논문 20-45-09-10 The Journal of Korean Institute of Communications and Information Sciences '20-09 Vol.45 No.09
https://doi.org/10.7840/kics.2020.45.9.1562

On BER-Based Calculation of Power Allocation under User Fairness for NOMA

Kyuhyuk Chung◆

ABSTRACT

In this letter, the authors derive a closed-form expression for the power allocation, which is based on the exhaustive search, under the constraint of the user fairness of the equal bit-error rates (BERs), in the binary phase shift keying (BPSK) non-orthogonal multiple access (NOMA). The power allocation calculation is based on the BER, not the achievable data rate. To the best of the authors' knowledge, this is the first analytical expression for the calculation of power allocation in the conventional BPSK NOMA.

Key Words : NOMA, user fairness, superposition coding, SIC, power allocation.

Ⅰ. Introduction

Recently, non-orthogonal multiple access (NOMA) has received considerable attention for the fifth generation (5G) mobile networks[1-6]. In NOMA, one frequency or time channel is allocated to several users within the cell. At the same time, in order to establish the user fairness, the power allocation between the users is significant.

The reason why the power optimization problem is significant, under the user fairness, because the two users share the channel resources, so that it might be desirable for the two users' performances to be equal.

In this letter, we derive a closed-form expression for the power allocation, under the user fairness of the equal bit-error rates (BERs). Then the analytical expression is validated by simulation results. Also, for the various channel gain scenarios, we present the analytical results of the power allocation.

The similar problem of the equal BERs was considered in [8]. By the sub-optimal algorithm, the solution was presented in [8], while in this letter, we solve the quartic equation, and obtain the exact solution.

Ⅱ. System and Channel Model

We consider a cellular downlink NOMA transmission system, in which two users are paired from a base station within the cell. The Rayleigh fading channel between the mth user and the base station is denoted by $h_m \sim \mathcal{CN}(0,\Sigma_m)$, $m=1,2$. The channels are sorted as $\Sigma_1 > \Sigma_2$. The base station will send the superimposed signal $x=\sqrt{\alpha P}s_1+\sqrt{(1-\alpha)P}s_2$, where s_m is the message for the mth user with unit power, α is the power allocation factor, with $0\le\alpha\le 1$, and P is the constant total transmitted power at the base station. The observation at the mth user is given by

$$r_m = |h_m| x + n_m, \tag{1}$$

where $n_m \sim \mathcal{N}(0, N_0/2)$ is additive white Gaussian noise (AWGN). Then, the perfect successive interference cancellation (SIC) BER performance of the 1st user is simply that of the binary phase shift keying (BPSK) modulation, which is given by

$$P_e^{(1;\, NOMA;\, perfect\ SIC;\, ideal)} = F\left(\frac{\Sigma_1 P\alpha}{N_0}\right), \tag{2}$$

where for the compact presentation of Rayleigh fading BER performance, we use the notation as

$$F(\gamma_b) = \frac{1}{2}\left(1-\sqrt{\frac{\gamma_b}{1+\gamma_b}}\right). \tag{3}$$

◆ First Author : (ORCID:0000-0001-5429-2254)Department of Software Science, Dankook University, khchung@dankook.ac.kr, 교수, 종신회원
논문번호 : 202005-114-A-LU, Received May 29, 2020; Revised June 28, 2020; Accepted July 1, 2020

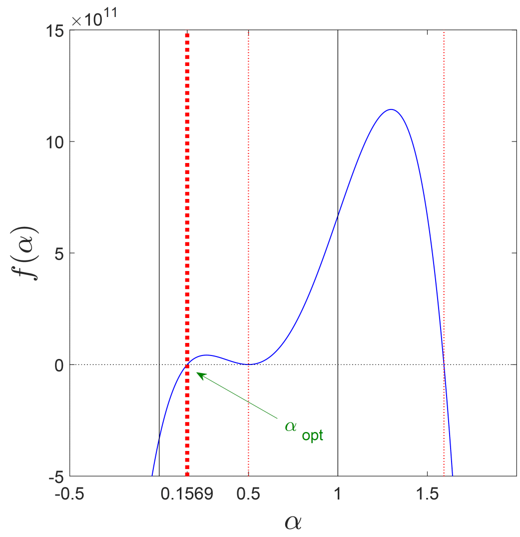

Fig. 1. Quartic function $f(\alpha)$ and real root α_{opt}.

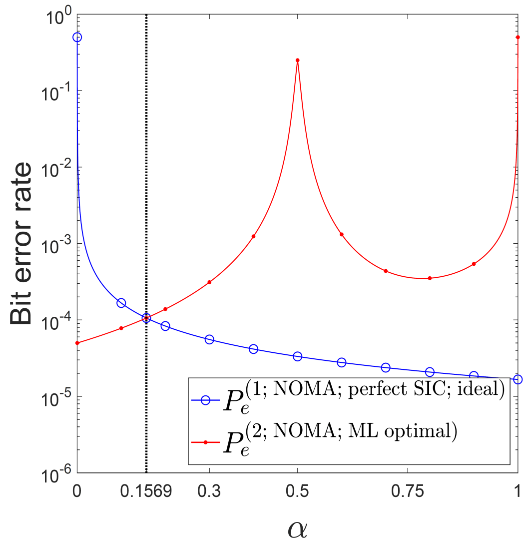

Fig. 2. Optimal power allocation $\alpha_{\text{opt}}^{(simulation)}$ by simulation.

In NOMA, the ideal perfect SIC is validated theoretically by the random channel coding theorem, practically such as low-density parity-check (LDPC) codes or turbo codes. And the optimal maximum likelihood (ML) BER performance of the 2nd user are given by [7], for $\alpha < 0.5$,

$$P_e^{(2;\,NOMA;\,ML\ optimal)} = \frac{1}{2}F\left(\frac{\Sigma_2 P\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2}{N_0}\right)+\frac{1}{2}F\left(\frac{\Sigma_2 P\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)^2}{N_0}\right). \tag{4}$$

Note that under the user fairness, we consider only $\alpha < 0.5$. Therefore, the equation is sufficient.

Ⅲ. Practical Power Calculation

The power calculation problem can be formulated as

$$\begin{aligned}&\text{Opimize } \alpha,\\ &\quad \text{s.t.}\\ &P_e^{(1;\,NOMA;\,perfect\ SIC;\,ideal)} = P_e^{(2;\,NOMA;\,ML\ optimal)}\end{aligned} \tag{5}$$

subject to $\alpha < 0.5$, for user fairness.

Then after some algebraic manipulation, we have the following quartic equation

$$f(\alpha) = \alpha^4 + p\alpha^3 + q\alpha^2 + r\alpha + s = 0, \tag{6}$$

where

$$p = \frac{-3\Sigma_1 P N_0 - 4\Sigma_1 P \Sigma_2 P - 32(\Sigma_2 P)^2}{16(\Sigma_2 P)^2}, \tag{7}$$

$$q = \frac{6N_0^2 + \Sigma_1 P(3N_0 + 4\Sigma_2 P) + 12\Sigma_2 P N_0 + 24(\Sigma_2 P)^2}{16(\Sigma_2 P)^2}, \tag{8}$$

$$r = -\frac{\Sigma_1 P(N_0 + \Sigma_2 P)^3}{16(\Sigma_2 P)^4} - \frac{-N_0^3 + 6\Sigma_1 P N_0^2 + 12\Sigma_1 P \Sigma_2 P N_0 + 8\Sigma_1 P(\Sigma_2 P)^2}{16\Sigma_1 P(\Sigma_2 P)^2}, \tag{9}$$

and

$$s = \frac{\Sigma_1\left(N_0 + \Sigma_2 P\right)^3 - \Sigma_2 N_0^3}{16\Sigma_1(\Sigma_2 P)^3}. \tag{10}$$

Then, for $0 \le \alpha \le 0.5$, we have only one real root

$$\alpha_{\text{opt}} = -\frac{p}{4} + \frac{1}{2}\sqrt{\frac{1}{4}p^2 - q + z_1} - \frac{1}{2}\sqrt{\frac{3}{4}p^2 - \left(\frac{1}{4}p^2 - q + z_1\right) - 2q + \frac{1}{4}\left(4pq - 8r - p^3\right)\frac{1}{\sqrt{\frac{1}{4}p^2 - q + z_1}}}, \tag{11}$$

where

$$z_1 = -\frac{(-q)}{3} + \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} + \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}, \tag{12}$$

with

$$a = \frac{1}{3}\left(3(pr - 4s) - (-q)^2\right),$$

$$b = \frac{1}{27}\left(2(-q)^3 - 9(-q)(pr - 4s) + 27\left(4qs - r^2 - p^2 s\right)\right). \tag{13}$$

Ⅳ. Results and Discussions

Assume $\Sigma_1 = 1.5$ and $\Sigma_2 = 0.5$, and the constant total transmitted signal power to noise power ratio $P/N_0 = 40\ \text{dB}$. First, in Fig. 1, we depict the quartic function $f(\alpha)$ in , and verify only one real root α_{opt}, i.e., the exact analytical optimal power allocation, in $0 \le \alpha \le 0.5$,

$$\alpha_{\text{opt}} = 0.156936577864388. \tag{14}$$

One additional comment on Fig. 1 is that $\alpha = 0.5$ looks like a root. However, $\alpha = 0.5$ is not a root, but the real part of the two complex conjugate non-real roots. Specifically, the quartic equation has two distinct real roots and two complex conjugate non-real roots, because the following condition holds

$$\Delta < 0, \tag{15}$$

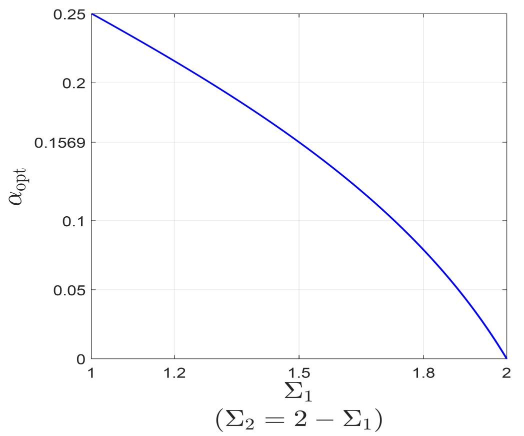

Fig. 3. Optimal power allocation α_{opt}, for various channels.

where

$$\begin{aligned}\Delta &= 256s^3 - 192prs^2 - 128qs^2 + 144qr^2s - 27r^4 \\ &+ 144p^2qs^2 - 6p^2r^2s - 80pq^2rs + 18pqr^3 + 16q^4s \\ &- 4q^3r^2 - 27p^4s^2 + 18p^3qrs - 4p^3r^3 - 4p^2q^3s + p^2q^2r^2.\end{aligned} \tag{16}$$

The calculated four roots are given by

$$\begin{aligned}\alpha_1 &= 1.593119671510678 + 0.000000000000000i \\ \alpha_{\text{opt}} &= 0.156936577864388 + 0.000000000000000i \\ \alpha_3 &= 0.500000000312466 - 0.007500239561129i \\ \alpha_4 &= 0.500000000312466 + 0.007500239561129i.\end{aligned} \tag{17}$$

Then, in Fig. 2, we validate α_{opt} with the value $\alpha_{\text{opt}}^{(simulation)} = 0.1569$ of the optimal power allocation obtained by the simulation. As shown in Fig. 2, $\alpha_{\text{opt}}^{(simulation)}$ is well consistent with α_{opt} in . Also, in Fig. 3, for all the channel gains with the energy conservation of the unit average variance, we depict α_{opt}. As shown in Fig. 3, for the equal channel gains, i.e., $\Sigma_1 = \Sigma_2 = 1$, we obtain $\alpha_{\text{opt}} = 0.25$, and as the difference of the channel gains increases, α_{opt} decreases, i.e., the more power is allocated to the weaker channel user, in order to establish the user fairness between the users.

We mention on the limitation of this letter; for the higher order modulation and many users, the proposed calculation will be more complex.

Lastly, we use the exact BER expressions for both users, so that the authors in this letter think that the results could be reliable, without Monte Carlo simulations.

Ⅴ. Conclusion

A closed-form expression for the calculation of power allocation, under the user fairness of the equal BERs, was derived in this letter. First, the analytical expression was validated by the simulation results. Then, the proposed power allocation was obtained, for all the channel gains of the unit average variance. For future researches, the proposed calculation for the higher order modulation and the number of users more than two will be interesting.

References

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE 77th VTC Spring*, pp. 1-5, 2013.

[2] Z. Ding, P. Fan, and H. V. Poor, "Impact of user pairing on 5G nonorthogonal multiple-access downlink transmissions," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6010-6023, Aug. 2016.

[3] S. R. Islam, J. M. Kim, and K. S. Kwak, "On non-orthogonal multiple access (NOMA) in 5G systems," *J. KICS*, vol. 40, no. 12, pp. 2549-2558, Dec. 2015.

[4] M. B. Uddin, M. F. Kader, A. Islam, and S. Y. Shin, "Power optimization of NOMA for multi-cell networks," *J. KICS*, vol. 43, no. 7, pp. 1182-1190, Jul. 2018.

[5] H. Lim, M. Lee, and J. Lim, "An application of NOMA for CRDSA performance improvement in random access satellite networks," *J. KICS*, vol. 44, no. 6, pp. 1064-1071, Jun. 2019.

[6] J. O. Kim, D. K. Hendraningrat, and S. Y. Shin, "A study of NOMA-SSK for solving similar channel gain problem of NOMA users," *J. KICS*, vol. 45, no. 3, pp. 474-481, Mar. 2020.

[7] K. Chung, "Impact of fading on performance of NOMA weak channel user," *J. KICS*, vol. 44, no. 10, pp. 1851-1855, Oct. 2019.

[8] T. Assaf, A. Al-Dweik, M. E. Moursi, and H. Zeineldin, "Exact BER performance analysis for downlink NOMA systems over Nakagami-*m* fading channels," *IEEE Access*, vol. 7, pp. 134539-134555, Sep. 2019.