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Achievable Rate Volume of Three-User CIS NOMA

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ABSTRACT

Non-orthogonal multiple access (NOMA) enables 5G mobile networks to transmit correlated information sources (CISs) simultaneously, in contrast to orthogonal multiple access. Recently, the achievable data rates were derived for NOMA with CIS. However, only the achievable rate region was investigated for two-user CIS NOMA. Therefore, in this study, we address the achievable rate volume of three-user CIS NOMA. First, the closed-form expression for the achievable rate volume of three-user CIS NOMA is derived. Second, the exact power allocation ranges of the rate volume of three-user independent information source (IIS) NOMA to be achieved by three-user CIS NOMA are derived. Then, it is shown that only a portion of the rate volume of the three-user IIS NOMA scheme can be achieved by the three-user CIS NOMA scheme.

Key Words : NOMA, correlated information sources, correlation coefficient, superposition coding, SIC, power allocation

Ⅰ. Introduction

BROADCASTING simultaneously independent information sources (IISs) in non-orthogonal multiple access (NOMA) has reduced the latencies in 5G networks[1-6]. Recently, splitting large intelligent surfaces (LIS) was studied in NOMA[7]. In addition, the principle of simultaneous broadcasting for a lower latency was investigated for correlated information sources (CISs) in NOMA[8]. However, only 2-user CIS NOMA was studied in [8]; hence, this study investigates three-user CIS NOMA. First, the closed-form expressions for the achievable rate volume are formally derived; then the exact power allocation ranges of the rate volume of three-user IIS NOMA that are to be achieved by three-user CIS NOMA are derived. The numerical results show that only a portion of the rate volume of the three-user IIS NOMA scheme can be achieved by the three-user CIS NOMA scheme.

Ⅱ. System and Channel Model

In a downlink NOMA system, three users are assumed to experience block fading in a narrow band; a base station and three users are considered within a cell. The complex channel coefficient between the mth user and base station is denoted by h_m, $m=1,2,3$. The channels are sorted as $|h_1|\geq|h_2|\geq|h_3|$, the base station transmits the superimposed signal $x=\sum_{m=1}^{3}\sqrt{\beta_m P_A}c_m$, where c_m is the message for the mth user, β_m is the power allocation coefficient for the CIS (we use α_m for IIS) with $\sum_{m=1}^{3}\beta_m=1$, and P_A is the total average allocated power. The average power of the message c_m for the mth user is normalized as a unit of power. The correlation coefficient between the ith and jth users is denoted by $\rho_{i,j}=\mathbb{E}\left[c_i c_j^*\right]$, where $\rho_{i,j}=\mathrm{Re}\{\rho_{i,j}\}\in[0,1)$, $\forall i,j,\ i\neq j,\ 1\leq i,j\leq 3$. Then, given the average total transmitted power P at the base station, P_A is expressed as

$$P_A\sum_{i=1}^{3}\sum_{j=1}^{3}\rho_{i,j}\sqrt{\beta_i\beta_j}=P. \tag{1}$$

The observation at the mth user is given by

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$$y_m = h_m x + n_m, \tag{2}$$

where $n_m \sim CN(0,\sigma^2)$ is the additive white Gaussian noise at the mth user. Then, the achievable rate volume of IIS NOMA is given by

$$R_1^{(\mathrm{IIS})} = \log_2\left(1+\frac{|h_1|^2 P\alpha_1}{\sigma^2}\right), \tag{3}$$

$$R_2^{(\mathrm{IIS})} = \log_2\left(1+\frac{|h_2|^2 P\alpha_2}{|h_2|^2 P\alpha_1+\sigma^2}\right), \tag{4}$$

and

$$R_3^{(\mathrm{IIS})} = \log_2\left(1+\frac{|h_3|^2 P\alpha_3}{|h_3|^2 P\alpha_1+|h_3|^2 P\alpha_2+\sigma^2}\right). \tag{5}$$

Ⅲ. Derivation of Rate Volume for CIS NOMA

Before presenting the derivations, we first discuss a numerical result to explain the derivations easily. We investigate the three-user NOMA scenario. To ensure the typical channel code rate of 1/2 for even the weakest channel gain user, the average total transmitted signal-to-noise power ratio (SNR) is chosen as $P/\sigma^2 = 50$, under the assumption of channel gains $|h_1| = \sqrt{2}$, $|h_2| = 1$, and $|h_3| = 0.1$. We assume that the correlation coefficients, $\rho_{1,2}$, $\rho_{1,3}$, and $\rho_{2,3}$ are set to equal value of $\sqrt{5}$, which is a moderate quantity, that is, $\rho_{1,2}^2 = 0.5$.

We then derive the achievable data rates $R_1^{(\mathrm{CIS})}$, $R_2^{(\mathrm{CIS})}$, and $R_3^{(\mathrm{CIS})}$. First, using conditional mutual information, $R_1^{(\mathrm{CIS})}$ for the first user rate is

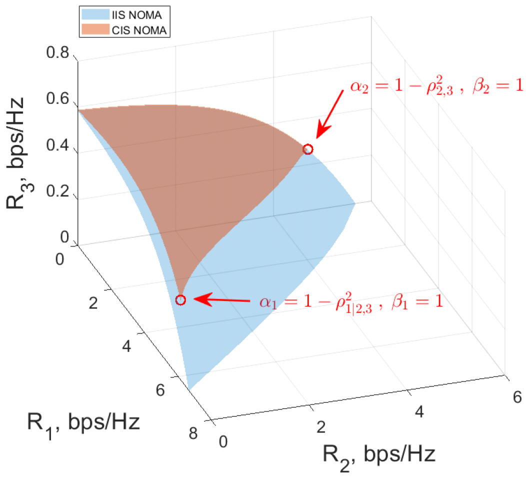

Fig. 1. Achievable rate volumes of IIS NOMA and CIS NOMA with $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = \sqrt{0.5}$, ($P/\sigma^2 = 50$, $|h_1| = \sqrt{2}$, $|h_2| = 1$, and $|h_3| = 0.1$).

derived as

$$\begin{aligned} &R_1^{(\mathrm{CIS})} \\ &= I(y_1;c_1 \mid c_2,c_3) \\ &= h(y_1 \mid c_2,c_3) - h(y_1 \mid c_1,c_2,c_3) \\ &= h(h_1\sqrt{P_A\beta_1}c_1 + h_1\sqrt{P_A\beta_2}c_2 + h_1\sqrt{P_A\beta_3}c_3 + n_1 \mid c_2,c_3) \\ &-h(h_1\sqrt{P_A\beta_1}c_1 + h_1\sqrt{P_A\beta_2}c_2 + n_1 \mid c_1,c_2,c_3) \\ &= h(h_1\sqrt{P_A\beta_1}c_1 + n_1 \mid c_2,c_3) - h(n_1 \mid c_1,c_2,c_3) \\ &= h(h_1\sqrt{P_A\beta_1}c_1 + n_1 \mid c_2,c_3) - h(n_1), \end{aligned} \tag{6}$$

where c_2 and c_3 cannot be removed as in the case of IIS because c_1 is correlated with both c_2 and c_3. Thus, using the conditional variance, we have

$$\begin{aligned} &R_1^{(\mathrm{CIS})} \\ &= \log_2 \pi e\left(|h_1|^2 P_A\beta_1\left(1-\rho_{1|2,3}^2\right)+\sigma^2\right) - \log_2 \pi e\sigma_2 \\ &= \log_2\left(\frac{|h_1|^2 P_A\beta_1\left(1-\rho_{1|2,3}^2\right)+\sigma^2}{\sigma_2}\right) \\ &= \log_2\left(1+\frac{|h_1|^2 P_A\beta_1\left(1-\rho_{1|2,3}^2\right)}{\sigma_2}\right), \end{aligned} \tag{7}$$

where the conditional correlation coefficient $\rho_{1|2,3}^2$ is derived as

$$\begin{aligned}&\sigma_{1|2,3}^2\\&=\left[\rho_{12}\sqrt{\beta_2}\quad \rho_{13}\sqrt{\beta_3}\right]\times\\&\begin{bmatrix}\beta_2 & \rho_{23}\sqrt{\beta_2}\sqrt{\beta_3}\\ \rho_{32}\sqrt{\beta_3}\sqrt{\beta_2} & \beta_3\end{bmatrix}^{-1}\begin{bmatrix}\rho_{21}\sqrt{\beta_2}\\ \rho_{31}\sqrt{\beta_3}\end{bmatrix}\\&=\frac{\left(\rho_{12}-\rho_{13}\right)^2+2\rho_{12}\rho_{31}\left(1-\rho_{23}\right)}{1-\rho_{23}^2}.\end{aligned}\tag{8}$$

Notably, by comparing $R_1^{(\mathrm{CIS})}$ in and $R_1^{(\mathrm{IIS})}$ in , $R_1^{(\mathrm{CIS})}$ is seen to decrease with respect to $R_1^{(\mathrm{IIS})}$, as shown in Fig. 1.

Second, $R_2^{(\mathrm{CIS})}$ for the second user is derived as

$$\begin{aligned}&R_2^{(\mathrm{CIS})}\\&=I(y_2;c_2\mid c_3)\\&=h(y_2\mid c_3)-h(y_2\mid c_2,c_3)\\&=h(h_2\sqrt{P_A\beta_1}c_1+h_2\sqrt{P_A\beta_2}c_2+h_2\sqrt{P_A\beta_2}c_3+n_2\mid c_3)\\&-h(h_2\sqrt{P_A\beta_1}c_1+h_2\sqrt{P_A\beta_2}c_2+h_2\sqrt{P_A\beta_2}c_3+n_2\mid c_2,c_3)\\&=h(h_2\sqrt{P_A\beta_1}c_1+h_2\sqrt{P_A\beta_2}c_2+n_2\mid c_3)\\&-h(h_2\sqrt{P_A\beta_1}c_1+n_2\mid c_2,c_3)\\&=\log_2\pi e\left(\left|h_2\right|^2P_A\begin{pmatrix}\beta_1\left(1-\rho_{1,3}^2\right)+\beta_2\left(1-\rho_{2,3}^2\right)\\+2\sqrt{\beta_1}\sqrt{\beta_2}\left(\rho_{1,2}-\rho_{1,3}\rho_{2,3}\right)\end{pmatrix}+\sigma^2\right)\\&-\log_2\pi e\left(\left|h_2\right|^2P_A\beta_1\left(1-\rho_{1|2,3}^2\right)+\sigma^2\right)\\&=\log_2\left(\frac{\left|h_2\right|^2P_A\begin{pmatrix}\beta_1\left(1-\rho_{1,3}^2\right)+\beta_2\left(1-\rho_{2,3}^2\right)\\+2\sqrt{\beta_1}\sqrt{\beta_2}\left(\rho_{1,2}-\rho_{1,3}\rho_{2,3}\right)\end{pmatrix}+\sigma^2}{\left|h_2\right|^2P_A\beta_1\left(1-\rho_{1|2,3}^2\right)+\sigma^2}\right).\end{aligned}\tag{9}$$

In contrast to $R_1^{(\mathrm{CIS})}$ and $R_1^{(\mathrm{IIS})}$, although a comparison of $R_2^{(\mathrm{CIS})}$ in and $R_2^{(\mathrm{IIS})}$ in does not clearly indicate an increase or a decrease, we observe from Fig. 1 that $R_2^{(\mathrm{CIS})}$ decreases with respect to $R_2^{(\mathrm{IIS})}$.

Third, $R_3^{(\mathrm{CIS})}$ for the third user is derived as

$$\begin{aligned}&R_3^{(\mathrm{CIS})}=I(y_3;c_3)=h(y_3)-h(y_3\mid c_3)\\&=\log_2\pi e(\left|h_3\right|^2P+\sigma^2)\\&-\log_2\pi e\left(\left|h_3\right|^2P_A\begin{pmatrix}\beta_1\left(1-\rho_{1,3}^2\right)+\beta_2\left(1-\rho_{2,3}^2\right)\\+2\sqrt{\beta_1}\sqrt{\beta_2}\left(\rho_{1,2}-\rho_{1,3}\rho_{2,3}\right)\end{pmatrix}+\sigma^2\right)\\&=\log_2\left(\frac{\left|h_3\right|^2P+\sigma^2}{\left|h_3\right|^2P_A\begin{pmatrix}\beta_1\left(1-\rho_{1,3}^2\right)+\beta_2\left(1-\rho_{2,3}^2\right)\\+2\sqrt{\beta_1}\sqrt{\beta_2}\left(\rho_{1,2}-\rho_{1,3}\rho_{2,3}\right)\end{pmatrix}+\sigma^2}\right).\end{aligned}\tag{10}$$

By comparing $R_3^{(\mathrm{CIS})}$ in and $R_3^{(\mathrm{IIS})}$ in , $R_3^{(\mathrm{CIS})}$ increases with respect to $R_3^{(\mathrm{IIS})}$, as shown in Fig. 1, in contrast to $R_1^{(\mathrm{CIS})}$ and $R_1^{(\mathrm{IIS})}$.

Ⅳ. Derivation of Power Allocation Ranges

In this section, the derivation for the power allocation ranges of the rate volume of IIS NOMA to be achieved by CIS NOMA are shown.

First, by comparing and , we obtain the following equation:

$$P\alpha_1=P_A\beta_1\left(1-\rho_{1|2,3}^2\right).\tag{11}$$

Then, when $\beta_1<1$, we obtain the power allocation range as

$$\alpha_1<1-\rho_{1|2,3}^2.\tag{12}$$

Similarly, by comparing and, we obtain the following equation:

$$P\alpha_1 + P\alpha_2 = P_A \begin{pmatrix} \beta_1 \left(1-\rho_{1,3}^2\right) + \beta_2 \left(1-\rho_{2,3}^2\right) \\ +2\sqrt{\beta_1}\sqrt{\beta_2}\left(\rho_{1,2}^2 - \rho_{1,3}\rho_{2,3}\right) \end{pmatrix}. \quad (13)$$

Then, when $\beta_2 < 1$, we obtain the power allocation range as

$$\alpha_2 < 1-\rho_{2,3}^2. \quad (14)$$

Therefore, the power allocation ranges of the rate volume of IIS NOMA that are to be covered by CIS NOMA are given by

$$\left(\alpha_1, \alpha_2, 1-\alpha_1-\alpha_2\right), \text{ for } \alpha_1 < 1-\rho_{1|2,3}^2 \text{ and } \alpha_2 < 1-\rho_{2,3}^2. \quad (15)$$

As shown in Fig. 1, only a portion of the rate volume of IIS NOMA can be achieved by CIS NOMA (i.e., the rate volume of CIS NOMA is smaller than that of IIS NOMA). Specifically, based on , the power allocation ranges of the rate volume of IIS NOMA to be achieved by CIS NOMA are given as

$$\left(\alpha_1, \alpha_2, \alpha_3\right), \text{ for } \alpha_1 < 0.4142, \ \alpha_2 < 0.5, \text{ and } \alpha_3 > 0.0858. \quad (16)$$

These power allocation ranges are independent of the channel gains and SNR, that is, they are dependent only on the correlation coefficients of the CISs.

Ⅴ. Conclusion

In this work, we derived the closed-form expression for the achievable rate volume of CIS NOMA for arbitrary correlation coefficients. We also derived the exact power allocation ranges of the rate volume of three-user IIS NOMA that are to be achieved by three-user CIS NOMA. Then, it was shown that only a part of the rate volume of IIS NOMA can be achieved by CIS NOMA. From the results, such observations should be considered when transmitting CSIs in 5G networks of the three-user NOMA schemes.

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