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A Proof of Random Channel Coding Theorem for Channel Capacity of Additive Correlated Gaussian Noise Channels

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ABSTRACT

Recently, the authors of this letter introduced a novel channel, namely additive correlated Gaussian noise (ACGN) channel, in non-orthogonal multiple access (NOMA) with correlated information sources (CIS), and derived the achievable rate of such channels. However, the rigorous proof for the achievability is still missing. Therefore, in this letter, we prove the random channel coding theorem for the channel capacity of an ACGN channel.

Key Words : NOMA, superposition coding, SIC, power allocation, correlation coefficient.

Ⅰ. Introduction

In non-orthogonal multiple access (NOMA), information sources are usually assumed to be independent, i.e., independent information sources (IIS)[1-6]. Sometimes, however, the common information needs to be transmitted. For example, many mobile game users are enjoying together the mobile interactive game, in the same cell of 5G networks, where some users are near the base station and the others are far from the base station. Then the most of data is common to the users. Such data can be modeled as correlated information sources (CIS). Recently, the authors of this letter derived the achievable data rate for CIS[7]. However, the rigorous proof for the achievability is still missing. Therefore, in this letter, we prove the random channel coding theorem for achievable data rate of an ACGN channel.

Notation: The superscript $*$ stands for the complex conjugate. $\mathbb{E}[\cdot]$ denotes the expectation. $CN(\mu,\sigma^2)$ represents the distribution of circularly-symmetric complex Gaussian (CSCG) random variable (RV) with mean μ and variance σ^2.

Ⅱ. System and Channel Model

In a downlink NOMA system, all users are assumed to be experiencing block fading, in a narrow band. A base station and M users are within the cell. The complex channel coefficient between the mth user and the base station is denoted by h_m. The channels are sorted as $|h_1| \geq \dots \geq |h_M|$. The base station transmits the superimposed signal $x = \sum_{m=1}^{M} \sqrt{\beta_m P_A} s_m$, where s_m is the message for the mth user, β_m is the power allocation coefficient for CIS (we use α_m for IIS), with $\sum_{m=1}^{M} \beta_m = 1$, and P_A is the total allocated power. The power of the message s_m for the mth user is normalized as unit power, denoted by $\rho_{m,m} = \mathbb{E}[s_m s_m^*] = \mathbb{E}[|s_m|^2] = 1$, $\forall m,\ 1 \leq m \leq M$. The correlation coefficient between the ith user and the jth user is denoted by $\rho_{i,j} = \mathbb{E}[s_i s_j^*]$, with $\mathrm{Re}\{\rho_{i,j}\} \in [0,1)$, $\forall i,j,\ i \neq j,\ 1 \leq i,j \leq M$. Due to the correlation, the power of the superimposed signal x is larger than P_A. Thus, given the constant total transmitted power P at the base station, P_A is effectively scaled down

$$P_A \left(\sum_{i=1}^{M} \sum_{j=1}^{M} \rho_{i,j} \sqrt{\beta_i \beta_j} \right) = P. \tag{1}$$

The observation at the mth user is given by

$$y_m = h_m x + n_m, \tag{2}$$

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where n_m is the additive white Gaussian noise (AWGN) at the mth user, $n_m \sim CN(0,\sigma^2)$.

Ⅲ. Brief Review of Achievable Data Rate for NOMA with IIS and CIS

In this section, we present the achievable rates for the real channels, which is decomposed from the complex channel. Consider 2-user NOMA. The well-known achievable data rate for IIS of the mth user in the conventional SIC NOMA system is expressed by

$$R_{1,r}^{(\text{SIC; IIS})} = \frac{1}{2}\log_2\left(\frac{|h_1|^2 P\alpha_1 + \sigma^2}{\sigma^2/2}\right),$$
$$R_{2,r}^{(\text{SIC; IIS})} = \frac{1}{2}\log_2\left(\frac{|h_2|^2 P + \sigma^2/2}{|h_2|^2 P\alpha_1 + \sigma^2/2}\right). \tag{3}$$

Then, the achievable data rate for CIS of the mth user in the conventional SIC NOMA system is expressed by [7]

$$R_{1,r}^{(\text{SIC; CIS})} = \frac{1}{2}\log_2\left(\frac{|h_1|^2 P_A\beta_1\left(1-|\rho_{1,2}|^2\right) + \sigma^2/2}{\sigma^2/2}\right),$$
$$R_{2,r}^{(\text{SIC; CIS})} = \frac{1}{2}\log_2\left(\frac{|h_2|^2 P + \sigma^2/2}{|h_2|^2 P_A\beta_1\left(1-|\rho_{1,2}|^2\right) + \sigma^2/2}\right). \tag{4}$$

Ⅳ. Proof for Achievability of $R_2^{(\text{SIC; CIS})}$.

For the stronger channel gain user, after SIC, the channel reduces simply to an AWGN channel. Thus, $R_1^{(\text{SIC; CIS})}$ can be proved by the conventional random channel coding theorem in [8]. However, for the weaker channel gain user, we encounter a novel channel, i.e., additive correlated Gaussian noise (ACGN) channel. Even though the achievable rate for such channels was derived in [7], the rigorous proof for the achievability is still missing. Thus, in this letter, we prove the achievability of $R_2^{(\text{SIC; CIS})}$.

We will use the same ideas as in the proof of the random channel coding theorem[8] in the case of AWGN channels, namely, random codes and joint typicality decoding. However, we must make some modifications to take into account CIS and ACGN; Specifically, for CIS, the random codes are generated according to a jointly Gaussian distribution with the same correlation coefficient as CIS. And we decode the codewords with respect to ACGN, not AWGN. In this case, the noise variance is obtained by the conditional variance.

1. *Generation of the codebook:* We generate a codebook in which all the codewords satisfy the power constraint. To ensure this, we generate the codewords with each element i.i.d. according to a jointly Gaussian distribution with the constant total transmitted power $P-\varepsilon$, and the correlation coefficient $\rho_{1,2}$. Let $x^n(w_1,w_2) \in \mathcal{R}^n$ be codewords, i.e.,

$$x^n(w_1,w_2) = \left(x_1(w_1,w_2), x_2(w_1,w_2), \cdots, x_n(w_1,w_2)\right) \tag{5}$$

where $1 \le w_j \le 2^{nR_j}$, $j=1,2$, with some rates R_1 and R_2.

2. *Encoding:* After the generation of the codebook, the codebook is revealed to both the base station and the receiver of the weaker channel gain user. To send the message index (w_1,w_2), the base station sends the (w_1,w_2)th codeword $x^n(w_1,w_2) \in \mathcal{R}^n$ in the codebook.

3. *Decoding:* The receiver of the weaker channel gain user looks down the list of codewords $\{x^n(*,w_2)\}$, where $*$ denotes an arbitrary index of the stronger channel gain user, and searches for one that is jointly typical with the received vector y_2^n.

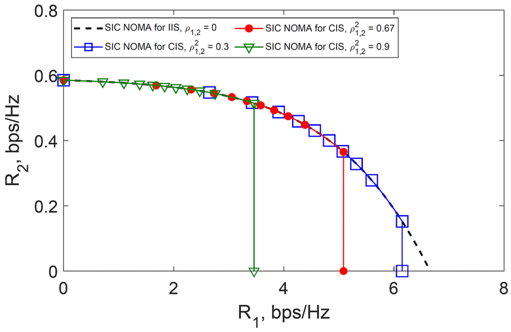

Fig. 1. Achievable rate regions of conventional SIC NOMA for IIS and CIS with $\rho_{1,2}^2 = 0, 0.3, 0.67, 0.9$, ($P/\sigma^2 = 50$, $|h_1| = \sqrt{2}$, $|h_2| = 0.1$, and $M = 2$).

If there is one and only one such codeword, the receiver declares it to be the transmitted codeword. Otherwise the receiver declares an error. The receiver also declares an error if the chosen codeword does not satisfy the power constraint.

4. *Probability of error:* Without loss of generality, assume that codeword $x^n(1,1)$ was sent. Thus

$$y_2^n = h_2^n x^n(1,1) + n_2^n. \tag{6}$$

Define the following events:

$$E_0 = \left\{ \sum_{i=1}^{n} x_i^2(1,1) > nP \right\} \tag{7}$$

and

$$E_{1,j} = \left\{ \left(x^n(1,j), y_2^n \right) \text{is in the jointly typical set} \right\}. \tag{8}$$

Then an error occurs if E_0 occurs (the power constraint is violated) or $E_{1,1}^c$ occurs (the transmitted codeword and the received sequence are not jointly typical) or

$$E_{1,2} \cup E_{1,3} \cup \cdots \cup E_{1,2^{nR_2}} \tag{9}$$

occurs (some wrong codeword is jointly typical with the received sequence). Let $\mathcal{E}$ denote the event $(1,\hat{w}_2) \neq (1,w_2)$. Hence

$$\begin{aligned} \mathbb{P}(\mathcal{E}) &= \mathbb{P}\left(E_0 \cup E_{1,1}^c \cup E_{1,2} \cup E_{1,3} \cup \cdots \cup E_{1,2^{nR_2}}\right) \\ &\le \mathbb{P}(E_0) + \mathbb{P}\left(E_{1,1}^c\right) + \mathbb{P}\left(E_{1,2} \cup E_{1,3} \cup \cdots \cup E_{1,2^{nR_2}}\right) \\ &\le \mathbb{P}(E_0) + \mathbb{P}\left(E_{1,1}^c\right) + \sum_{j=2}^{2^{nR_2}} \mathbb{P}\left(E_{1,j}\right), \end{aligned} \tag{10}$$

by the union of events bound for probabilities. By the law of large numbers, $\mathbb{P}(E_0) \to 0$ as $n \to \infty$. Now, by the joint asymptotic equipartition property (AEP), $\mathbb{P}\left(E_{1,1}^c\right) \to 0$, and hence

$$\mathbb{P}\left(E_{1,1}^c\right) \le \varepsilon \text{ for } n \text{ sufficiently large.} \tag{11}$$

Since by the code generation process, $x^n(1,1)$ and $x^n(1,j)$ are independent, so are y_2^n and $x^n(1,j)$. Hence, the probability that $x^n(1,j)$ and y_2^n will be jointly typical is $\le 2^{-n\left(R_{2,r}^{(\mathrm{SIC;CIS})} - 3\varepsilon\right)}$ by the joint AEP. Hence

$$\begin{aligned} \mathbb{P}(\mathcal{E}) &\le \varepsilon + \varepsilon + \left(2^{nR_2} - 1\right) 2^{-n\left(R_{2,r}^{(\mathrm{SIC;CIS})} - 3\varepsilon\right)} \\ &\le \varepsilon + \varepsilon + 2^{3n\varepsilon} 2^{-n\left(R_{2,r}^{(\mathrm{SIC;CIS})} - R_2\right)} \\ &\le 3\varepsilon \end{aligned} \tag{12}$$

for n sufficiently large and $R_2 < R_{2,r}^{(\mathrm{SIC;CIS})} - 3\varepsilon$.

This proves the existence of a good code. *Q.E.D.*

V. Results and Discussions

Even though in the previous section, we proved the achievable rates for the real channels, which is decomposed from the complex channel, in this section, we depict the achievable rates for the complex channels, which is calculated from the achievable rates for the real channels by the following relationship

$$R_2^{(\mathrm{SIC;\,CIS})}\left(\frac{P}{\sigma^2}\right)=2R_{2,r}^{(\mathrm{SIC;\,CIS})}\left(\frac{P}{\sigma^2/2}\right). \tag{13}$$

We investigate the two-user NOMA scenario, $M=2$. The constant total transmitted signal-to-noise power ratio (SNR) is $P/\sigma^2=50$, and the channel gains $|h_1|$ and $|h_2|$ are assumed to be $\sqrt{2}$ and 0.1, respectively. In Fig. 1, for the various values of the correlation coefficient, $\rho_{1,2}^2=0, 0.3, 0.67,$ and 0.9, the achievable rate regions are depicted. As shown in Fig. 1, the achievable rate region of SIC NOMA with CIS is no larger than that of SIC NOMA with IIS. Especially, the maximum rate of $R_1^{(\mathrm{SIC;\,CIS})}$ is less than that of $R_1^{(\mathrm{SIC;\,IIS})}$. Specifically, when $\alpha_1=1$ and $\beta_1=1$, we have

$$R_1^{(\mathrm{SIC;\,IIS})}=\log_2\left(\frac{|h_1|^2P+\sigma^2}{\sigma^2}\right),$$
$$R_1^{(\mathrm{SIC;\,CIS})}=\log_2\left(\frac{|h_1|^2P\left(1-|\rho_{1,2}|^2\right)+\sigma^2}{\sigma^2}\right). \tag{14}$$

Therefore, we can have $R_1^{(\mathrm{SIC;\,CIS})}<R_1^{(\mathrm{SIC;\,IIS})}$, with $|\rho_{1,2}|^2\neq 0$.

In addition, we comment on the case for the number of users more than two; Based on the results of the two-user case, for the strongest channel gain user, we have

$$R_1^{(\mathrm{SIC;\,CIS})}\ll R_1^{(\mathrm{SIC;\,IIS})}. \tag{15}$$

It should be noted that for the weakest channel gain user, we have

$$R_M^{(\mathrm{SIC;\,CIS})}\gg R_M^{(\mathrm{SIC;\,IIS})}. \tag{16}$$

Ⅵ. Conclusion

In this letter, we proved the random channel coding theorem for the channel capacity of ACGN channel. Even though we followed the same ideas as in the proof of the random channel coding theorem in the case of AWGN channels, we made some modifications to take into account CIS and ACGN, which are our main contributions, specifically random codes generated according to a jointly Gaussian distribution.

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