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# Impact of Multilevel Modulation on Performance of NOMA Weak Channel User

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## ABSTRACT

New multiple access technique, non-orthogonal multiple access (NOMA), has been proposed recently for increasing the system capacity. Moreover, to increase the capacity, naturally, we consider the multilevel modulation to NOMA. In this paper, the impact of the multilevel modulation, such as 4-ary pulse amplitude modulation (4PAM), on the performance for the NOMA weak channel user is presented. The analysis of this paper is complete in that the performance is analyzed for the entire range of the power allocation factor. It is shown how much the NOMA performance with 4PAM degrades, compared to the orthogonal multiple access (OMA). In result, there are gain and loss; the gain is that two users can use the same channel resources, i.e., the system capacity becomes double and the loss is the performance degradation, which is shown analytically in this paper.

**Key Words** : Non-orthogonal multiple access, optimal detection, maximum likelihood, 4-ary pulse amplitude modulation, power allocation.

## Ⅰ. Introduction

To increase the system capacity for fifth generation (5G) and beyond networks, non-orthogonal multiple access (NOMA) has been considered[1-6]. Further to increase the rate of transmission, naturally we can consider the multilevel modulation, such as the 4-ary pulse amplitude modulation (4PAM), of which the quadrature version, e.g., quadrature amplitude modulation (QAM), is usually used in the practical cellular mobile radio multiple access networks. In [6], the single level modulation NOMA performance, such as binary phase shift keying (BPSK), is presented for the weaker channel user. In this paper, it is shown how much the NOMA maximum likelihood (ML) performance with 4PAM degrades, compared to the orthogonal multiple access (OMA). Before this paper starts, we should mention the practical considerations in NOMA; the NOMA principle is based on the fact that the more power is allocated to the weaker channel users and the less power is allocated to the stronger channel users, so that the user fairness is established.

The paper is organized as follows. Section II defines the system and channel model. In Section III, the performance of 4PAM NOMA is derived analytically for the user with the weaker channel condition. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

## Ⅱ. System and Channel Model

Assume that the total transmit power is $P$, the power allocation factor is $\alpha$ with $0 \le \alpha \le 1$, and the channel gains are $h_1$ and $h_2$ with $|h_1| > |h_2|$. Then $\alpha P$ is allocated to the user-1 signal $s_1$ and $(1-\alpha)P$ is allocated to the user-2 signal $s_2$, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The expectation notation $\mathbb{E}[u]$ is defined as

$$\mathbb{E}[u] = \int_{-\infty}^{\infty} u p_U(u)\, du \tag{1}$$

where $p_U(u)$ is the probability density function (PDF). The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{2}$$

After the successive interference cancellation (SIC) is performed on the user-1 with the better channel condition, the received signal of the strong channel user is given by

$$r_1 = |h_1| \sqrt{\alpha P} s_1 + n_1 \tag{3}$$

where $n_1 \sim \mathcal{N}(0, N_0/2)$ is additive white Gaussian

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noise (AWGN). The notation $\mathcal{N}(\mu,\Sigma)$ denotes the normal distribution with mean $\mu$ and variance $\Sigma$ and $N_0$ is one-sided power spectral density. The SIC is not performed on the user-2 with the worse channel condition. Then the received signal of the weak channel user is given by

$$\begin{aligned} r_2 &= \left|h_2\right|\left(\sqrt{\alpha P}s_1 + \sqrt{(1-\alpha)P}s_2\right) + n_2 \\ &= \left|h_2\right|\sqrt{(1-\alpha)P}s_2 + \left(\left|h_2\right|\sqrt{\alpha P}s_1 + n_2\right) \end{aligned} \tag{4}$$

where $n_2 \sim \mathcal{N}(0, N_0/2)$ is AWGN. We consider 4PAM, with

$$s_1, s_2 \in \left\{+\frac{3}{\sqrt{5}}, +\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -\frac{3}{\sqrt{5}}\right\} \tag{5}$$

## Ⅲ. 4PAM Receiver Performance Derivations

We derive the optimal receiver. The optimum detection is made, based on the ML, as

$$\hat{s}_2 = \underset{s_2 \in \left\{+\frac{3}{\sqrt{5}}, +\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -\frac{3}{\sqrt{5}}\right\}}{\arg\max} p_{R_2|S_2}(r_2 \mid s_2) \tag{6}$$

where the likelihoods are expressed by

$$\begin{aligned} & p_{R_2|S_2}(r_2 \mid s_2) \\ &= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P}s_2 - |h_2|\sqrt{\alpha P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P}s_2 - |h_2|\sqrt{\alpha P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P}s_2 + |h_2|\sqrt{\alpha P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P}s_2 + |h_2|\sqrt{\alpha P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}}. \end{aligned} \tag{7}$$

For $0 < \alpha < 0.1$, the decision region is given by,

$$\begin{cases} \text{for } s_2 = +\frac{3}{\sqrt{5}}, & \left|h_2\right|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_2 \\ \text{for } s_2 = +\frac{1}{\sqrt{5}}, & 0 < r_2 < \left|h_2\right|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\ \text{for } s_2 = -\frac{1}{\sqrt{5}}, & -\left|h_2\right|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_2 < 0 \\ \text{for } s_2 = -\frac{3}{\sqrt{5}}, & r_2 < -\left|h_2\right|\sqrt{\alpha P}\frac{2}{\sqrt{5}}. \end{cases} \tag{8}$$

with the one exact decision boundary, $r_2 = 0$, and the two approximate decision boundaries, $r_2 \simeq \pm\left|h_2\right|\sqrt{\alpha P}\frac{2}{\sqrt{5}}$. Then for $0 < \alpha < 0.1$,

$$P_e^{(2;\, M=4;\, NOMA;\, optimal\ ML)} \simeq 6 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \sum_{j=-2}^{1} q^{(1;2j+1)} \tag{9}$$

where for the simplification, we define the notation as

$$q^{(I;A)} = Q\left(\frac{\left|h_2\right|\sqrt{P}\left(\sqrt{(1-\alpha)}\frac{I}{\sqrt{5}} + \sqrt{\alpha}\frac{A}{\sqrt{5}}\right)}{\sqrt{N_0/2}}\right). \tag{10}$$

and $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz$. Note that the approximate sign happens due to the approximate decision boundaries for the ML detection. The decision boundary error is small and tolerable, resorting to the $68 - 95 - 99.7$ rule, for $\mathcal{N}(0, 1^2)$,

$$\begin{aligned} Q(3) &= \int_3^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz \simeq 0.0015 \\ Q(2) &= \int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz \simeq 0.025 \\ Q(1) &= \int_1^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz \simeq 0.16. \end{aligned} \tag{11}$$

Similarly, for $0.1 < \alpha < 0.2$,

$$\begin{aligned} & P_e^{(2;\, M=4;\, NOMA;\, optimal\ ML)} \\ & \simeq 6 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \left(\sum_{j=-1}^{1}\left(2q^{(1;2j)} - q^{(1;2j+1)}\right) + q^{(1;4)} + \sum_{j=3}^{4}(-1)^{j+1} q^{(-1;j)}\right) \end{aligned} \tag{12}$$

for $0.2 < \alpha < \frac{4}{13} \simeq 0.307$,

$$\begin{aligned} & P_e^{(2;\, M=4;\, NOMA;\, optimal\ ML)} \\ & \simeq 2 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \left(\sum_{j=-1}^{1}\left(3q^{(1;2j+1)} - 3q^{(1;2j+2)} + 2q^{(2;2j-1)}\right)\right. \\ & +2q^{(2;3)} + q^{(1;3)} + \sum_{j=2}^{3}(-1)^j q^{(-1;j)} + \sum_{j=2}^{4} 3(-1)^j q^{(-1;j)} \\ & \left. -2q^{(-1;5)} + 2q^{(2;-1)} + \sum_{j=-1}^{0}\left(2q^{(1;2j+1)} - 3q^{(1;2j+2)}\right)\right) \end{aligned} \tag{13}$$

for $\frac{4}{13} \simeq 0.307 < \alpha < 0.5$,

$$\begin{aligned} & P_e^{(2;\, M=4\ NOMA;\, optimal\ ML)} \\ & \simeq 2 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \left(\sum_{j=-1}^{1}\left(5q^{(1;2j+1)} - 2q^{(2;2j+1)}\right) + 3q^{(1;5)} + q^{(-1;5)}\right. \\ & + \sum_{j=-1}^{0}\left(4q^{(1;2j+1)} - 2q^{(1;2j+2)}\right) + 2q^{(-1;2)} - 6q^{(-1;3)} \\ & \left. +q^{(-1;4)} - 2q^{(-1;5)} + 2q^{(-2;3)} + q^{(1;3)} - q^{(1;4)}\right) \end{aligned} \tag{14}$$

for $0.5 < \alpha < \frac{9}{13} \simeq 0.693$,

$$\begin{aligned}
&P_e^{(2;\, M=4\ NOMA;\, optimal\ ML)} \\
&\simeq 2 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \Bigg( \sum_{j=-1}^{1} \left( 2q^{(3;2j)} - 2q^{(1;2j+3)} + 2q^{(-1;2j+3)} + q^{(+2;2j+3)} \right) \\
&+ \sum_{j=1}^{6} \left( (-1)^{j+1} q^{(-1;j)} + (-1)^{j} q^{(1;j)} \right) + \sum_{j=-1}^{0} (-1)^{j} 2q^{(1;-j)} \\
&+ q^{(3;4)} - q^{(-3;4)} + 3q^{(1;-1)} - q^{(1;1)} + q^{(2;-1)} \\
&+ \sum_{j=-1}^{0} \left( 2q^{(-1;2j+3)} - 2q^{(1;2j+3)} + q^{(2;2j+1)} - q^{(-2;2j+5)} + q^{(-1;2j+3)} - q^{(1;2j+3)} \right) \Bigg)
\end{aligned} \tag{15}$$

for $\frac{9}{13} \simeq 0.693 < \alpha < 0.8$,

$$\begin{aligned}
&P_e^{(2;\, M=4\ NOMA;\, optimal\ ML)} \\
&\simeq 2 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \Bigg( 2q^{(2;5)} + \sum_{j=-1}^{1} \left( 4q^{(2;2j+1)} - q^{(3;2j+2)} - q^{(1;2j+3)} + q^{(-1;2j+3)} \right) \\
&+ \sum_{j=1}^{6} \left( (-1)^{j+1} q^{(-1;j)} + (-1)^{j} q^{(1;j)} \right) - q^{(+1;-2)} - q^{(+3;-2)} + q^{(+2;-3)} + 2q^{(1;0)} + 2q^{(-1;1)} - 2q^{(1;1)} \\
&+ \sum_{j=-1}^{0} \left( 2q^{(2;2j+1)} - 2q^{(-2;2j+5)} - q^{(3;2j+2)} + q^{(-3;2j+4)} - q^{(1;2j+3)} + q^{(-1;2j+3)} \right) \Bigg)
\end{aligned} \tag{16}$$

for $0.8 < \alpha < 0.9$,

$$\begin{aligned}
&P_e^{(2;\, M=4\ NOMA;\, optimal\ ML)} \simeq \\
&2 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \Bigg( \sum_{i=0}^{1} \sum_{j=-1}^{1} \left( -(-1)^{i} 2q^{((-1)^{i} 2;2j+3)} \right) + q^{(3;5)} + q^{(1;6)} \\
&+ \sum_{j=-1}^{1} \left( 2q^{(3;2j+1)} + 2q^{(1;2j+2)} + q^{(1;2j+4)} - 2q^{(-1;2j+4)} \right) \\
&+ 3q^{(1;0)} - 2q^{(2;1)} - q^{(+2;-1)} + q^{(+1;-2)} - q^{(-3;3)} + q^{(-2;1)} \\
&+ \sum_{j=-1}^{0} \left( q^{(3;2j+1)} - 2q^{(2;2j+3)} \right) + \sum_{j=1}^{2} \left( q^{(1;2j-1)} - q^{(-1;2j)} \right) \\
&+ \sum_{j=1}^{2} \left( 2q^{(-2;2j-1)} - q^{(-3;2j+1)} \right) \Bigg)
\end{aligned} \tag{17}$$

for $0.9 < \alpha < 1$,

$$\begin{aligned}
&P_e^{(2;\, M=4;\, NOMA;\, optimal\ ML)} \\
&\simeq 2 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \Bigg( \sum_{j=1}^{3} \left( 3q^{(1;2j-2)} - 3q^{(-1;2j)} + q^{(-3;2j-1)} - q^{(3;2j-1)} \right) \\
&+ \sum_{j=-1}^{0} \left( 3q^{(1;2j+2)} - q^{(3;2j+3)} \right) + \sum_{j=0}^{3} 3q^{(1;2j)} \\
&+ \sum_{j=1}^{2} \left( q^{(-3;2j-1)} - 3q^{(-1;2j)} \right) + 3q^{(1;0)} - 3q^{(-1;2)} - q^{(3;1)} + q^{(-3;1)} \Bigg) \\
&\simeq 2 \cdot \frac{1}{4} \cdot \frac{1}{4} \Big( \\
&+q^{(+1;+0)} - q^{(+3;+1)} + q^{(+1;+2)} - q^{(+3;+3)} + q^{(+1;+4)} - q^{(+3;+5)} + q^{(+1;+6)} \\
&-q^{(-1;+2)} + q^{(-3;+1)} + q^{(+1;+0)} - q^{(+3;+1)} + q^{(+1;+2)} - q^{(+3;+3)} + q^{(+1;+4)} \\
&-q^{(-1;+4)} + q^{(-3;+3)} - q^{(-1;+2)} + q^{(-3;+1)} + q^{(+1;+0)} - q^{(+3;+1)} + q^{(+1;+2)} \\
&-q^{(-1;+6)} + q^{(-3;+5)} - q^{(-1;+4)} + q^{(-3;+3)} - q^{(-1;+2)} + q^{(-3;+1)} + q^{(+1;+0)} \\
&+q^{(+1;+0)} + q^{(+1;+0)} - q^{(-1;+2)} + q^{(+1;+2)} - q^{(-1;+4)} + q^{(+1;+4)} - q^{(-1;+6)} + q^{(+1;+6)} \\
&+q^{(+1;+2)} - q^{(-1;+2)} + q^{(+1;+0)} + q^{(+1;+0)} - q^{(-1;+2)} + q^{(+1;+2)} - q^{(-1;+4)} + q^{(+1;+4)} \\
&+q^{(+1;+4)} - q^{(-1;+4)} + q^{(+1;+2)} - q^{(-1;+2)} + q^{(+1;+0)} + q^{(+1;+0)} - q^{(-1;+2)} + q^{(+1;+2)} \\
&+q^{(+1;+6)} - q^{(-1;+6)} + q^{(+1;+4)} - q^{(-1;+4)} + q^{(+1;+2)} - q^{(-1;+2)} + q^{(+1;+0)} + q^{(+1;+0)} \Big).
\end{aligned} \tag{18}$$

Note that in the above equation (18) , we include the non-simplified expression on purpose, for having the better intuition about the equations, which are derived for each power allocation interval.

## Ⅳ. Results and Discussions

Assume that the channel gain of the user-2 is $|h_2| = 0.9$. For the fair comparison of 4PAM NOMA to 2PAM NOMA, we define the total transmit signal power per bit or per symbol as $E_b^{(total)}$ or $E_s^{(total)}$, respectively. Then one 4PAM symbol transmits two bits, so that $E_s^{(total)} = 2E_b^{(total)}$. Therefore, if for 2PAM or BPSK, $P = E_b^{(total)}$, then for 4PAM, $P = E_s^{(total)} = 2E_b^{(total)}$, where $P$ is the actual total transmit power per channel use. Furthermore, for the probability of errors, if we assume Gray mapping for 4PAM, the dominant 4PAM symbol error results in one bit error. In this case, we can compare two modulation schemes fairly. Assume that the total transmit signal power per bit to one-sided power spectral density ratio is $E_b^{(total)} / N_0 = 30$. The probability of errors for 2PAM NOMA $P_e^{(2;\, M=2\ NOMA;\, optimal\ ML)}$ is presented in [6]. In addition, we also compare NOMA to OMA; for 2PAM OMA,

$$P_e^{(2;\, M=2;\, OMA;\, optimal\ ML)} = Q\left( \frac{|h_2| \sqrt{(1-\alpha)P}}{\sqrt{N_0/2}} \right) \tag{19}$$

and for 4PAM OMA,

$$P_e^{(2;\, M=4;\, OMA;\, optimal\ ML)} = 6 \cdot \frac{1}{4} \cdot Q\left( \frac{|h_2| \sqrt{(1-\alpha)P}\, \frac{1}{\sqrt{5}}}{\sqrt{N_0/2}} \right). \tag{20}$$

Then the probabilities of errors are compared in Fig. 1. It is observed that the 4PAM performance degradation of NOMA over OMA is larger than the 2PAM performance degradation. Note, however, that NOMA serves two users on the same channel resources, while OMA serves only a single user on the given channel resources.

Lastly, as we mention in Section I., the NOMA

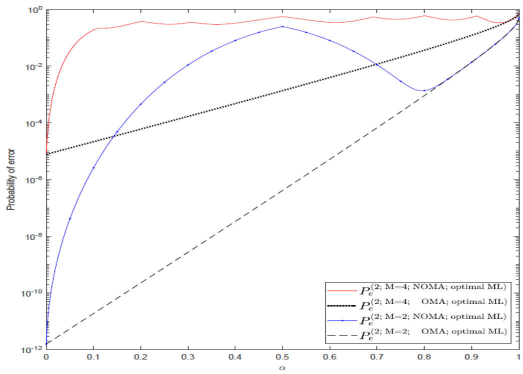


Fig. 1. Comparisons of probabilities of errors for various receivers.

operates on the user fairness principle, in which the weaker channel user with the more power faces with the stronger channel user with the less power; in that case, the inter user-2 interference, i.e., the weak power interference is practically ignored. This paper, however, analyzes the entire range of the power allocation. The authors are very careful for such analyses to mislead the readers to overlook the major NOMA principle.

## Ⅴ. Conclusion

First we derived the optimal ML receiver for 4-PAM in NOMA. Then we investigated the effect of the multilevel modulations to NOMA systems. It was shown how much the NOMA performance with 4PAM degrades, compared to the OMA. Consequently, there were gain and loss; the gain was that two users could use the same channel resources, i.e., the system capacity became double and the loss was the performance degradation, which was shown analytically in this paper.

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