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Performance Analysis on Non-SIC ML Receiver for NOMA Strong Channel User (Part II): 4PAM under Rayleigh Fading Channel

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ABSTRACT

In non-orthogonal multiple access (NOMA), successive interference cancellation (SIC) is the essential foundation for NOMA techniques. This paper derives the optimal performance for NOMA with 4-ary pulse amplitude modulation (4PAM) under Rayleigh fading channels. Such performances are calculated by first deriving the maximum-likelihood (ML) performance, conditioned on the channel gain, and then averaging the conditioned performance over the channel gain. As opposed to the standard SIC, which is by nature the hard-decision based SIC, we present the optimal ML performance, which can be viewed as the soft-decision based SIC, namely, the optimal soft SIC, because it is general that the soft-decision performance is better than the hard-decision performance. It is shown that the severe performance degradation of the optimal soft SIC can be mitigated, if we avoid the power allocation factors about 10%, 20%, 30%, 50%, 70%, 80%, 90%, under Rayleigh fading channel environments. In future researches, it is meaningful to achieve the perfect SIC performance for NOMA, because the NOMA capacity guarantees the perfect SIC performance for one of two users, regardless of the power allocation, i.e., for the entire range of the power allocation factor, from 0% to 100%.

Key Words : NOMA, Rayleigh fading channel, successive interference cancellation, 4-ary pulse amplitude modulation, maximum-likelihood, power allocation

Ⅰ. Introduction

5G mobile communication services have been commercialized in Korea, April 3, 2019, for the first time in the world. However, Samsung plans to develop its own 5G modem chip, and the international standardization is still in progress. One of key technologies in the 5G mobile network is non-orthogonal multiple access (NOMA)[1-6]. Recently, the performance on NOMA with binary phase shift keying (BPSK) has been analyzed[7,8]. In addition, in order to increase the system capacity, it is natural to consider the multilevel modulations, such as the 4-ary pulse amplitude modulation (4PAM)[9]. However, in [9], the performance of NOMA with 4PAM is presented for the fixed channel gain. This paper presents the optimal maximum-likelihood (ML) performance for the NOMA strong channel user with 4PAM under Rayleigh fading channels. Before we finish this introduction section, we should mention the practical issues on 4PAM; in practical communication systems, quadrature phase shift keying (QPSK) is used frequently. However, in the academic perspective, if we can calculate the performance of BPSK, of which the quadrature version is QPSK, we also obtain that of QPSK, i.e., the performance of BPSK with the half of power of QPSK (assuming Gray mapping). In the similar way, we can think 4PAM and 16-ary quadrature amplitude modulation (QAM). The paper is organized as follows. Section II defines the system and channel model. In Section III, the performance of 4PAM NOMA is derived analytically for the user with the stronger channel condition. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, ($0\% \le \alpha \le 100\%$), and the channel gains $h_1 \sim \mathcal{CN}(0, \Sigma_1)$

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and $h_2 \sim \mathcal{CN}(0,\Sigma_2)$ are Rayleigh faded, with $\Sigma_1 > \Sigma_2$. The notation $\mathcal{CN}(\mu,\Sigma)$ denotes the complex circularly-symmetric normal distribution with mean μ and variance Σ. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The expectation notation $\mathbb{E}[u]$ is defined as

$$\mathbb{E}[u] = \int_{-\infty}^{\infty} u p_U(u)\, du \tag{1}$$

where $p_U(u)$ is the probability density function (PDF). The superimposed signal is expressed by

$$x = \sqrt{\alpha P}s_1 + \sqrt{(1-\alpha)P}s_2. \tag{2}$$

Before the successive interference cancellation (SIC) is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} z_1 &= h_1\sqrt{\alpha P}s_1 + \left(h_1\sqrt{(1-\alpha)P}s_2 + w_1\right) \\ z_2 &= h_2\sqrt{(1-\alpha)P}s_2 + \left(h_2\sqrt{\alpha P}s_1 + w_2\right) \end{aligned} \tag{3}$$

where w_1 and $w_2 \sim \mathcal{CN}(0,N_0)$ are complex additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The coherent receivers of Rayleigh fading channels construct the following metrics from the received signals;

$$\begin{aligned} h_1^* z_1 &= |h_1|^2\sqrt{\alpha P}s_1 + \left(|h_1|^2\sqrt{(1-\alpha)P}s_2 + h_1^* w_1\right) \\ h_2^* z_2 &= |h_2|^2\sqrt{(1-\alpha)P}s_2 + \left(|h_2|^2\sqrt{\alpha P}s_1 + h_2^* w_2\right). \end{aligned} \tag{4}$$

Furthermore, the receivers process the above metrics one step more;

$$\begin{aligned} \frac{h_1^*}{|h_1|}z_1 &= |h_1|\sqrt{\alpha P}s_1 + \left(|h_1|\sqrt{(1-\alpha)P}s_2 + \frac{h_1^*}{|h_1|}w_1\right) \\ \frac{h_2^*}{|h_2|}z_2 &= |h_2|\sqrt{(1-\alpha)P}s_2 + \left(|h_2|\sqrt{\alpha P}s_1 + \frac{h_2^*}{|h_2|}w_2\right). \end{aligned} \tag{5}$$

Note that the noise $\frac{h_1^*}{|h_1|}w_1$ and $\frac{h_2^*}{|h_2|}w_2$ have the same statistics as w_1 and w_2, because $\frac{h_1^*}{|h_1|} = e^{j\theta}$ with θ uniformly distributed. Moreover, if the 1-dimensional modulation constellation is considered, the following metrics are sufficient statistics;

$$\begin{aligned} r_1 = \mathrm{Re}\left\{\frac{h_1^*}{|h_1|}z_1\right\} &= |h_1|\sqrt{\alpha P}s_1 + \left(|h_1|\sqrt{(1-\alpha)P}s_2 + \mathrm{Re}\left\{\frac{h_1^*}{|h_1|}w_1\right\}\right) \\ &= |h_1|\sqrt{\alpha P}s_1 + \left(|h_1|\sqrt{(1-\alpha)P}s_2 + n_1\right) \\ r_2 = \mathrm{Re}\left\{\frac{h_2^*}{|h_2|}z_2\right\} &= |h_2|\sqrt{(1-\alpha)P}s_2 + \left(|h_2|\sqrt{\alpha P}s_1 + \mathrm{Re}\left\{\frac{h_2^*}{|h_2|}w_2\right\}\right) \\ &= |h_2|\sqrt{(1-\alpha)P}s_2 + \left(|h_2|\sqrt{\alpha P}s_1 + n_2\right) \end{aligned} \tag{6}$$

where n_1 and $n_2 \sim \mathcal{N}(0,N_0/2)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu,\Sigma)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signal is given by, if the perfect SIC is assumed,

$$y_1 = r_1 - |h_1|\sqrt{(1-\alpha)P}s_2 = |h_1|\sqrt{\alpha P}s_1 + n_1. \tag{7}$$

We assume the 4PAM modulations for both users in the standard NOMA, i.e., the 4PAM/4PAM NOMA,

$$s_1, s_2 \in \left\{+\frac{3}{\sqrt{5}}, +\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -\frac{3}{\sqrt{5}}\right\}. \tag{8}$$

Ⅲ. 4PAM NOMA Performance Derivation

We first observe that the strong channel user performance can be obtained from the weak channel user performance by interchanging α and $(1-\alpha)$, with the proper changes of the channel parameters h_1 and h_2 h_2 [7,8]. Therefore the ML performance of the user-1, conditioned on the channel gain, can be represented as follows[9]; the optimum detection is made, based on the ML, as

$$\hat{s}_1 = \underset{s_1 \in \left\{+\frac{3}{\sqrt{5}},+\frac{1}{\sqrt{5}},-\frac{1}{\sqrt{5}},-\frac{3}{\sqrt{5}}\right\}}{\arg\max}\; p_{R_1|S_1}\left(r_1 \mid s_1\right) \tag{9}$$

where the likelihoods are expressed by

$$p_{R_1|S_1}\left(r_1 \mid s_1\right) = \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}\sum_{j=1}^{4} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}s_1 - |h_1|\sqrt{(1-\alpha)P}\frac{2j-5}{\sqrt{5}}\right)^2}{2N_0/2}}. \tag{10}$$

For $0 < \alpha < 0.1$, the decision region is given by, with the one exact decision boundary, r_1=0, and the fourteen approximate decision boundaries. Then for $0 < \alpha < 0.1$,

$$\begin{cases}
\text{for } s_1 = +\frac{3}{\sqrt{5}}, \\
\quad +|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 \\
\quad +|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 < +|h_1|\sqrt{(1-\alpha)P}\frac{2}{\sqrt{5}} \\
\quad -|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 < 0 \\
\quad -|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 < -|h_1|\sqrt{(1-\alpha)P}\frac{2}{\sqrt{5}} \\
\text{for } s_1 = +\frac{1}{\sqrt{5}}, \\
\quad +|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} < r_1 < +|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\
\quad +|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} < r_1 < +|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\
\quad -|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} < r_1 < -|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\
\quad -|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} < r_1 < -|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} + |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\
\text{for } s_1 = -\frac{1}{\sqrt{5}}, \\
\quad +|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 < +|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} \\
\quad +|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 < +|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} \\
\quad -|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 < -|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} \\
\quad -|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} < r_1 < -|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} \\
\text{for } s_1 = -\frac{3}{\sqrt{5}}, \\
\quad +|h_1|\sqrt{(1-\alpha)P}\frac{2}{\sqrt{5}} < r_1 < +|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\
\quad 0 < r_1 < +|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\
\quad -|h_1|\sqrt{(1-\alpha)P}\frac{2}{\sqrt{5}} < r_1 < -|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}} \\
\quad r_1 < -|h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}} - |h_1|\sqrt{\alpha P}\frac{2}{\sqrt{5}}
\end{cases} \tag{11}$$

$$\begin{aligned}
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4}\times\frac{1}{4}\times 2\times \Big[\\
&+q^{(+0;+1)} - q^{(+1;+3)} + q^{(+2;+1)} - q^{(+3;+3)} + q^{(+4;+1)} - q^{(+5;+3)} + q^{(+6;+1)} \\
&-q^{(+2;-1)} + q^{(+1;-3)} + q^{(+0;+1)} - q^{(+1;+3)} + q^{(+2;+1)} - q^{(+3;+3)} + q^{(+4;+1)} \\
&-q^{(+4;-1)} + q^{(+3;-3)} - q^{(+2;-1)} + q^{(+1;-3)} + q^{(+0;+1)} - q^{(+1;+3)} + q^{(+2;+1)} \\
&-q^{(+6;-1)} + q^{(+5;-3)} - q^{(+4;-1)} + q^{(+3;-3)} - q^{(+2;-1)} + q^{(+1;-3)} + q^{(+0;+1)} \\
&+q^{(+0;+1)} + q^{(+0;+1)} - q^{(+2;-1)} + q^{(+2;+1)} - q^{(+4;-1)} + q^{(+4;+1)} - q^{(+6;-1)} + q^{(+6;+1)} \\
&+q^{(+2;+1)} - q^{(+2;-1)} + q^{(+0;+1)} + q^{(+0;+1)} - q^{(+2;-1)} + q^{(+2;+1)} - q^{(+4;-1)} + q^{(+4;+1)} \\
&+q^{(+4;+1)} - q^{(+4;-1)} + q^{(+2;+1)} - q^{(+2;-1)} + q^{(+0;+1)} + q^{(+0;+1)} - q^{(+2;-1)} + q^{(+2;+1)} \\
&+q^{(+6;+1)} - q^{(+6;-1)} + q^{(+4;+1)} - q^{(+4;-1)} + q^{(+2;+1)} - q^{(+2;-1)} + q^{(+0;+1)} + q^{(+0;+1)} \Big]
\end{aligned} \tag{12}$$

where for the simplification, we define the notation as

$$q^{(I;A)} = Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{(1-\alpha)}\frac{I}{\sqrt{5}} + \sqrt{\alpha}\frac{A}{\sqrt{5}}\right)}{\sqrt{N_0/2}}\right) \tag{13}$$

and $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. Note that at $\alpha = 0.1$ and $r_1 = +|h_1|\sqrt{(1-\alpha)P}\frac{2}{\sqrt{5}}$, the dominant term of $p_{R_1|S_1}\left(r_1 \mid s_1 = +\frac{3}{\sqrt{5}}\right)$ is equal to that of $p_{R_1|S_1}\left(r_1 \mid s_1 = -\frac{3}{\sqrt{5}}\right)$, i.e.,

Thus now the decision region changes at $\alpha = 0.1$. Similarly, for $0.1 < \alpha < 0.2$,

Again note that at $\alpha = 0.2$ and $r_1 = +|h_1|\sqrt{\alpha P}\frac{1}{\sqrt{5}} + |h_1|\sqrt{(1-\alpha)P}\frac{2}{\sqrt{5}}$, the dominant term of $p_{R_1|S_1}\left(r_1 \mid s_1 = +\frac{3}{\sqrt{5}}\right)$ is equal to that of $p_{R_1|S_1}\left(r_1 \mid s_1 = -\frac{1}{\sqrt{5}}\right)$, i.e.,

Thus now the decision region changes at $\alpha = 0.2$.

$$\begin{aligned}
&p_{R_1|S_1}\left(r_1 \mid s_1 = +\frac{3}{\sqrt{5}}\right) \\
&\simeq \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\left(+\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(|h_1|\sqrt{\frac{9}{10}P}\frac{2}{\sqrt{5}} - |h_1|\sqrt{\frac{1}{10}P}\left(+\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{\frac{9}{10}P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}},
\end{aligned}
\qquad
\begin{aligned}
&p_{R_1|S_1}\left(r_1 \mid s_1 = -\frac{3}{\sqrt{5}}\right) \\
&\simeq \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\left(-\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(|h_1|\sqrt{\frac{9}{10}P}\frac{2}{\sqrt{5}} - |h_1|\sqrt{\frac{1}{10}P}\left(-\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{\frac{9}{10}P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}.
\end{aligned}
\tag{14}$$

$$\begin{aligned}
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4}\times\frac{1}{4}\times 2\times\Big[\\
&+q^{(+0;+1)} - q^{(+1;+2)} + q^{(+1;+3)} - q^{(+3;+2)} + q^{(+3;+3)} - q^{(+5;+2)} + q^{(+5;+3)} \\
&-q^{(+2;-1)} + q^{(+1;-2)} + q^{(-1;+3)} - q^{(+1;+2)} + q^{(+1;+3)} - q^{(+3;+2)} + q^{(+3;+3)} \\
&-q^{(+4;-1)} + q^{(+3;-2)} - q^{(+3;-3)} + q^{(+1;-2)} + q^{(-1;+3)} - q^{(+1;+2)} + q^{(+1;+3)} \\
&-q^{(+6;-1)} + q^{(+5;-2)} - q^{(+5;-3)} + q^{(+3;-2)} - q^{(+3;-3)} + q^{(+1;-2)} + q^{(-1;+3)} \\
&+q^{(+0;+1)} + q^{(+0;+1)} - q^{(+1;+2)} + q^{(+2;+1)} - q^{(+3;+2)} + q^{(+4;+1)} - q^{(+5;+2)} + q^{(+6;+1)} \\
&+q^{(+2;+1)} - q^{(+2;-1)} + q^{(+1;-2)} + q^{(+0;+1)} - q^{(+1;+2)} + q^{(+2;+1)} - q^{(+3;+2)} + q^{(+4;+1)} \\
&+q^{(+4;+1)} - q^{(+4;-1)} + q^{(+3;-2)} - q^{(+2;-1)} + q^{(+1;-2)} + q^{(+0;+1)} - q^{(+1;+2)} + q^{(+2;+1)} \\
&+q^{(+6;+1)} - q^{(+6;-1)} + q^{(+5;-2)} - q^{(+4;-1)} + q^{(+3;-2)} - q^{(+2;-1)} + q^{(+1;-2)} + q^{(+0;+1)} \Big].
\end{aligned}
\tag{15}$$

$$\begin{aligned}
&p_{R_1|S_1}\left(r_1 \mid s_1 = +\frac{3}{\sqrt{5}}\right) \\
&\simeq \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\left(+\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(|h_1|\sqrt{\frac{1}{5}P}\frac{1}{\sqrt{5}} + |h_1|\sqrt{\frac{4}{5}P}\frac{2}{\sqrt{5}} - |h_1|\sqrt{\frac{1}{5}P}\left(+\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{\frac{4}{5}P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}},
\end{aligned}
\qquad
\begin{aligned}
&p_{R_1|S_1}\left(r_1 \mid s_1 = -\frac{1}{\sqrt{5}}\right) \\
&\simeq \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\left(-\frac{1}{\sqrt{5}}\right) - |h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(|h_1|\sqrt{\frac{1}{5}P}\frac{1}{\sqrt{5}} + |h_1|\sqrt{\frac{4}{5}P}\frac{2}{\sqrt{5}} - |h_1|\sqrt{\frac{1}{5}P}\left(-\frac{1}{\sqrt{5}}\right) - |h_1|\sqrt{\frac{4}{5}P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}.
\end{aligned}
\tag{16}$$

Then, for $0.2 < \alpha < \frac{4}{13} \simeq 0.307$,

Remark that at $\alpha = \frac{4}{13}$ and $r_1 = +|h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}}$, the dominant term of $p_{R_1|S_1}\left(r_1 \mid s_1 = +\frac{3}{\sqrt{5}}\right)$ is equal to the dominant term of $p_{R_1|S_1}\left(r_1 \mid s_1 = -\frac{3}{\sqrt{5}}\right)$, i.e.,

Thus now the decision region changes at $\alpha = \frac{4}{13}$.

Then, for $\frac{4}{13} \simeq 0.307 < \alpha < 0.5$,

In the followings, the limits $\alpha = 0.5, \frac{9}{13}, 0.8, 0.9$ of intervals are similarly calculated. Then for $0.5 < \alpha < \frac{9}{13} \simeq 0.693$,

for $\frac{9}{13} \simeq 0.693 < \alpha < 0.8$,

for $0.8 < \alpha < 0.9$,

$$
\begin{aligned}
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times \Big[\\
&+q^{(+0;+1)} - q^{(+1;+1)} + q^{(+1;+2)} - q^{(+2;+3)} + q^{(+3;+2)} - q^{(+4;+3)} + q^{(+5;+2)} \\
&-q^{(+2;-1)} + q^{(+1;-1)} + q^{(-1;+2)} - q^{(+0;+3)} + q^{(+1;+2)} - q^{(+2;+3)} + q^{(+3;+2)} \\
&-q^{(+4;-1)} + q^{(+3;-1)} - q^{(+3;-2)} + q^{(+2;-3)} + q^{(-1;+2)} - q^{(+0;+3)} + q^{(+1;+2)} \\
&-q^{(+6;-1)} + q^{(+5;-1)} - q^{(+5;-2)} + q^{(+4;-3)} - q^{(+3;-2)} + q^{(+2;-3)} + q^{(-1;+2)} \\
&+q^{(+0;+1)} + q^{(+1;-1)} - q^{(+1;+1)} + q^{(+1;+2)} - q^{(+3;+1)} + q^{(+3;+2)} - q^{(+5;+1)} + q^{(+5;+2)} \\
&+q^{(+2;+1)} - q^{(+1;+1)} + q^{(+1;-1)} + q^{(-1;+2)} - q^{(+1;+1)} + q^{(+1;+2)} - q^{(+3;+1)} + q^{(+3;+2)} \\
&+q^{(+4;+1)} - q^{(+3;+1)} + q^{(+3;-1)} - q^{(+3;-2)} + q^{(+1;-1)} + q^{(-1;+2)} - q^{(+1;+1)} + q^{(+1;+2)} \\
&+q^{(+6;+1)} - q^{(+5;+1)} + q^{(+5;-1)} - q^{(+5;-2)} + q^{(+3;-1)} - q^{(+1;-2)} + q^{(+1;-1)} + q^{(-1;+2)} \Big].
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&p_{R_1|S_1}\left(r_1 \mid s_1 = +\frac{3}{\sqrt{5}}\right) \\
&\simeq \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\left(+\frac{3}{\sqrt{5}}\right) + |h_1|\sqrt{(1-\alpha)P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(+|h_1|\sqrt{\frac{9}{13}P}\frac{1}{\sqrt{5}} - |h_1|\sqrt{\frac{4}{13}P}\left(+\frac{3}{\sqrt{5}}\right) + |h_1|\sqrt{\frac{9}{13}P}\frac{1}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}},
\end{aligned}
\qquad
\begin{aligned}
&p_{R_1|S_1}\left(r_1 \mid s_1 = -\frac{3}{\sqrt{5}}\right) \\
&\simeq \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\left(-\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{(1-\alpha)P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(+|h_1|\sqrt{\frac{9}{13}P}\frac{1}{\sqrt{5}} - |h_1|\sqrt{\frac{4}{13}P}\left(-\frac{3}{\sqrt{5}}\right) - |h_1|\sqrt{\frac{9}{13}P}\frac{3}{\sqrt{5}}\right)^2}{2N_0/2}} \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}.
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times \Big[\\
&+q^{(+0;+1)} - q^{(+1;+1)} + q^{(+1;+2)} - q^{(+3;+1)} + q^{(+2;+3)} - q^{(+5;+1)} + q^{(+4;+3)} \\
&-q^{(+2;-1)} + q^{(+1;-1)} + q^{(-1;+2)} - q^{(+1;+1)} + q^{(+0;+3)} - q^{(+3;+1)} + q^{(+2;+3)} \\
&-q^{(+4;-1)} + q^{(+3;-1)} - q^{(+3;-2)} + q^{(+1;-1)} + q^{(-2;+3)} - q^{(+1;+1)} + q^{(+0;+3)} \\
&-q^{(+6;-1)} + q^{(+5;-1)} - q^{(+5;-2)} + q^{(+3;-1)} - q^{(+4;-3)} + q^{(+1;-1)} + q^{(-2;+3)} \\
&+q^{(+0;+1)} + q^{(+1;-1)} - q^{(+1;+1)} + q^{(+3;-1)} - q^{(+3;+1)} + q^{(+5;-1)} - q^{(+5;+1)} + q^{(+5;+2)} \\
&+q^{(+2;+1)} - q^{(+1;+1)} + q^{(+1;-1)} + q^{(+1;-1)} - q^{(+1;+1)} + q^{(+3;-1)} - q^{(+3;+1)} + q^{(+3;+2)} \\
&+q^{(+4;+1)} - q^{(+3;+1)} + q^{(+3;-1)} - q^{(+1;+1)} + q^{(+1;-1)} + q^{(+1;-1)} - q^{(+1;+1)} + q^{(+1;+2)} \\
&+q^{(+6;+1)} - q^{(+5;+1)} + q^{(+5;-1)} - q^{(+3;+1)} + q^{(+3;-1)} - q^{(+1;+1)} + q^{(+1;-1)} + q^{(-1;+2)} \Big].
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4}\times\frac{1}{4}\times 2\times 3\times\Big[\\
+q^{(+2;+1)} - q^{(+3;+1)} + q^{(+4;+1)} + q^{(+0;+1)} - q^{(+1;+1)} + q^{(+2;+1)} \\
+q^{(-2;+1)} - q^{(-1;+1)} + q^{(+0;+1)} - q^{(+4;-1)} + q^{(+3;-1)} + q^{(-2;+1)} \Big]
\end{aligned}
\tag{22}
$$

for $0.9 < \alpha < 1$,

Then the fading performance is calculated by

Here, $Q(x)$ can be represented as

$$
Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}} d\theta. \tag{25}
$$

Thus we can use the well-known Rayleigh fading integration formula,

$$
\begin{aligned}
&\int_0^\infty q^{(I;A)} \frac{1}{\gamma_b} e^{-\frac{\gamma}{\gamma_b}}\, d\gamma \\
&= \int_0^\infty Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{(1-\alpha)}\frac{I}{\sqrt{5}} + \sqrt{\alpha}\frac{A}{\sqrt{5}}\right)}{\sqrt{N_0/2}} \right) \frac{1}{\gamma_b} e^{-\frac{\gamma}{\gamma_b}}\, d\gamma \\
&= \int_0^\infty Q\left(\sqrt{2\frac{|h_1|^2 P\left(\sqrt{(1-\alpha)}\frac{I}{\sqrt{5}} + \sqrt{\alpha}\frac{A}{\sqrt{5}}\right)^2}{N_0}} \right) \frac{1}{\gamma_b} e^{-\frac{\gamma}{\gamma_b}}\, d\gamma \\
&= \int_0^\infty Q\left(\sqrt{2\gamma}\right) \frac{1}{\gamma_b} e^{-\frac{\gamma}{\gamma_b}}\, d\gamma = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right)
\end{aligned}
\tag{26}
$$

where the random variable (RV) γ is exponentially distributed and the mean of γ is defined as

$$
\begin{aligned}
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4}\times\frac{1}{4}\times 2\times\Big[\\
+ q^{(+1;+1)} - q^{(+2;+1)} + q^{(+3;+1)} - q^{(+3;+2)} + q^{(+5;+1)} \\
+ q^{(-1;+1)} - q^{(+0;+1)} + q^{(+1;+1)} - q^{(+1;+2)} + q^{(+3;+1)} \\
- q^{(+3;-1)} + q^{(+2;-1)} + q^{(-1;+1)} - q^{(-1;+2)} + q^{(+1;+1)} \\
- q^{(+5;-1)} + q^{(+4;-1)} - q^{(+3;-1)} + q^{(+3;-2)} + q^{(-1;+3)} \\
+q^{(-1;+1)} + q^{(+2;-1)} - q^{(+3;-1)} + q^{(+1;+1)} - q^{(+5;-1)} + q^{(+3;+1)} - q^{(+3;+2)} + q^{(+5;+1)} \\
+q^{(+1;+1)} - q^{(-0;+1)} + q^{(-1;+1)} + q^{(-1;+1)} - q^{(+3;-1)} + q^{(+1;+1)} - q^{(+1;+2)} + q^{(+3;+1)} \\
+q^{(+3;+1)} - q^{(+2;+1)} + q^{(+1;+1)} - q^{(+3;-1)} + q^{(-1;+1)} + q^{(-1;+1)} - q^{(-1;+2)} + q^{(+1;+1)} \\
+q^{(+5;+1)} - q^{(+4;+1)} + q^{(+3;+1)} - q^{(+5;-1)} + q^{(+1;+1)} - q^{(+3;-1)} + q^{(+3;-2)} + q^{(-1;+1)} \Big]
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4}\times\frac{1}{4}\times 2\times\Big[\\
+ q^{(+1;+1)} - q^{(+2;+1)} + q^{(+3;+1)} - q^{(+4;+1)} + q^{(+3;+2)} \\
+ q^{(-1;+1)} - q^{(+0;+1)} + q^{(+1;+1)} - q^{(+2;+1)} + q^{(+1;+2)} \\
- q^{(+3;-1)} + q^{(+2;-1)} + q^{(-1;+1)} - q^{(+0;+1)} + q^{(-1;+2)} \\
- q^{(+5;-1)} + q^{(+4;-1)} - q^{(+3;-1)} + q^{(+2;-1)} + q^{(-3;+2)} \\
+q^{(-1;+1)} + q^{(+2;-1)} - q^{(+3;-1)} + q^{(+4;-1)} - q^{(+2;+1)} + q^{(+3;+1)} - q^{(+4;+1)} + q^{(+3;+2)} \\
+q^{(+1;+1)} - q^{(-0;+1)} + q^{(-1;+1)} + q^{(+2;-1)} - q^{(+0;+1)} + q^{(+1;+1)} - q^{(+2;+1)} + q^{(+1;+2)} \\
+q^{(+3;+1)} - q^{(+2;+1)} + q^{(+1;+1)} - q^{(-0;+1)} + q^{(+2;-1)} + q^{(-1;+1)} - q^{(+0;+1)} + q^{(-1;+2)} \\
+q^{(+5;+1)} - q^{(+4;+1)} + q^{(+3;+1)} - q^{(+2;+1)} + q^{(+4;-1)} - q^{(+3;-1)} + q^{(+2;-1)} + q^{(-3;+2)} \Big]
\end{aligned}
\tag{21}
$$

$$
P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \simeq \frac{1}{4}\times\frac{1}{4}\times 2\times 3\times\Big[+ q^{(+3;+1)} + q^{(+1;+1)} + q^{(-1;+1)} + q^{(-3;+1)} \Big]. \tag{23}
$$

$$
P_{e}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} = \mathbb{E}_{h_1}\left[P_{e|h_1}^{(1;\, M=4;\, NOMA;\, optimal\ ML)} \right]. \tag{24}
$$

$$\gamma_b = \mathbb{E}[\gamma] = \mathbb{E}\left[\frac{|h_1|^2 P\left(\sqrt{(1-\alpha)}\frac{I}{\sqrt{5}} + \sqrt{\alpha}\frac{A}{\sqrt{5}}\right)^2}{N_0}\right]$$
$$= \frac{\Sigma_1 P\left(\sqrt{(1-\alpha)}\frac{I}{\sqrt{5}} + \sqrt{\alpha}\frac{A}{\sqrt{5}}\right)^2}{N_0}. \quad (27)$$

Ⅳ. Results and Discussions

Assume that $\Sigma_1 = (2.0)^2$ and the total transmit signal power to one-sided power spectral density ratio $P / N_0 = 50\text{ dB}, 30\text{ dB}$. In Fig. 1 and 2, we compare $P_e^{(1;\, M=4;\, NOMA;\, optimal\ ML)}$ to $P_e^{(1;\, M=4;\, NOMA;\, ideal\ perfect\ SIC)}$, which is given by

$$P_e^{(1;\, M=4;\, NOMA;\, ideal\ perfect\ SIC)} = \frac{1}{2}\left(1 - \sqrt{\frac{\frac{\alpha P \Sigma_1}{N_0}}{1 + \frac{\alpha P \Sigma_1}{N_0}}}\right). \quad (28)$$

As shown in Fig. 1 and 2, if the power allocation factors $\alpha \simeq 0.1, 0.2, 0.3, 0.5, 0.7, 0.8, 0.9$ are avoided, the severe performance degradation of the optimal ML receiver can be mitigated. The performance degradation with respect to the ideal perfect SIC receiver is due to the fact that in NOMA, two users share the channel resources, so that each user signal acts as the inter user interference to the other user signal. And the performance changes of the optimal ML receiver according to the power allocation α in Fig. 1 and 2 occur, because the decision boundaries of the ML detection also change irregularly, dependent on the power allocation α.

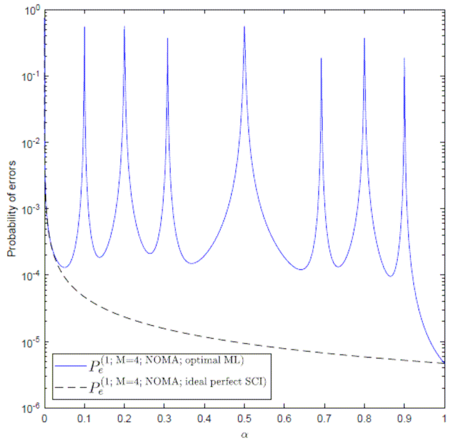

Fig. 1. Comparison of probabilities of errors for optimal ML receiver and ideal perfect SIC receiver, ($P / N_0 = 50\text{ dB}$).

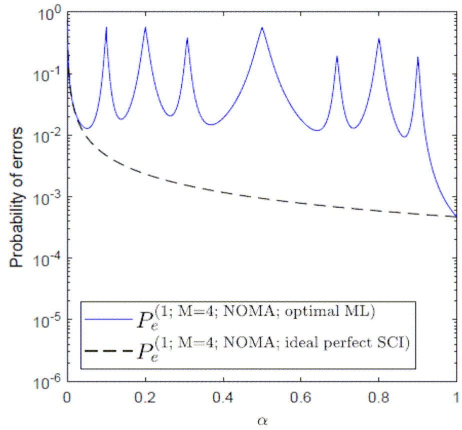

Fig. 2. Comparison of probabilities of errors for optimal ML receiver and ideal perfect SIC receiver, ($P / N_0 = 30\text{ dB}$).

Ⅴ. Conclusion

First, we derived the average optimal ML performance of the strong channel user in non-SIC NOMA, under Rayleigh fading channels. Then the optimal ML performance was compared to the ideal perfect SIC performance. In result, some power allocation factors should be avoided, due to the severe performance degradation.

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