ &\begin{cases} s_2(b_2 = 0 \mid b_1 = 0) = +\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 0) = 0 \end{cases} \quad \begin{cases} s_2(b_2 = 0 \mid b_1 = 1) = -\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 1) = 0. \end{cases} \end{aligned} \tag{9}$$

Ⅳ. Quadrature Expansion of POOK

We finish the brief review of POOK in the 1-dimensional constellation. Now it is extended into the 2-dimensional constellation, i.e., QPOOK. The key idea for such expansion is that "If there exists interference, polar OOK gets away from interference in the direction from the origin to interference as far as possible." The picture is now in 2-dimension; Let us start the 2-dimensional extension of POOK to obtain QPOOK. Assume the quadrature phase shift keying (QPSK) modulation for the user-1, with

$$s_1 \in \left\{+\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}, +\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}, -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}, -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right\}. \tag{10}$$

The normalized power is as follows, with

$$\begin{aligned} &\mathbb{E}\left[|s_1|^2\right] \\ &= \frac{1}{4}\left(\sqrt{\left(+\frac{1}{\sqrt{2}}\right)^2 + \left(+\frac{1}{\sqrt{2}}\right)^2}\right)^2 + \frac{1}{4}\left(\sqrt{\left(+\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}\right)^2 \\ &+ \frac{1}{4}\left(\sqrt{\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(+\frac{1}{\sqrt{2}}\right)^2}\right)^2 + \frac{1}{4}\left(\sqrt{\left(-\frac{1}{\sqrt{2}}\right)^2 - \left(+\frac{1}{\sqrt{2}}\right)^2}\right)^2 \\ &= 1. \end{aligned} \tag{11}$$

Then QPOOK, with

$$s_2 \in \left\{0, +1, -1, +j, -j, +1 + j, +1 - j, -1 + j, -1 - j\right\} \tag{12}$$

is the inter user interference s_1 dependent OOK. The power is normalized as

$$\begin{aligned} \mathbb{E}\left[|s_2|^2\right] &= \frac{4}{16}\left(\sqrt{(0)^2 + (0)^2}\right)^2 \\ &+ \frac{1}{16}\left(\sqrt{(+1)^2 + (0)^2}\right)^2 + \frac{1}{16}\left(\sqrt{(-1)^2 + (0)^2}\right)^2 \\ &+ \frac{1}{16}\left(\sqrt{(0)^2 + (+1)^2}\right)^2 + \frac{1}{16}\left(\sqrt{(0)^2 + (-1)^2}\right)^2 \\ &+ \frac{2}{16}\left(\sqrt{(+1)^2 + (+1)^2}\right)^2 + \frac{2}{16}\left(\sqrt{(+1)^2 + (-1)^2}\right)^2 \\ &+ \frac{2}{16}\left(\sqrt{(-1)^2 + (+1)^2}\right)^2 + \frac{2}{16}\left(\sqrt{(-1)^2 + (-1)^2}\right)^2 \\ &= 4 \times \frac{1}{16} + 4 \times \frac{2}{16} \times 2 \\ &= 1 \end{aligned} \tag{13}$$

If there exists the inter user interference s_1, QPOOK gets away from the interference in the direction from the origin to interference as far as possible. Therefore we give polarity to QPOOK, as

$$\begin{cases} s_2\left(s_1 = +\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = 0 \\ s_2\left(s_1 = +\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = +1 \\ s_2\left(s_1 = +\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = +j \\ s_2\left(s_1 = +\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = +1+j \end{cases} \begin{cases} s_2\left(s_1 = +\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = 0 \\ s_2\left(s_1 = +\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = +1 \\ s_2\left(s_1 = +\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = -j \\ s_2\left(s_1 = +\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = +1-j \end{cases} \tag{14}$$

$$\begin{cases} s_2\left(s_1 = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = 0 \\ s_2\left(s_1 = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = -1 \\ s_2\left(s_1 = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = +j \\ s_2\left(s_1 = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = -1+j \end{cases} \begin{cases} s_2\left(s_1 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = 0 \\ s_2\left(s_1 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = -1 \\ s_2\left(s_1 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = -j \\ s_2\left(s_1 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = -1-j. \end{cases}$$

V. Results and Discussions

Assume that the channel gains are $|h_1| = 1.1$ and $|h_2| = 0.9$ and the total transmit signal power to one-sided power spectral density ratio is $P / N_0 = 15$. For the probability of errors, if we assume Gray mapping for QPSK or QPOOK, the dominant QPSK or QPOOK symbol errors result in one bit error per symbol. In this case, we can compare two modulation schemes fairly with the probability of (symbol or bit) errors. In addition, since the 2-dimensional modulation amounts to the two 1-dimensional modulations in quadrature with the power evenly divided between them, we calculate the probabilities of errors for QPSK or QPOOK with the following equation,

$$P_e^{(QPSK \text{ or } QPOOK)}\left(P\right) = P_e^{(BPSK \text{ or } POOK)}\left(\frac{P}{2}\right). \tag{15}$$

Note that when we calculate the probabilities of errors of BPSK/POOK, we use $P_e^{(BPSK \text{ or } POOK)}\left(P\right)$, not $P_e^{(BPSK \text{ or } POOK)}\left(\frac{P}{2}\right)$ in the equation (15). Therefore, the actual comparisons are made with

$$\begin{cases} P_e^{(BPSK \text{ or } POOK)}\left(P\right) \\ \qquad \updownarrow \\ P_e^{(QPSK \text{ or } QPOOK)}\left(P\right) = P_e^{(BPSK \text{ or } POOK)}\left(\frac{P}{2}\right). \end{cases} \tag{16}$$

From the above equation (16), it is obvious that the performance of the 2-dimaensional modulations is worse than that of the 1-dimensional by 3 dB (a factor of $\frac{1}{2}$). Note that one QPSK or one QPOOK symbols transmit two bits per symbol, while one BPSK or one POOK symbols transmit one bit per symbol. Before the comparisons, let us clarify what systems are compared; for the standard NOMA, BPSK/BPSK NOMA in the 1-dimensional constellation and QPSK/QPSK NOMA in 2-dimensional constellation; for the orthogonal NOMA (O NOMA), BPSK/POOK NOMA [6] in the 1-dimensional constellation and QPSK/QPOOK NOMA (in this paper) in the 2-dimensional constellation. For more clarification, we summarize such systems in Table I. The probabilities of errors for the six receivers in Table I for the user-1 are compared in Fig. 1, with different power allocations, $0 \leq \alpha \leq 1$. As shown in Fig. 1, for all α, the performance of QPSK in O NOMA is better than that of QPSK in NOMA for the user-1, even better than the ideal QPSK performance. Such outstanding QPSK performance in O NOMA is the result of transforming effectively the inter user interference s_2 into the meaningful signal by QPOOK user-2. We also compare the probabilities of errors for the four receivers in Table I for the user-2, in Fig. 2. As shown in Fig. 2, the probability of errors of QPOOK for the user-2 is better than that of QPSK in NOMA for the user-2 for the power allocation factor greater than about 10% and less than about 70%. It is ordinary that NOMA operates usually on $10\% \leq \alpha \leq 70\%$. An additional comment on $10\% \leq \alpha \leq 70\%$ is that even if we increase the power enormously, the QPSK performance in NOMA never improve in the vicinity of $\alpha = 50\%$. However, the QPOOK performance in O NOMA improves linearly, as the power allocation factor decreases, i.e., the allocated power to the user-2 increases. Lastly, we prove that the BPSK/QPSK

Table 1. Modulations for NOMA / O NOMA

NOMA	modulation/receiver/reference	
	user-1	user-2
standard	BPSK/non-SIC ML/[7] BPSK/perfect SIC /[7]	BPSK/ML/[8]
orthogonal	BPSK/ML/[6]	POOK/ML/[6]
standard	QPSK/non-SIC ML/[7] QPSK/perfect SIC /[7]	QPSK/ML/[8]
orthogonal	QPSK/ML/[this paper]	QPOOK/ML/[this paper]

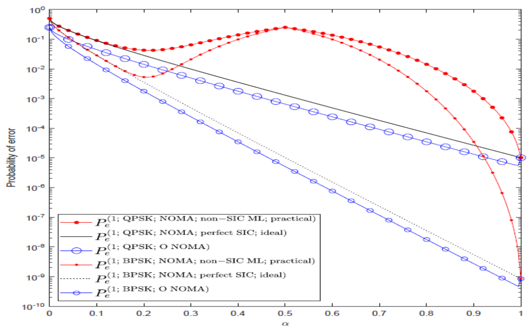

Fig. 1. Probabilities of errors of BPSK/QPSK in NOMA / O NOMA for the user-1.

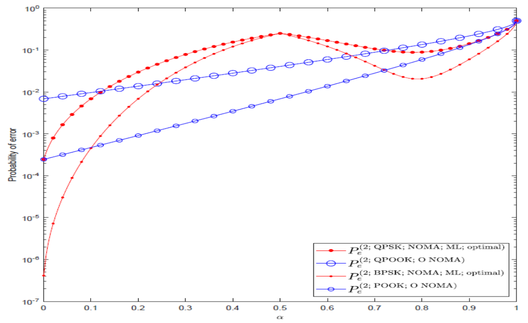

Fig. 2. Probabilities of errors of BPSK or POOK / QPSK or QPOOK in NOMA / O NOMA for the user-2.

performance in the standard NOMA for both the user-1 and the user-2 never improve in the vicinity of $\alpha = 50\%$.

Theorem 1: The probabilities of errors for the BPSK/QPSK in the standard NOMA for both the user-1 and the user-2 do not improve at the power allocation factor $\alpha = 0.5$, even though the total transmit power P approaches the infinity, i.e., $P = \infty$. In the limit, the probability of errors is $\frac{1}{4} = 0.25$.

Proof) First, let us consider the user-1.

The probability of error $P_e^{(1;\,BPSK;\,NOMA;\,non\text{-}SIC\ ML;\,practical)}$ for $\alpha > 0.5$ is given in [7], as

$$P_e^{(1;\,BPSK;\,NOMA;\,non\text{-}SIC\ ML;\,practical)} = \frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{\alpha}+\sqrt{(1-\alpha)}\right)}{\sqrt{N_0/2}}\right) \tag{17}$$

where $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz$.

We plug $\alpha = 0.5$ into the equation (17) and have the following constant specific value,

$$P_e^{(1;\,BPSK;\,NOMA;\,non\text{-}SIC\ ML;\,practical)} = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(2\sqrt{\frac{1}{2}}\right)}{\sqrt{N_0/2}}\right). \tag{18}$$

Then we have the limit as

$$\lim_{P\to\infty} P_e^{(1;\,BPSK;\,NOMA;\,non\text{-}SIC\ ML;\,practical)} = \frac{1}{4}. \tag{19}$$

The proof of $P_e^{(1;\,QPSK;\,NOMA;\,non\text{-}SIC\ ML;\,practical)}$ is straightforward based on the equation (15).

Next, let us consider the user-2.

The probability of error $P_e^{(2;\,BPSK;\,NOMA;\,ML;\,optimal)}$ for $\alpha < 0.5$ is given in [8], as

$$P_e^{(2;\,BPSK;\,NOMA;\,ML;\,optimal)} = \frac{1}{2}Q\left(\frac{|h_2|\sqrt{P}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{|h_2|\sqrt{P}\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right). \tag{20}$$

We plug $\alpha = 0.5$ into the equation (20) and have the following constant specific value,

$$P_e^{(2;\,BPSK;\,NOMA;\,ML;\,optimal)} = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}Q\left(\frac{|h_2|\sqrt{P}\left(2\sqrt{\frac{1}{2}}\right)}{\sqrt{N_0/2}}\right). \tag{21}$$

Then we have the limit as

$$\lim_{P\to\infty} P_e^{(2;\,BPSK;\,NOMA;\,ML;\,optimal)} = \frac{1}{4}. \tag{22}$$

The proof of $P_e^{(2;\,QPSK;\,NOMA;\,ML;\,optimal)}$ is straightforward based on the equation (15). *Q.E.D.*

Ⅵ. Conclusion

This paper expanded POOK into QPOOK. It was shown that POOK can be extended in quadrature and still preserves the outstanding features over the standard modulation schemes, such as QPSK. Consequently, NOMA with the help of QPOOK could be considered for 5G and beyond mobile networks.

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